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Writing to Learn College Algebra

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WRITING TO LEARN COLLEGE ALGEBRA

by

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This dissertation, submitted by Patricia A. Mower in partial fulfillment of the requirements for the Degree of Doctor of Philosophy from the University of North Dakota, has been read by the Faculty Advisory Committee under whom the work has been done and is hereby approved.

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Signature Patricia A. Mower

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ABSTRACT

The purpose of this study of writing to learn college algebra was to explore how writing in a college algebra course affected the learning experiences of a group of undergraduate students. The study included an analysis of the existing literature on the writing to learn process. Several writing to learn techniques suggested by the literature were used in the teaching of the course. The students' perceptions of these processes were recorded and analyzed for the study. This research was important because it allowed students' voices to be heard regarding writing to learn mathematics as a form of pedagogical and curricular reform.

Phenomenology, a form of qualitative research involving data collection and analysis, was the methodology employed in this study. Journal writings and exit interviews provided the principal forms of qualitative data. The data were analyzed and categorized into three areas: student effects, teacher effects, and student and teacher effects.

The findings from the study included both benefits and disadvantages for student writers and teacher reader. Ten outcomes held true for most of the participants of the study:

- Writing promoted student comprehension of mathematics.
- Writing facilitated students in making personal meaning of mathematical concepts.
- Writing became a vehicle for an ongoing dialogue between student and teacher.

- Writing allowed for student reflection regarding the learning of mathematics.
- Writing fostered teacher analysis of the rhythms of learning mathematics.
- Writing brought moments of clarity and of genius.
- Writing permitted authentic and alternative assessment.
- Writing provided a forum for disclosure for both student and teacher.
- Writing assisted in the development of a community of learners by encouraging classroom disclosure.
- Writing caused disadvantages for some students and the teacher.

The findings from this study will enlarge and enhance the understanding of how students learn mathematics and how writing to learn mathematics might be used as an effective form of mathematics educational reform. Several recommendations are given regarding future courses and studies involving writing to learn mathematics.

CHAPTER I

INTRODUCTION

> The primary question was not "What do we know?," but "How do we know it?"
> Aristotle, 350 BC

In 1989 the National Council of Teachers of Mathematics published the Curriculum and Evaluation Standards for School Mathematics because many educators were concerned about the quality of mathematics education. The NCTM standards called for the reform of mathematics instruction at all levels, elementary through postsecondary, citing change in the world as the rationale for this need for educational change:

> All industrialized countries have experienced a shift from an industrial to an information society, a shift that has transformed both aspects of mathematics that need to be transmitted to students and the concepts and procedures that they must master if they are to be self-fulfilled, productive citizens in the next century (p. 3).

Because technology had reduced the time and skills needed for computation, the NCTM recommended the new focus in mathematics instruction be on conceptualization and communication as opposed to the mechanics of mathematics. Many college mathematics instructors and researchers heeded the call for reform by exploring and using different instructional techniques in the mathematics classroom (Davidson, 1994; Gopen & Smith, 1990; Miller, 1988).

Traditionally, lower level undergraduate mathematics instruction included a mixture of lecture and demonstration or what commonly has been referred to as chalk-and-talk.

College mathematics students passively watched the teacher explain concepts and demonstrate problems. Some time later, the students took tests on this material. While some students fared well with this educational format which emphasized memorization and recitation, others did not.

Many educators believe that students must become actively involved in the classroom for learning to take place (Freire, 1970; Fulwiler, 1986). In 1986 Fulwiler noted that traditional mathematics instruction and content also seemed to lack relevancy to the experiences of the students. Understandably, not all mathematics will be relevant to the students' daily lives, but some effort at connecting mathematics to the real world is needed.

Another component missing in the traditional format was the teaching of skills for communicating students' new mathematical knowledge. Many learning skills besides memorization and recitation are required for effective mathematical communication and conceptualization. Memorizing theorems and definitions does little to assist the student in understanding complex mathematical concepts. Students must reflect on and discuss mathematical concepts to become actively involved in thinking and in learning.

Since the latter 1980s, efforts to include writing within the mathematics curriculum have begun to address these concerns. Many educators see writing as an avenue for emphasizing mathematical conceptualization over memorization. The Writing Across the Curriculum movement which began in the 1970s has aroused great interest and curiosity in mathematics educators. Proponents of this movement stress that students learn all subjects through writing, speaking, and thinking (Kenney, 1992).

Problems such as time constraints and grading policies, however, have kept many instructors from incorporating writing into their mathematics instruction. Other instructors are not convinced of the significance of students writing to learn mathematics and see it as a waste of time and effort (Kenney, 1992).

Several studies have examined the effectiveness of writing programs in college mathematics classrooms (Johnson, 1991; Maher, 1992; McDonald, 1992; Rose, 1989; Stempien, 1990). These writing programs employed several different writing strategies including journals, autobiographical writings, written proofs, and written examination questions. All of the studies concluded that writing in the mathematics classroom was beneficial in varying ways to the students and the instructors.

The qualitative study that is the subject of this dissertation was pursued in order to enhance the understanding of the effects of writing in mathematics. This study looked through the eyes of the students at an intensive writing component incorporated into a college algebra course. The student participants of the study were undergraduates and mostly non-mathematics majors; some liked mathematics and others did not. The writing activities of the course placed an emphasis on students writing to learn mathematical concepts and to foster reflective learning. Throughout the course, students learned to evaluate their own mathematical progress by using techniques developed by Angelo and Cross (1993) and MacGregor (1993). Some of these techniques included reflective journal writing, autobiographical sketches, and assessment summaries.

This writing intensive college algebra course was evaluated through the students' experiences and voices. Too often, the success of a new instructional method is measured

only by the grades students earn. This study asked the broad qualitative question: What were the learning experiences of a group of students enrolled in a writing intensive college algebra course? Through the researching of this question came the answers to how writing benefits or does not benefit the mathematics student and teacher.

Purpose of the Study

The purpose of the study was to explore how writing in a college algebra course affected the learning experiences of a group of undergraduate college students. One secondary purpose of the study was to expand and enhance the understanding of how students learn and make meaning of mathematics through writing. Another purpose of the study was to allow students' voices to be heard regarding the assessment and effect of writing in a mathematics course.

The methodology of the study was selected for the explicit purpose of assessing the writing component of the college algebra course through the students' words. The research methodology employed was phenomenology, a form of qualitative research which uses interactive techniques to discover meanings of specific phenomena. The interactive techniques employed in the study included student and teacher journals, student individual and group writing assignments, and exit interviews. The final analysis of the research included the coding and categorizing of the data in an effort to establish significant patterns of the effects of the writing to learn component of the course.

Need for the Study

Many studies involving curricular reform focus on quantitative measurements of student success and failure. Successful learning should not be measured through quantitative

measures or grading only. This study sought to discover how the students thought and felt about writing as a means of learning algebra. It is important that students' voices be heard, as well as educators', to allow for a more holistic view of any educational reform.

Several studies have looked at particular writing activities in college mathematics courses. This study examined a writing intensive college algebra course which included daily writing exercises. Upon review of the literature, I found few qualitative studies involving writing intensive mathematics courses. Therefore, there is a need to discover what benefits and disadvantages particular students and teachers experience in these courses. This study should expand mathematics educators' understanding of how writing affects the learning experiences of certain students.

Limitations of the Study

The study looked at only one college algebra class over one semester. Thus, the data and findings refer to just this one group of students during one period of time at one institution of higher education.

The population involved in this study included some students who were inexperienced and inept at expressing or assessing themselves. Also, some students approached the writing assignments with more seriousness and thoughtfulness than others. Thus, some data were impossible to attain through written form or interviews.

There was potential for bias in the design, conduct, and analysis of this study due to the fact that I was the instructor of the course and wanted all of the students in the course to succeed. Another instructor and/or teaching style might have resulted in different data. Collection and analysis of data by someone else also might have produced different results.

The mortality of this study (student attrition) was a slight problem, as nine of the original 36 students dropped the course and disappeared with partial journals. (The normal attrition rate for college algebra courses at this university was 15 to 25% of the students originally enrolled.)

Assumptions of the Study

The study was based on the assumption that writing to learn is an instructional process which addresses the call for mathematics education reform. I assumed that writing encourages and promotes conceptualization and communication of mathematical ideas.

Three assumptions gleaned from the literature also shaped the direction of this study:

1. Writing to learn and learning to write mathematics enhance mathematical understanding (Rose, 1992).

2. Reflective or expressive writing helps students to appreciate and value difficult subjects (MacGregor, 1993).

3. Allowing students' voices to be heard regarding curricular assessment and reform brings about student ownership of the content (Kenney, 1992).

Definition of Terms

College Algebra is the first algebra course for which the student may earn mathematics credit at the University of North Dakota. The course content includes real numbers, linear equations, graphing techniques, functions, theory of polynomials, logarithms, and systems of equations.

Constructivism is an educational philosophy which espouses discovery learning where the learner is guided to uncover and construct knowledge as well as personal meaningfulness and relevance of knowledge.

Critical thinking skills are cognitive abilities that encourage the view of thinking as a "continuous, self-correcting process" (Clarke, 1993, p. 10) and as flexible enough to promote individual choice in opinion and action .

Epistemology is the study of how knowledge is constructed by the learner.

Expository mathematics is the detailed writing of words and symbols that clearly explain mathematical concepts.

Expressive writing is a writer's exploration of the meaning of and feelings about experiences and subjects.

Freewriting is a form of writing where the writer writes whatever comes to mind, disregarding grammar, spelling, and punctuation.

Journals are collections of writings by one person kept together in one place; similar to diaries.

Mathematical literacy is the ability of students to understand and use mathematical content.

Metacognition is the act of thinking about thinking or learning.

Phenomenological research is the human scientific study of phenomena or lived experience; this research employs interactive techniques such as participant observation and interviewing to discover meaning regarding specific phenomena

Qualitative research is a methodology design employing various techniques for data collection and analysis and entailing the use of data from interviews, observations, and written

documents. Qualitative research does not quantify or measure data and focuses on particularization rather than generalization in an effort to enlarge and enhance the understanding of certain experiences or subjects (Glesne & Peshkin, 1992).

Reflective writings are writings which call for self exploration or examination of the writer's thoughts or feelings.

Rhythms of learning are the highs, lows, and plateaus of emotions experienced by the student during the learning process.

Small Group Instructional Diagnosis (SGID) is an assessment process involving the use of an outside assessor who conducts a small group, student analysis of a particular course in an effort to strengthen the teaching and learning of the coursework. The instructor is absent during the process but receives a written assessment report soon afterward.

Student self-evaluation (SSE) is a process and/or a product of student reflection and writing about individual educational progress.

Transactional writing is writing that "aims to inform, persuade or instruct an audience in clear, conventional, and concise prose" (Britton, 1986, p. 23).

Triangulation is the process of using multiple data sources to demonstrate the trustworthiness of the data.

Writing Across the Curriculum (WAC) - an educational movement which began in the mid 1970s and integrates writing within various disciplines in an effort to promote quality writing.

Writing to learn is a teaching and learning method which employs student writing efforts as the primary tools for the learning of certain concepts.

Organization of the Study

The study is organized into six chapters. The first chapter introduces the study. The second chapter presents the review of the literature pertinent to the study. The third chapter gives a description of the phenomenological methodology used. The fourth chapter consists of a narrative description of the course including the participants, setting, and coursework. The data of the study accumulated from the students' words are presented in the fifth chapter. In the sixth chapter, the data are analyzed and synthesized into the findings and conclusions of the study. This final chapter also presents several recommendations for the creation of writing to learn mathematics courses and for further research on writing in the mathematics classroom.

CHAPTER II

REVIEW OF LITERATURE

> Learning, in the proper sense, is not learning things,
> but the *meanings* of things, and the process involves
> the use of signs, or language in its generic sense.
> John Dewey, 1910, p. 176

The review of literature is divided into three sections. The first section gives consideration to the call for pedagogical reform in mathematics education and introduces writing to learn as an instrument for this reform. Section two presents the history and objectives of the writing to learn movement called Writing Across the Curriculum. The third and final section ties together the reform of mathematics education and writing across the curriculum by giving a description of the ways in which writing to learn has been used in the teaching and learning of college mathematics.

The Call for Reform in Mathematics Education

In 1970 the Brazilian educator Freire used "banking and ranking" to describe what he perceived to be occurring in the mathematics classroom. He wrote that students banked information and teachers ranked students according to how much information they banked. In many lower level undergraduate mathematics classrooms, the primary instructional methodology used has been chalk-and-talk, a mixture of teacher-generated lecture and demonstration. Students listen and watch as teachers talk about and work out computational problems, and, at some point, the students spew back this information on tests and

homework. Regarding mathematics instruction in the 1970s, Doyle wrote critically of students being treated as if they were "passive recipients of instructional treatments" (1979, p. 203). Yet, many students of the 1980s and 1990s were still being asked to compute and calculate with little regard for their comprehension of mathematical concepts. Smith and Gopen from Duke University summed up lower division college mathematics instruction in this manner: "We have created memory-based courses, driven by efficient means of testing, which is defined in terms of calculational skill. We may rationalize that accuracy in computation implies a previous mastery of concepts, but we all know better" (1990, p. 2).

During interviews with mathematics students in 1986 and 1988, Miller (1991) found that most students perceived mathematics as a subject with right or wrong answers and felt anxious about finding the right answers. In 1992 Brandau suggested that educators change their mathematics "stories" and beliefs about mathematical education: "If teachers believe mathematics consists of memorization of rules and procedures, then their teaching and lessons will reflect this belief; it is likely their students will also have this belief" (1992, p. 73). As Dewey (1902) said, students and teachers grow accustomed to their chains, even when those chains foster a lack of success in learning.

In 1989 the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards called for the reform of mathematics instruction and suggested new goals for mathematics education:

1) that students learn to value mathematics,

2) that students become confident in their ability to do mathematics,

3) that students become mathematical problem solvers,

4) that students learn to communicate mathematically, and

5) that students learn to reason mathematically. (p. 5)

The NCTM Standards also advocate a change in the assessment of student performance by recommending more use of oral and written reports and of small group tests.

Simon (1995), a constructivist and mathematics educator, wrote that students' thinking and understanding must become the focus for designing and implementing the new mathematics curriculum demanded by the NCTM standards. He recommended that teachers learn about how students think and understand mathematics in order to help students develop the critical thinking skills called for by the standards.

In 1991 LeGere wrote that a forum for dialogue and the interchange of ideas between educators and students was required to change the state of mathematics education. An atmosphere of collaboration might then permeate the mathematics classroom, lessening the anxiety felt by some students. LeGere surmised that students would then become more involved in the learning process and use skills at a higher level than memorization.

Many educators employed writing to learn activities in their classrooms in an effort to bring about better student understanding of mathematical concepts. Brandau (1992) wrote that "writing is central to the understanding process in general. We often do not know what we think until we need to find the words to put to paper" (p. 76). Another mathematics teacher committed to writing as a means of learning mathematics described mathematics as

a "place for revision and refinement" (Buerk, 1992, p. 78). When students must describe in words what they know about mathematics, they obtain an ownership of these concepts in a way not realized by echoing what the teacher has told or shown them about mathematics. Patton (1990) also wrote of writing as a vehicle for students to make personal meaning of mathematics in their world:

> Writing is a kind of self-making or forming. To write is to measure the depth of things, as well as to come to a sense of one's own depth. . . Writing abstracts our experience, yet it also concretizes our understanding of the world. (p. 126)

To summarize, since 1970 several mathematics educators have written about the need for change in the instruction of mathematics (Brandau, 1992; Doyle, 1979; Freire, 1970; Miller, 1991; Smith & Gopen, 1990). The new NCTM standards suggested that mathematics instruction focus on certain critical thinking skills. Many educators believe that writing to learn mathematics successfully addresses this call for reform. The next section gives the background for the writing to learn process.

Writing Across the Curriculum

The writing to learn concept evolved through the Writing Across the Curriculum (WAC) movement initiated in the mid 1970's. Writing Across the Curriculum is a broadly based program which promotes and facilitates disciplinary writing projects on campuses throughout the United States. One of its early proponents defined WAC as a "synthesis of writing and learning" (Kenney, 1992, p. 17). The first WAC faculty workshops took place during the summers of 1974 and 1975 at Carleton College in Northfield, Minnesota. The early agenda of WAC included the provision for a means of "incorporating student responses into teaching" (McLeod & Soven, 1992, p. xiii). Initially, the main objective of the movement

was to improve students' writing. By the latter 1980s, however, Writing Across the Curriculum began to serve as an instrument to assist learning within several different disciplines (1992).

WAC became a popular program on many college campuses throughout the latter 1970s and the 1980s. By 1991 one-third to one-half of all postsecondary institutions in the United States had an active WAC program. Many of these institutions had facilities and faculty dedicated solely to the writing to learn efforts of the Writing Across the Curriculum program (Whitis, 1991).

The theory behind writing to learn has been linked to cognitive psychology and the research of Piaget and Bruner. In the 1950s, Piaget described the learner as an active agent constructing knowledge through observation and thought. During the same decade, Bruner wrote of learning as an active process where learners discover, structure, and organize knowledge through experience and symbolism (Goetz, Alexander, & Ash, 1992). WAC leaders have taken the position that the writing process fosters the construction of knowledge and promotes the discovery learning espoused by Piaget and Bruner, as well as improving the writing skills of students (Whitis, 1991).

Britton (1986) classified three types of writing important to the learning process: transactional writing which "aims to inform, persuade or instruct an audience in clear, conventional, and concise prose," poetic or creative writing, and expressive writing which includes the use of journals, diaries, letters, or any papers written to oneself (p. 23). He advocated the use of expressive writing or thinking out loud on paper, stating that this form of writing encourages the writer to think about learning. As early as 1962, linguistic theorist

Vygotsky suggested that language and writing are needed to transform data into meaningful concepts. In the late 1970s, educational theorists Emig and Bruner acknowledged that because language is required for intellectual development, the use of journals and expressive writing in the classroom would be an effective way for students to make sense of complex subjects (Selfe, 1986). Since the 1960s, Freire (1970) has promoted a liberating education which focuses on the development of critical thinking skills. He recommended shifting the emphasis from transactional writing to expressive writing which advances thought by generating ideas.

Throughout the 1980s and early 1990s, many colleges offered writing intensive courses or courses which required extensive writing on a regular basis. These courses were often used as vehicles for discipline specific curricular or pedagogical reform. Writing intensive courses were an attempt to move away from Freire's "banking" model of education and to promote critical thinking skills. Studies regarding writing intensive courses revealed that the writing assignments reinforced students' writing skills and that students' writing improved most when they wrote for their major area courses (Farris & Smith, 1992).

In 1992 Farris and Smith suggested several guidelines for the design and implementation of writing intensive courses:

1) Class size should be limited to 15 to 25 students.

2) A fixed number of papers or words should be required of each student.

3) Guidelines for how the writing will affect the final grade should be given to each student at the onset of the course.

4) Writing assignments should be varied, yet, several should be sequential.

5) Support services for instructors should be available. (p. 74)

Descriptions of two colleges' inclusion of writing intensive courses in their curriculum follow.

In the 1990s, Kean College, one of New Jersey's eight state colleges, offered both general education and major area courses as writing-emphasis courses. To graduate, students at Kean College completed two writing-emphasis courses, including one from their major areas. Instructors for each course agreed to assign a variety of writing exercises, one per week with three to five being sequential. Writing-emphasis course instructors also agreed to give timely feedback on all writing assignments. Instructors and students participating in the writing-emphasis courses reported positive teaching and learning experiences (Goldberg, 1992).

By 1992 Swarthmore College in Pennsylvania had a Writing Across the Curriculum program in which all departments participated. Each discipline offered at least one Primary Distribution Course (PDC), and students were required to take a certain number of PDC's, according to each major's requirements. Primary Distribution Courses were either general education or major area courses. Each course incorporated reading, analyzing, arguing, and writing into the coursework. Usually, two major papers were assigned per term. However, the specific requirements of each discipline's PDC were different. For example, the Primary Distribution Course in mathematics focused on the proper use of definitions, theorems, and notation, and on the creation of "good expository mathematics" (Maurer, 1992, p. 22).

Several drawbacks have been cited regarding the inclusion of writing intensive programs in general education courses: motivation is often low and resistance is often high; class size may be much larger than in major areas; courses are introductory and

interdisciplinary so students know little about the language and methods of the subject area; classes include mainly freshmen or sophomores who are less comfortable or confident with the course's content than older students; faculty who teach general education courses often have less experience and support than other faculty; and goals of the course may be vague and idealistic (Thaiss, 1992). Fulwiler (1986) compiled the following list of problems in both general education and major area courses requiring writing:

1) Attitude (motivation, interest)
2) Mechanical skills (spelling, punctuation)
3) Organizational skills (how to piece it together)
4) Style (conventions appropriate to the task and audience)
5) Reasoning ability (thinking, logic)
6) Knowledge (something to write about) (p. 26)

However, proponents of writing to learn have not allowed these problems to keep them from developing writing intensive curricula for general education and major courses (Rose, 1992; Sipka, 1992; Thaiss, 1992). They believe that students should be asked to write continuously throughout their college experiences. Fulwiler (1986) wrote that the more students write, the stronger and more confident they become in these areas. The next section ties together the reform of mathematics education and the writing to learn principles and trends by exploring the use of writing within the mathematics curriculum.

Writing Across the Mathematics Curriculum

Several mathematics departments in colleges throughout the United States began including writing intensive courses in their curriculum as vehicles for educational reform in

the 1980s (Sipka, 1992). Mathematics educator Sipka (1992) summarized the reasons for this curricular reform:

> Writing . . . assignments add variety to the typical lecture-oriented math class; writing assignments help students become autonomous learners . . . and writing assignments provide an accurate assessment of a student's level of understanding/confusion. (p. 13)

Mathematics instructors designed writing assignments for the college mathematics classroom to meet several purposes, such as reinforcing daily material, allowing the student to step back and look at content so as to "make it their own," and leading the student on a path "tangential to the course content" (Snow, 1992, p. 113-114). Snow (1992) discovered that writing assignments also provided a forum for teaching the student how to communicate the solutions to mathematical problems. Another professor's rationale for assigning writing exercises was based on his past experiences with bright mathematics students who seemed to solve problems in a gestalt without analyzing what they were doing and with less bright students who relied on memorization and had given up trying to understand what they were doing (Henriksen, 1992). As early as 1954, Hadamard wrote that students of mathematics often expressed that something was lacking even when they knew the correct steps to or processes for solving a mathematical problem. Writing about mathematics has been described as a means of helping students to fill this void by forcing the "construction of understanding, because we cannot write coherently about something we do not understand" (Talman, 1992, p. 107).

Some mathematics faculty expressed skepticism about the pedagogy of writing to learn mathematics. These hesitant educators described the inclusion of writing as making mathematics into a "soft science" and as a waste of time and effort (Kenney, 1992, p. 17).

Some felt unqualified to teach or assess writing skills, and others were unsure of how to include a writing component in their curriculum. However, teachers who regularly incorporated writing in their mathematics courses said the extra time and effort were well worth the rewards they yielded (Rishel, 1992; Rose, 1992). Rose (1992) reported both affective and cognitive benefits to the teacher and student participants of her writing in mathematics courses. These benefits included better communication, greater interaction, more valuable feedback, and more productive relationships between students and teacher.

Educators who employed writing to learn activities in the mathematics classroom often cited student resistance as an initial obstacle to overcome. Students themselves told of struggling with not knowing what to write and not having the vocabulary required to write about certain mathematical concepts. Other weaknesses found in students' writing included their failure to support conclusions with evidence and denial of or rationalization for the mistakes they made (Gopen & Smith, 1990).

Teachers and students found that writing in mathematics was much different than writing in an English or history course. Both formal and informal writing exercises have been assigned in the mathematics classroom. Formal writing assignments included exam questions, proofs, process papers, summaries of journal articles, solutions to journal problems, research papers, and lecture notes. Informal writing assignments fell into the two categories of focused (explicitly defined) and unfocused (free) writing and included mathematics autobiographies, journals, reading logs, or letters (Sipka, 1992).

Small group interaction and peer teaching were natural outgrowths of the writing to learn process. Sibly from St. John's University in Minnesota assigned papers to his

mathematics students regularly and used student peer refereeing as a teaching and assessing aid. As peer referees, students critiqued their peers' writings before composing final drafts. Sibly cited his greatest reward from his student writers as "seeing them learning mathematics and learning how to learn" (1992, p. 53). Other educators assigned group writing of problem solving strategies, word problem creations, and explanations of mathematical concepts (Sipka, 1992). In 1992 Kenney wrote generally about writing in groups as a means of building "trust in the student's own ability to communicate mathematical ideas" (p. 18).

Writing in mathematics often takes the form of written responses to homework problems. In 1989, Price assigned homework that involved written responses to his Number Theory students and also gave them explicit guidelines regarding these assignments. The guidelines included such basic principles as the purpose of the assignment, examples of good and bad writing, and grammatical suggestions such as "avoid pronouns" and "write in sentences" (p. 394-395). He stated that "our system trains students to go home, zip through their ten or twenty . . . exercises as quickly as possible, tear the paper out of their spiral notebooks, and hand it in with a sigh of relief . . . homework should be an exercise in learning, organizing, and explaining, not a certificate in drudgery" (p. 394). He found that 50 to 60 percent of his students in all of the courses where he assigned writing improved significantly in writing skills.

Two general goals for writing assignments were suggested by Brown (1992):

1) The assignment must be an integral part of the coursework.
2) The assignment must require the student to make a significant effort to communicate the results in writing. (p. 131)

Other educators gave more specific purposes for asking their students to write about mathematics. Because he felt compartmentalization was overemphasized in mathematics courses and texts, Rauff (1992) assigned comprehensive essays to his students. He designed the essay assignments to force his students to integrate the pieces and construct their own "cognitive maps" (p. 138) of the course content. Hartz (1992) incorporated the use of written abstracts for his upper level mathematics students to force his students to review weekly and to put the content in their own words. Each abstract was a one- to two-page summary and analysis of the material covered over the previous week.

The calculus reform movement of the latter 1980s and early 1990s provided a natural forum for writing in the mathematics classroom. The proponents of calculus reform updated the study of calculus by incorporating various teaching and learning techniques such as computer applications and written projects in their calculus courses. Although calculus reform took on a variety of forms at different institutions, one general objective was the enhancement of students' understanding of calculus concepts. Many calculus instructors regularly used writing assignments to achieve this goal (Gopen & Smith, 1990; Stoughton, 1992).

In the latter 1980s, the mathematics department of Duke University, a leader in calculus reform, included an intensive writing component in its calculus labs. During labs, calculus students wrote lab reports and answered essay exam questions. Because technology (calculators and computers) reduced the amount of time and attention previously spent on computational techniques, a new emphasis on conceptualization unfolded in Duke's calculus courses. Writing assignments forced students to think about and express mathematical

concepts. Regarding students writing in the calculus lab, Duke faculty discovered the following facts: (a) "thought and expression of thought" regarding mathematics concepts became the new focus, (b) writing improved student comprehension of mathematics, (c) teachers saw significant success without a lot more effort, and (d) students produced better writing and became better writers (Gopen & Smith, 1990, p. 3).

Another example of the use of writing in the calculus classroom occurred at Hope College in Michigan where calculus instructor Stoughton required his Calculus I students to write term papers. Each semester he assigned two term papers, one on a precalculus topic and one on a calculus topic. His goal was to allow his students to make connections between algebra and calculus concepts. Some of his recommended topics included a discussion about absolute value or an analysis of the limit concept (1992).

Many statistics teachers also have assigned writing to learn exercises. In an effort to make her statistics course more relevant to her students, Lubecke (1992) assigned writings on readings portraying real life statistical problems. From this experience, she discovered that when her students realized she was actually interested in what they were thinking and writing about, they displayed more thoughtful work. She described the reading and writing assignments as causing the students to think about mathematics in ways they had never thought before and "having their eyes opened to an entirely new world" (p. 47).

The use of written questions on mathematics exams grew significantly in the early 1990s. Britton (1992) cited several reasons for students experiencing difficulties in writing on mathematics exams: they have little past experience in writing on mathematics exams; they have seen few models of how to write about mathematics; and they have no concept of who

their audience should be and how much they should write. Regarding the construction of written exam questions, Hayden (1992), a statistics instructor, suggested starting slowly and that a one-sentence answer might meet the goals of the class. His goal in assigning written exam questions was to de-emphasize mechanics and encourage thought. Before his exams, he gave sample test questions so the students were aware of his expectations. Another mathematics instructor insisted that students answering written exam questions "write for an audience other than the course instructor" and thus "eliminate the student writer's tendency to rely on the reader to correctly interpret incomplete or unclear explanations" (Brown, 1992, p. 131).

The NCTM standards caused many mathematics educators to reexamine the types of written or word problems presented to students. After questioning his mathematics students, Marks (1994) found that they often viewed word problems as questions that would never be asked in the real world. He concluded that word problems should "connect algebra with real life" (p. 610) and should be sensible and relevant to the student's experience:

> Algebra is the crowning achievement in a quest that has gone on for as long as the human has had the power of thought: a quest for the ability to solve problems that the real world creates for its inhabitants. (p. 610)

Asking students to create word problems was one way instructors found to merge problem solving with the students' real world experiences.

As the writing to learn process became more familiar on campuses, some mathematics instructors began requiring their students to keep journals in their courses. Journal assignments varied from course to course. While Rose (1989) advocated freewriting, Bell

and Bell (1985) promoted expository writing, and Miller and England (1989) used writing prompts in their journal assignments. Other instructors were more explicit in their expectations of what journal entries should include. Britton (1992) required his students to write weekly journal entries, and each entry had to include a summary of the past week's material and any questions or comments the student had about the course or coursework. Talman (1992) also assigned weekly journal entries. Entries handed in to him had to include a short summary of the week's topics, a short report of the student's activities over the past week, and a long analysis of the week's work, including an analysis of at least one problem not solved in class.

Teachers who assigned journal writing in their classrooms initially grappled with such problems as whether or not to grade each entry and whether or not to use prompts suggesting journal topics (MacGregor, 1993). Yet, once these details were worked out, most teachers reported dramatic benefits for both themselves and the student writers. Buerk (1992) wrote that through the reading of her students' journal entries, she became aware of students' survival strategies which, in turn, allowed her to find ways to help them feel empowered in the mathematics classroom. In 1991 Miller reported that reading student journal entries helped her to become a better teacher as she became more aware of and receptive to her students' perceptions of mathematics.

In 1993 Wajngurt suggested four steps required for mathematical learning to take place: apprehension, acquisition, storage, and retrieval. She further stated that journal writing, where students write out processes for solving mathematics problems, was an effective pedagogical tool to aid students in learning these steps. She believed that writing

helped students conceptualize the problem and organize their thoughts which allowed them to solve the problem.

In the classes where she assigned journals, Brandau (1992) found that the journaling process personalized mathematics and compelled students to engage actively in problem solving. Moreover, students who were hesitant to ask questions in her classes because of shyness and not wanting to appear stupid to their peers became willing to write out these questions in their journals. After conducting a study on the positive effects of journal writing, Rose (1992) wrote that journal writing promoted understanding, facilitated reasoning and problem solving, stimulated questions, served as a study tool, helped retention, promoted independent learning, improved writing skills, and generally benefited students personally. Britton (1992) also gave some advantages to assigning journal writing: journals forced students to keep up with their work; journals provided increased communication between students and teacher; and journals served as a way of taking attendance.

Mathematics journals provide a forum for student self assessment in many classrooms. MacGregor (1993) described this form of evaluation as both a process, "reflecting, composing, and writing," (p. 9) and a product, the written document. She gave five characteristics demonstrated by student self evaluations: attitude of inquiry, integration of learning, meaning and relevance, voice and authority, and self-directedness. Angelo and Cross (1993) suggested the use of a one-minute summary in a journal at the end of each class as a means for students to assess their understanding of the day's coursework. The rationale for requiring students to assess their own progress dates back to Bloom's (1969) concept of critical thinking as an integration of analysis, synthesis, and evaluation. Student self

evaluations should provide the teacher with a better understanding of student learning which should improve the overall quality of education. MacGregor (1993) wrote that "learning involves making an action out of knowledge, using knowledge to think, judge, decide, discover, interact, create; learning succeeds to the degree that it gradually assists the learner to take control of his or her own learning process" (p. 55). Journals are an excellent place for students to learn to assess their mathematical progress and, thus, feel some sense of control regarding the learning of mathematics.

In a 1986 study at Michigan Technological University to examine the effects of journal writing, researchers used experimental and control groups in three sections of Analytic Geometry and Calculus. The results were assessed using the Student Attitude Survey (SAS), Writing Apprehension Test (WAT), journal writings, and open-ended questions. The following findings were reported:

1. Students in the experimental section performed equally as well on content area tests as the students in the control sections.
2. The series of journal writing assignments in the experimental section did not change students' attitudes toward writing activities as identified by the SAS.
3. The series of journal writing assignments completed by the experimental section did not reduce students' writing apprehension as measured by the WAT.
4. In the experimental section, some students used their journals to think and learn about mathematical concepts and problem solving.

5. The teacher of the experimental section felt that the journals were beneficial in helping most students understand mathematical concepts and solving mathematics problems.
6. Generally, the students in the experimental sections felt that journals were a positive addition. Some students commented that the journals helped them think and understand the mathematical concepts they learned in class (Selfe, 1986, p. 203).

The findings from this study illustrated the fact that not all journal writers experience the same effects.

Britton (1992) gave the following reasons for mathematics educators expecting undergraduate students to write about mathematics:

1. If students are to thoroughly learn the material, then they should be able to write an explanation of the concepts, procedures, and key results of that material in order to demonstrate what they have learned.
2. The act of writing, as well as the study and preparation necessary to do that writing, are activities which aid the students' learning. It should help strengthen their mastery of the material. (p. 104)

He also suggested several "areas for further investigation" regarding writing in mathematics: does writing increase mathematics retention and problem solving abilities, does grading journals impact writing in the journal, what are the cumulative effects of extensive writing in mathematics classes, and does the use of writing affect mathematics apprehension?

In summary, many types of writing assignments have been used in the mathematics classroom to promote student comprehension of mathematical concepts and procedures. The review of literature painted a promising future for those instructors looking to incorporate writing as an effective teaching and learning technique in the mathematics classroom. Chapter III will present the methodology used in this study of the effects of writing in a particular mathematics course.

CHAPTER III

RESEARCH METHODOLOGY

> My theme is that doing research is a similar exercise
> to going on a journey of discovery, and that we undertake
> scholarly studies to make the familiar strange.
> Sara Delamont, 1992, p. vii

The purpose of this study was to explore the learning experiences of a group of undergraduate students enrolled in a writing intensive mathematics course. The study examined student experiences and perceptions of writing in a college algebra course. Specifically, it focused on the multiple realities of the participants and the analysis of their lived experiences in this setting.

The methodology employed in this study was phenomenological research, which is a form of qualitative inquiry. Qualitative inquiry entails an emergent research design and places the researcher in a continuous discovery mode (Delamont, 1992). Phenomenology is the human scientific study of phenomena or lived experience and employs interactive techniques such as participant observation and interviewing to discover meaning regarding specific phenomena. Qualitative research originated in an effort to allow persons who had little or no voice to be heard (Erickson, 1988). Students themselves often have little voice concerning curricular reform. Qualitative research, therefore, seemed a natural choice of research methodology for this study involving writing to learn mathematics as a form of curriculum reform. In 1979 Doyle wrote that students in the classroom should not be

"passive recipients of instructional treatments" (p. 203). This study allowed students to voice their thoughts and perceptions of their experiences in this particular classroom and to become active participants in their own education.

Van Manen (1990) described qualitative inquiry as "the search for what it means to be human" (p.12). He further stated that "theory enlightens practice. Practice (or life) always comes first and theory comes later as a result of reflection" (p. 15). The research direction and underlying theory of the following study emerged during observations of the student participants' lived experiences and analysis of their written and verbal reflections regarding the writing to learn process.

Three conceptual frameworks or assumptions helped inform my research:

1. Writing to learn and learning to write mathematics enhances mathematical understanding (Rose, 1992).

2. Reflective or expressive writing helps students to appreciate and value difficult subjects (MacGregor, 1993).

3. Allowing students' voices to be heard regarding curricular assessment and reform brings about student ownership of the content (Kenney, 1992).

These theories were used to reshape the research design as the study evolved. The central question which guided my phenomenological research was, "What were the learning experiences of this particular group of students in this writing intensive college algebra course?" Van Manen (1990) has described phenomenology as asking "the simpler question, what is it like to have a certain experience, for example, an educational experience?" (p. 45).

Qualitative research relies on the use of multiple data collection methods that verify one another. This practice, called triangulation, contributes to the trustworthiness or internal validity of the data by providing multiple pieces of data to support each conclusion. Qualitative researchers observe and interview participants, collect written documents, and analyze the results. The results of qualitative research are not generalizable but do extend the understanding of particular phenomena (Glesne, 1992).

In this study, I chose five qualitative methods of data collection: weekly student journal entries, daily teacher journal entries, student writing to learn exercises, interviews, and a Small Group Instructional Diagnosis (SGID) assessment. Analysis of the data took place at all stages of the study, allowing for the formulating and reformulating of research paths. The final analysis of the data was a systematic categorization of emergent themes and patterns. Some descriptive statistics are displayed in order to illuminate the student participants and their perspectives.

The Setting

The setting for this qualitative research project was a freshman mathematics class at a midwestern university. The subject was college algebra, the first mathematics course for which students receive college mathematics credit at this university. Because college algebra is required for numerous majors, the students enrolled in this course have varying mathematical, writing, and other intellectual abilities. For this reason, this student sample came as close to being a random sample as any college course could. Also, because this course is an initial college mathematics course, it seemed appropriate to introduce curricular

reform at this early level, further encouraging these students to continue to reflect and to write about mathematics in their future studies.

As the instructor for this course, I had the added advantage of being a full-time participant observer. I have taught college algebra several times and have used isolated writing assignments to enhance student understanding. This study was my first effort at an intensive writing program incorporated throughout the algebra course.

The study began during Fall Semester 1994 with 36 students originally enrolled in the college algebra course. By the end of the fall term, 27 student participants remained in the class. All student participants found out about the writing to learn component on the first day of class and were given the option to switch to another section of college algebra. The class met each Monday, Wednesday, and Friday from one to two o'clock in the afternoon for sixteen weeks. The topics covered in the course included the usual array of college algebra topics including sets of numbers, functions, graphing techniques, systems of equations, theory of polynomials, and logarithms. For a more detailed description of the course and the participants, see Chapter 4.

Data Collection

Several different methods of data collection were used in this study: journal entries, writing assignments, observation notes, interview transcripts, and a SGID assessment. The variety of methods allowed for triangulation of the results in an effort to address threats to the validity of the findings.

Journals

The primary data used in this study were the students' reflective journal entries. Each Friday, the students were asked to write in their journals at the end of or after class. Each entry was one to two pages in length. I encouraged the students to write about the course and themselves in the course. The entries often became a form of student self evaluation (SSE). The purpose of the journals as a method through which to gain a better understanding of student learning was emphasized so as to foster more "thoughtful and serious" self assessments (MacGregor, 1993, p. 44).

Although the journal assignment was designed purposely to capture the students' immediate thoughts and feelings regarding mathematics, I found some students needed more direction. Thus, a list of open-ended prompts was available for the times when they needed a focus for their writings. The list included the following:

1. What problems did you encounter in class this week?
2. Write about your mathematical strengths and weaknesses in terms of concepts and processes you are comfortable or uncomfortable using.
3. How do you feel about writing in a mathematics course?
4. Write about what you learned this week in this class for an absent student.
5. Evaluate your progress in mastering the concepts and processes of this unit.
6. Describe some concept you learned this week to a friend's child.

The first journal entry assigned was a mathematics autobiography where each student detailed her or his mathematical studies, adventures, and concerns. These historical writings served to guide the direction of the course and my teaching practices.

Each week's journal entry was turned in on the following Monday in part to help decrease the amount of data lost by accident or by students who dropped the course and disappeared (subject attrition.) Points were given for each entry. Because the entries were considered freewriting, grammatical or spelling errors were not considered.

My goal was to receive immediate and continuous feedback from the student participants regarding their learning and their ability to make meaning of mathematics through writing. Thus, the use of journals as the primary vehicle for data gathering proved especially significant to this study.

Writing Assignments

Various other written assignments were used to accumulate data for the study. Each day in class, students were asked to write out mathematical processes, theorems, definitions, solutions, and concepts in words. Because these efforts were lessons in writing to learn mathematics, they were not always used for numerical grading. However, periodically I did examine these writings to establish strengths and weaknesses of the students' skills and to help direct further writing assignments. The students wrote often about these exercises in their journals.

Most of the homework assigned and handed in for grading included a writing to learn component. These writings were assessed for both content and clarity. Also, each of the four examinations included questions which called for written responses. All assigned writings were used as data for this study.

Observations

Erickson (1988) referred to qualitative research as an effort to uncover the "invisibility of everyday life" (p. 121). Being in the classroom actively and busily involved with teaching every day, I had difficulty in stepping aside and addressing this basic qualitative disrobement. One significant form of validity for qualitative research is descriptive validity, which calls for a thorough and disciplined examination and description of the acts rather than the meanings assigned to the acts (Maxwell, 1992). As researcher and instructor, as well as observer and student advocate, I experienced a certain sense of imbalance when attempting to increase the validity of my data.

In an effort to address the validity issue and to answer the question of what was actually happening in the class, I initially planned to videotape the class three times. However, because one student vehemently objected to this idea, I decided to forego the taping and opted for another less objective method. Each day after class, I went directly to my office and wrote at length in my own journal about my classroom observations. I wrote about the salient events of each class period and about my highs and lows regarding my research. Although, my writings were my own perceptions and understandings of the classroom activities, they formed a counterbalance to the students' journal reflections of the same happenings.

Interviews

Phenomenology is concerned with the "local meanings that happenings have for the people involved in them" (Erickson, 1988, p. 121). In this particular study, the unique perspectives of the students involved in the writing of college algebra became the

phenomenological data. To clarify my interpretation and analysis of the data I had already amassed, I knew I needed to hear the students' voices regarding this experience. Patton (1990) wrote, "The purpose of interviewing, then, is to allow us to enter into the other person's perspective. Qualitative interviewing begins with the assumption that the perspective of others is meaningful, knowable, and able to be made explicit" (p. 49). I interviewed each of the participants at the end of the semester after the students had received their grades. To further strengthen the validity of my research, another interviewer conducted one-third of the student interviews. Each interview lasted approximately thirty minutes and included such questions as the following: Would you take another mathematics course which involved writing? Would you recommend this course to another student? (For a more complete list of the interview questions, see Appendix D.) Other interview questions were formulated from the responses of the individual interviewees.

Small Group Instructional Diagnosis

At the middle of the semester, a small group evaluation was conducted by a Small Group Instructional Diagnosis (SGID) team member who was a professor from the College of Business. The SGID team consists of faculty members from different departments on campus who are trained to conduct small group student assessments of college courses. During the process, students meet in groups of three or four to discuss and list the strengths and weaknesses of a course and its instruction. The SGID team member then compiles a written report detailing the findings to give to the instructor. The instructor is not allowed to be present during the process, and the written report does not give the students' names. The process allows the students freedom to be forthright about their assessment. The SGID

team asks that faculty members discuss the results with their classes soon afterward and agree to implement any reasonable suggestions.

The SGID team member facilitated the small group assessment process on a Friday, and the results were given to me on the following Monday before class. (See Appendix C.) The anonymity of the process contributed to the study's validity and provided important data for the research.

Data Analysis

Good phenomenological description forms what Van Manen (1990) calls a "validating circle of inquiry," that is, it "is validated by lived experience and validates lived experience" (p. 27). Through the use of multiple methods of data collection, I attempted to paint a collage of individual experiences and meanings into a holistic portrait of the students' lived experiences in writing to create mathematical meaning and understanding. Phenomenological researchers "strive to understand a phenomenon as a whole" (Patton, 1990, p. 49).

The analysis of the data was an ongoing process. I endeavored to apply a sense of "disciplined subjectivity" to all phases of the research through continuous questioning and reevaluation (McMillan, 1989, p. 192). This effort allowed for the forming and reforming of some journal prompts, writing assignments, interview questions, and research paths.

In the final analysis of the student writings, journal entries, and interview transcripts, I entered what Glesne (1992) referred to as the "code mines" (p. 132), searching for buried treasure hidden in the reams of data. I used Erickson's (1988) method of categorizing the coded data and of allowing the categorizes to illuminate the patterns and threads woven

through the data. The patterns formed a mosaic consisting of the three major sections of findings: student effects, teacher effects, and student and teacher effects.

Qualitative and phenomenological researchers are concerned with "particularizability" (Erickson, 1988, p. 130) rather than generalizability. Thus, my goal in this study was to look closely at this specific case of writing throughout the college algebra course and its effects upon the participants. The analysis of the data from this study should enhance and enlarge the understanding of how particular students learn mathematics and how the writing of mathematics and about mathematics facilitates that learning.

Summary

The research methodology of this study was a qualitative inquiry into the learning experiences of the students of a particular college algebra class which included an intensive writing component. The methods employed in this study included extensive data collection and analysis.

Qualitative researchers assume " meaningfulness of human actions depends on the contexts or situations in which these actions, feelings, and perceptions occur" (McMillan, 1989, p. 187). This study examined closely the experiences and perceptions of the student participants. The phenomenological findings or what Van Manen (1990) described as "poetizing activity" (p. 13) comprise the next three chapters. Chapter IV gives a narrative description of the course taken directly from my journal entries and recollections. Lastly, the findings and conclusions of the study are detailed in Chapters V and VI.

CHAPTER IV

COURSE DESCRIPTION

> The goal . . . is to change these students' conceptions of mathematics as a discipline and of themselves as learners of mathematics.
>
> Buerk, 1992, p. 78

The study exploring the learning experiences of a group of undergraduate students enrolled in a writing intensive college algebra course took place during Fall Semester 1994. Thirty-six students were present the first day of class, but only 27 completed the course. During the first week, five students dropped the course, citing scheduling problems or the fact that the coursework seemed too advanced. Four other students dropped the course around midterm. Two of these students left because they felt the coursework was too easy and the other two because they were doing failing work.

The remaining 27 students made up a diverse group of undergraduate students majoring in several different disciplines. (See table 1.) Of this group, 17 were freshmen, seven were sophomores, two were juniors, and one was a senior. Seventeen of the students took the course because it was required for their majors. The other ten enrolled in the course for general education credit or because they thought they might need college algebra as a prerequisite for future courses. Sixteen of the students needed and planned to take another mathematics course in the future.

Table 1

Declared Majors of Student Participants in a Writing to Learn College Algebra Course

Major/Field	Number of Students
Aviation	1
Business	3
Communications	1
Computer Science	1
Elementary Education	2
Industrial Technology	1
Mathematics	2
Medicine	6
Philosophy	1
Political Science	3
Undecided	6

Note. The category of medicine includes the majors pre-occupational therapy, pre-nursing, and pre-medical technology.

The student participants of this study wrote weekly in mathematics journals, detailing their progress and struggles in the course, summarizing content, and generally asking and responding to questions about the material and the course. Although the students' journal entries contributed significantly to this study, they were not the only written data amassed.

Recalling Glesne and Peshkin's (1992) recommendation that qualitative researchers write extensive memos to themselves as an early beginning to the analysis process, I also kept a journal. Immediately after each class, I returned to my office and put on paper all thoughts and perceptions of the class activities related to the study. My journal became what Van Manen (1990) referred to as an attempt to "grasp the pedagogical essence of this experience" (p. 78). The following narration is my version of the story of writing to learn college algebra taken from my journal entries and from my memory.

Fall Semester 1994 began with its usual mixture of warm, rainy days and campus halls filled with noisy, excited students searching for their assigned classrooms. A certain sense of newness mixed with the smell of damp autumn leaves seemed to make the gray afternoon of August 28 brighter. An hour before the first meeting of my Writing to Learn College Algebra course, I sat in my small, mostly empty and newly assigned office feeling excited, anxious, and somewhat unprepared. Although I had taught mathematics for seven years, I was a novice regarding research in mathematics education. Only one year earlier I had begun my doctoral studies in education and up to this point had piloted only one short qualitative study. Knowing I must make the most of each class period for my research to yield significant findings only added to my slightly agitated and humble state. I found some comfort in recalling the warning of Glesne and Peshkin (1992) that "experiencing insecurity throughout all stages of the research process is common" (p. xv).

At two o'clock I entered a room with 36 student desks, each with a student looking expectantly at me. The walls were a pale green and partially obscured by three sets of blackboards which ran the lengths of the three walls. A panel of large windows made up most

of the fourth wall. The room was like all the other semi-barren classrooms I had been in over the six years I had taught for the Mathematics Department. I introduced myself to the class and described the adventure on which we were to embark, an educational journey integrating writing with the learning of algebra. A few minutes into the hour, I found that the words came easily because I believed what I was saying, and I discovered my nervousness had vanished. Perhaps Van Manen (1990) was right when he wrote that when we speak we discover what it is we wish to say. I was further encouraged when only one student expressed concern about adding the dimension of writing to a mathematics course. This particular student remained and did very well in the course.

On that first day, the students signed informed consent forms agreeing to participate in my research and allowing me to write and publish my findings. I distributed the syllabus outlining the course objectives and schedule and the guidelines describing the weekly journal entries. (See Appendix A.) The students asked a few questions about the logistics of the course but seemed to accept the writing component. One student objected to being videotaped, so I decided on the spot to forego the taping. Reminded of Delamont's caution about the necessity of flexibility in qualitative research methods, I went with my alternative plan of recording the day's happenings in my journal immediately after each class. Hoping to set the tone for future classroom participation, I had the students turn to their neighbors and introduce themselves. For five minutes the room was filled with noisy chatter which seemed to have a calming effect on the class as a whole.

After the preliminary introductions, I gave a brief review of the real number system and its properties. Then, so as to give a glimpse into what the semester would hold, I asked the students to write out in words what the following expression meant to them.

$$\frac{a}{0} = \varnothing$$

During the last few minutes of that first class, the students shared some of their responses:

"You can't divide a number by zero."

"A number divided by zero is not rational."

Their responses portrayed a mixture of the various ways students read and comprehend mathematical terms and symbols, for example, the ability to generalize from "a" to "a number" and the incorrect use of the term "rational." As Dewey (1910) once said, learning cannot take place unless there is immaturity or uncertainty at the start. Overall, the first day went well and I felt as though the students had some idea of how writing would fit into the course. After class, a male student approached me to say he was in the wrong section of college algebra but wanted to stay in my class instead of the one assigned to him. Taking this as a good omen, I promised to facilitate the section change.

Over the semester the students participated in several writing to learn activities. All of the exercises are displayed in the following concept map. (See Figure 1.) The narrative below describes and gives examples of each writing exercise.

During the semester, the students spent the last 10 to 15 minutes of each Friday writing weekly journal entries while exploring their thoughts and opinions regarding the course. These writings allowed for a continuous dialogue between student and teacher where

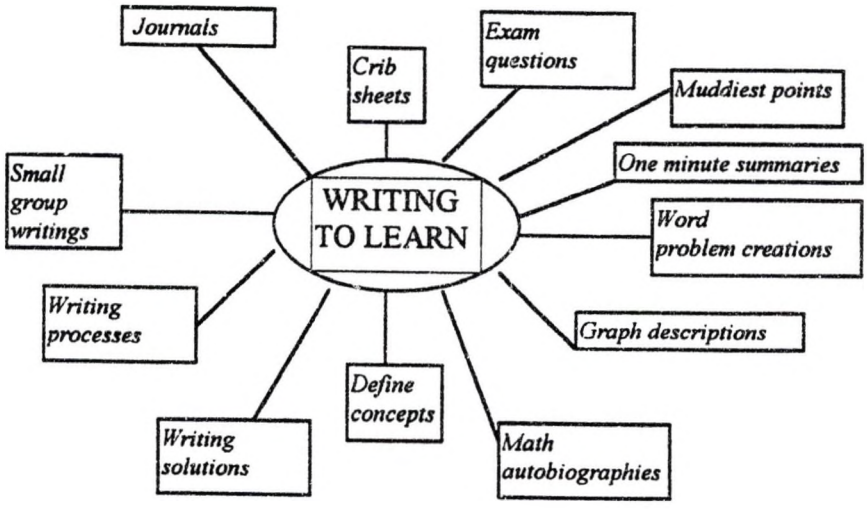


Figure 1. Writing to learn mathematics exercises used during the course.

students asked and I answered questions, as well as serving as data for my research. The first entry was a one- to two-page mathematics autobiography in which each student told about her or his history concerning the study or use of mathematics. As Brandau (1992) wrote, "We all have a story to tell of our mathematical lives . . . We all have also created a story of mathematics, our own perception of what mathematics is, [and] our own understanding of mathematical concepts" (p. 73). Reading these entries gave me immediate feedback on the perceived ability, fear, and mathematical maturity of each student.

On some class days when I would explain or demonstrate a mathematical concept, I noticed that some of the students looked bored. When this happened, I would catch myself speeding up. I had to make a conscious effort not to teach to only these students. Through

the students' journal entries I found that I was not always reading these looks correctly. Many of the students whom I assumed were bored because the material was too easy actually were struggling with the content.

The personalities of the students came alive for me through their writings. For example, Jay was a bright student who often expressed in writing his need to be challenged. Then, there were Adrian and Annie who both smiled at me whenever I looked their way and wrote about their personal lives in their journals. I also discovered other sides to students. Both Connie, who always looked so serious in class, and Debra, who was older than average and an overachiever, displayed much lighter personalities in their journal writings. Tim's writings depicted his feelings of insecurity about himself as a learner, reminding me of Van Manen's (1990) statement: "To write is to exercise self-consciousness" (p. 129).

Each day in class, the students and I wrote definitions, methods of operation (M.O.s), and step-by-step directions for solving particular problems. Some examples of these activities follow:

1) Write out in words the steps for solving the following equation for an absent student:

$$\sqrt{x^2-1}=4$$

2) Write out how you would start to graph

$$y = \frac{x^2 - 1}{x - 1}$$

3) Define the term circle in your own words.

This last exercise was given immediately before the lesson on graphing the circle and elicited some intuitive and revealing responses in terms of the disparity in students' understanding of geometry. Students described a circle in the following terms:

Geometric object with no beginning or end.

A set of points equal distance from a given point, called the center.

After this exercise Sara, an energetic and spirited freshman, challenged me in class by asking what became her constant question regarding ungraded writing exercises: "What is the point for doing this?" Although I was unsure if she were merely curious or if she felt she needed some tangible reason for having to write, I was careful after that day to give an objective for every piece of ungraded writing I asked of the students. I was determined to apply Dewey's (1910) theory that education must be relevant to the student's experience.

Over the semester, I assigned twelve 20-point homework assignments which included problems requiring written solutions. Some of the problems focused on honing students' mathematical communication skills, and others emphasized writing to learn and to make meaning of algebraic concepts. Examples included the following:

1) Write out precisely what is wrong with this statement:

   $|3x - 2| < 0$.

2) Construct a crib sheet for an absent student consisting of the material covered in class today.

3) Construct a crib sheet for chapter 5.

The crib sheets were detailed accounts of a unit of material taken from either the text or the students' notes.

Each of the four exams also included questions which called for a written response. (See Appendix B.) These questions asked students to rewrite or explain false statements, define concepts, or describe graph characteristics.

In small groups, the students created and solved algebraic word problems, reviewed for exams, and composed lists of clearly written algebraic facts. During one class, each group composed a list of eight to ten qualities and facts about the logarithm function. From these lists, I amassed a longer list made up of the "good" qualities and gave each student a copy. We also discussed what was wrong with the "not-so-good" qualities. On another day, the students created and solved word problems involving systems of two equations with two variables and then presented these to the class. Many of the students initially expressed confusion and uncertainty before they realized that to construct a workable problem they must work backwards. After writing their problems on the blackboards, several students stayed after class to explore their peers' creations further. Upon entering the room and seeing the students milling about, the teacher who was teaching the next period kidded: "What kind of teaching is this?"

Other short writing activities allowed me to gauge the progress of the students on a daily basis. Once most of the students moved past the notion that they should receive a score or reward for everything they wrote, they seemed to embrace these activities wholeheartedly. Using the concept of the muddiest point (Angelo and Cross, 1993), I frequently asked students to write the most confusing topic of that day's lesson or exercise on a three by five

index card. I would then attempt to address the most common complaint at the beginning of the next class. On three occasions I asked the students to write one-minute summaries of what they had just learned, allowing me to assess their progress authentically.

Another activity allowed students to assess their own progress in using and comprehending certain graphing terms. In this exercise the students counted off in ones and twos. The twos left the room, and the ones remained in class. The ones then were asked to write a description of a graph so their counterpart "twos" could construct the graph using the description. This exercise was done twice, once before the section on graphing and again after that section was completed. Most students were able to see their progress in terms of the ability to use correct graphing terms and to interpret graphs. Two major drawbacks to this activity were that it was time consuming and that not all students were able to be writers. The greatest problem I encountered throughout the semester was a lack of time. I often wished for more time to develop and explore each writing to learn component of the course.

Throughout the term, the students were asked to evaluate the course and their progress repeatedly. The weekly journal entries served as a forum for this assessment, and the writers became quite verbose about expressing their likes and dislikes about the coursework, as well as evaluating their own work. About midway through the semester, in response to several students writing that they were running out of "stuff to write about," I began suggesting journal entry topics like, "How did you feel about Monday's exam" and "Describe yourself today as a learner of mathematics." The last journal entry was a handout asking the students for their personal reactions to and comments about each of the writing to

learn exercises. By the end of the course the students had become quite adept at course and self assessment.

At midterm, a small group evaluation was conducted by a Small Group Instructional Diagnosis (SGID) team member, a professor from the College of Business. At this institution, the SGID process is conducted by a team of faculty members from several different academic departments. Each team member is trained to conduct small group student assessments of college courses. The assessment usually takes place part way through the course and always at the invitation of the instructor. A written report is given to the instructor detailing the findings after the assessment process. At my invitation, a SGID team member came to my class and oversaw the small group evaluation process. I was not in the room during the process, which allowed the students to be candid in their analysis and comments. During the small group process, the students described the strengths and weaknesses of the course. The SGID team member then compiled a list of these attributes in a written report. (See Appendix C). Because I was not present and the comments were anonymous in the report which I received immediately afterward, the findings were especially salient to my research.

By the end of the semester, I felt somewhat guilty about the amount of assessment I had asked of my students regarding the course. I wondered if they had ever thought or written in such great detail about any other teaching and learning methodology. However, when reading and categorizing my data, I discovered that most of the students liked being part of the process.

The climax of the semester for me came just as everyone was leaving on the last day of class prior to the final examination. Of the original 36 students who had been present the first day of class, 27 were present on that day. A tenseness permeated the room that afternoon as the students asked questions and expressed anxiety about the final examination scheduled for the following Monday. At the end of the hour, students began to file past me, talking anxiously and excitedly to each other. Although I would see the students for their final examination and again for their exit interviews, I stood in the front of the room feeling sad that the semester was coming to an end. I recalled Glesne and Peshkin's (1992) words describing the leaving of the fieldwork site of a study as a "bittersweet time" (p. 60). Suddenly the sharp, tinkling crash of broken glass exploded through the commotion, and the noise in the room suddenly died. Sara had dropped a bottle of bright maroon nail polish, splattering brownish red paint across the floor and filling the room with a toxic smell. As soon as the last tinkle of glass was heard, the room erupted in a wave of nervous laughter, and I felt a weight lifted off my chest. I had developed what Delamont termed "genuine feelings" for these students. Together we had enjoyed a rapport I will not forget.

The next chapter presents the data from the study. The data came from the words written and spoken by the student participants and my observations of the class.

CHAPTER V

PRESENTATION OF DATA

> Learning in the proper sense, is not learning things, but the *meanings* of things, and this process involves the use of signs, or language in its generic sense.
>
> John Dewey, 1910, p. 176

The purpose of this study was to explore how writing in a college algebra course affected the learning experiences of a group of undergraduate students. The findings of this study focus on three major areas: student effects, teacher effects, and student and teacher effects. Each area is developed based on the writing to learn exercises, written examination questions, journal writings, memos to myself, evaluation data, and interview transcriptions. Because of the qualitative inquiry process, many of the findings overlap the three areas.

The inclusion of an intensive writing program in this college algebra course engendered a transformation in how the participants understood mathematics education. Moreover, the rhythms of the learning process, the highs and lows experienced by the learners, became apparent as the course evolved. Therefore, the following discussion, while focusing primarily on the effects of writing on the participants of a college algebra course, also paints a revealing portrait of what it meant for these students to be learners of mathematics.

Student Effects

The 27 student participants who remained in the course over the entire semester were asked repeatedly to evaluate this course and their progress in this course. In most cases, they did so with great enthusiasm and earnestness. Their perspectives in assessing the content and organization of this course were extremely revealing and, at times, somewhat unsettling. From their writings, four main categories of student effects emerged: students' understanding of course content, students' feelings regarding the learning of mathematics, students' assessment of individual progress, and students' assessment of the course and instruction.

Students' Understanding of Course Content

Proponents of writing in mathematics courses cite writing as a significant learning tool for understanding mathematical concepts (Rose, 1992; Kenney, 1992; Brandau, 1992; Gopen & Smith, 1990). There was much support for this idea in the students' weekly journal entries. Regarding functions, one student wrote the following:

> I was unaware of the numerous types of functions . . . I wrote each down individually and was surprised at the number of types of problems which fall under the label of a function. Up until now I was just doing the problems . . . wondering what the whole picture was in the building process. Now I am enlightened . . . of the correlation between all functions.

Another student, while listing the shapes of graphs of functions, "$y = x$ is a line and $y = x^2$ is a parabola," came to the realization that functions "$y = x^{n>2}$ must have the shape of a wave!" Other realizations regarding functions came in the form of questions:

> I understood the examples of years in college increase your salary, or weight varies with age . . . could this be a function: Your interest in functions increases your understanding?

> I was a little confused about the horizontal asymptote . . . it would cover the whole line . . . from right to left, without any holes . . . So, a line could cross the H.A., but would it be a point . . . or an open circle?

One student worked out her own question after listing the operations of functions in one of her journal writings:

> . . . You can take a function of a function. I wonder if you can take $f \circ g \circ h$, like $f(g(h))$. I'll check, if $f = 2$, $g = 3$, and $h = 4$. $f(g(h)) = f(g(4)) = f(12) =$ wait, doesn't make sense-try again; $f = x+1$, $g = 2x+1$, $h = x+2$, $f(g(h)) = f(g(x+2)) = f(2(x+2)+1) = f(2x+4+1) = f(2x+5) = (2x+5) + 1 = 2x+6$ . . . seems like you could do that.

This student's journal entry provided her with a forum to think on paper, reinforcing her understanding of the content, a phenomenon supported by the literature on writing to learn mathematics (Snow, 1992).

The writing in this course promoted student comprehension of certain mathematical concepts. One student wrote about slope on a crib sheet constructed for a fictitious absent student:

> Slope = steepness of a line . . . definition of a slope does not apply to all lines, m is undefined when it requires division by zero.

Constructing written explanations for certain logarithm laws, students wrote the following:

> If the logs with the same bases equal each other, then the arguments will be equal to each other.
>
> If the argument of a log is the same as its base, then the log is equal to one.
>
> A log is an exponent.

Although some of the preceding facts were merely the laws written out in words, the student writers reported that this exercise helped them to clarify what the notation of the laws meant.

The following student definitions of a circle were written before the material regarding circles was explored in the course:

> A neverending line that curves perfectly to meet at the start and continues in the same pattern.
>
> A circle is a perfection, no end and no beginning. The shape of the largest objects in the universe. The sun, the stars. A perfect curve. A two-dimensional view of a sphere.

Through their writings, students were able to attach personal meaningfulness to some mathematical concepts.

Often, the journals were the locus of ongoing dialogue between student and teacher: students asked questions about course content and received replies when the journals were handed back. One of the most common types of dialogue concerned the usefulness of certain algebraic concepts in the "real world." One student wrote, "I'm not really sure what a log is except an exponent. What is it good for? What would we use it for in the real world?" Repeatedly, the question would be asked, "where will I ever use this in the real world?" That education in general and mathematics in particular should be relevant to the student's present or future experiences was a common theme for these students. In 1910, Dewey wrote, "The material furnished by way of information should be relevant to a question that is vital in the student's own experience" (p. 199). Although I usually responded to these questions with various "real world" applications, I was fully aware that many of these students would never see this material again. I am equally certain that Dewey's solution would have been to omit much of the irrelevant content, a luxury in which most undergraduate mathematics instructors

dare not indulge. Many mathematics courses are sequential, and students are assumed to have learned a certain amount of material in each course.

However, once in a while a student would make a connection between the algebraic content and their real world. A chemistry major wrote the following:

> . . . in chemistry . . . like where you have so many grams of calcium over here . . . you use basic algebra, like solving for grams . . .

and

> I have shadowed doctors and they still use algebra . . . in their figuring out of prescriptions and stuff like that, so I think it will be useful.

Journal writing assisted this student's construction of personal relevance regarding mathematics and algebra in general.

Another topic that surfaced often in the journal entries was the solving of word problems. Students who liked working on word problems described the solving of these types of problems as a "challenge" or a "contest." They often referred to the fact that word problems involved real-life situations. Reminding me of Daniel Marks' (1994) admonition that instructors should create word problems that "connect algebra to real life" (p. 610), one student summarized:

> Most problems to me are busy work. I don't like busy work. Word problems make me feel like I [found] a solution to a real problem.

However, many more students expressed aloud their disdain and fear of word problems.

Twice during the semester students created their own word problems. The first construction was done individually as homework and then given to a peer to solve. Two of the more creative problems follow:

> Baby Sumba and Mufassa leave the cave at the same time, but in opposite directions. If Sumba runs at 4 MPH and Mufassa runs at 10 MPH, how long will it take for them to be 24 miles apart?
>
> The FMIPL (Fat Men in Pink Leotards) is holding its 7th annual "Skip-a-thon" tomorrow. If Gorgio skips at an average pace of 5 MPH and Frank skips at an average of 2 MPH, how long will Gorgio have to wait for Frank at the end of the 45 mile race?

Although the students tended to use the same formats as assigned in practice problems from the text, they demonstrated both a creativity and an ability to construct a solvable problem.

The second word problem creation exercise occurred much later in the term and was a small group effort. Each problem was to involve the solving of two equations with two variables. Several of the creations are given below:

> If a case contained 2 gallons of beer and a keg contained 18 gallons of beer, and the students drank 1200 gallons of beer, how many kegs and cases of beer did they drink?
>
> An elephant on a moped traveled 90 miles in three hours with a tail wind. On his way home it took him 6 hours. Find the rate of the moped and the tail wind.
>
> Sally found 50 pink and aqua sea shells while vacationing in Mexico. Each pink shell weighed 2 ounces and each aqua shell weighed 3 ounces. The total weight of the shells is 8 pounds. How many of each colored shell did she find?
>
> If a person once had some blue fish and some green fish and had a total of 500 fish [sic]. Each blue fish weighed 1.5 lbs and each green fish weighed 3 lbs, he had a total of 1500 lbs of fish. After he went on vacation and forgot to feed them, how many blue fish and green fish did he have left?

Because this was a classroom activity and time was limited, the clarity and mechanics of the writing of these problems were poorer than those on the first set. Once again the students relied on the standard formats given by the text. However, in a classroom discussion after this exercise, the students cited several realizations they had during the creation process:

1) You must work backwards to get the numbers to work out nicely.

2) You must assume an ideal mathematical world where the wind does not change and fish don't die from starvation.

3) You must understand why the problem works, not just how it works.

Over the semester I found several written accounts of moments of either brilliance or clarity in the students' journals. One student's inspired attempt to connect algebra to chaos theory to Jurassic Park ended in the following paragraph:

> Every situation or object or group of objects or the entire universe no matter how random it may seem in its form and function follows a strict pattern of structure and order. Which if chaos theory is basically true, it could mean that everything is fated and we haven't as much control over our lives as we thought.

This particular student dropped the course at midterm, and I sincerely missed reading his often brilliant, and always bizarre, writings. I only regret that I did not get an opportunity to find out if he was a reader or writer of science fiction.

Other writers shared more down-to-earth, yet significant insights in their entries, illustrating the fact that the "medium of words" allows the writer to "obtain ownership of ideas" (Kiltinen & Mansfield, 1992, p. 93). After grappling with the concept of a graph, one student wrote of what he had come to understand a graph "really" meant:

> The graph of a function is a representation of the particular solutions for a particular equation. Every point on a graph is a solution and can be plugged into the equation.

Clearly, this student had come to a correct understanding of what a graph represents and was prepared to address the mathematical correctness of his definition. In his journal, I pointed out the further restrictions for a graph of a function, which at this point he was ready and able to learn.

Another student demonstrated the need for coming up with his own method of operation for solving word problems (the class had constructed one together the day prior to this writing.) His M.O. follows:

> 1) assign variable
> 2) list all info mathematically
> 3) construct equation that illustrates the problem
> 4) solve the equation
> 5) check it

It became very clear that many of these students wanted a recipe for each type of problem and, if necessary, could and would construct one of their own.

Students often referred to the act of writing in mathematics as helping them to construct and memorize procedures. A female student spoke about this during her interview:

> I like this course more than any of my other math courses because we learned more . . . because we wrote things down and we did things differently and it helped me remember things.

During a brainstorming activity in one class, a small group of two females and one male wrote about the importance of writing to the study and memorization of mathematics:

> When you write things down it is like encoding it. A person is more likely to remember when they write it . . . the most helpful thing writing has done for us is that it helps us remember the steps and the rules from our lectures.

It was therefore no great surprise that the activity most students liked over the semester was the construction of crib sheets which could then be used to study and memorize for the test. I must admit to feeling that I had let my students down at times because some of the writing activities seemed to encourage the students to memorize rather than conceptualize algebraic ideas. However, there is a place for memorization in the learning of mathematics.

As Fulwiler (1986) suggested, the process of writing reinforces several different critical thinking skills. Moreover, the students' comments supported writing as a means of developing relevance, clarity, creativity, and ownership in mathematical content.

Students' Feelings Regarding the Learning of Mathematics

Students wrote in their journals most often about their feelings regarding this course, the writing in this course, or the learning of mathematics in general. The journal became a forum for students to express their fears, anxieties, confusion, and joy. For most of the students, keeping a mathematical journal was a new experience, and it seemed natural that they would write expressively about the effect of this experience. The use of expressive writing is cited in the literature as a means of encouraging students to feel and think about the process of learning (Britton, 1986).

Many students used the journal to write about their feelings of frustration and confusion:

> Today was the first day of new material and I feel overwhelmed. I thought it was hard to stay caught up when I already had an idea of how to do the problems! The material we covered wasn't that hard, but there are just a lot of little "rules" to remember.

> Last week seemed like a rollercoaster for me as far as understanding. During your lecture, I felt as if I had a decent idea of concepts and methods . . . my problem is that once I leave the class I forget.

The rhythms of learning became quite apparent as students began to use the journal to describe for me and for themselves the incessant and often rushed nature of college life:

> Another week, another whatever. Everything just seems to be bleeding into one. I can't distinguish one day from the next. Between school, work, baseball, it doesn't feel like I have a week-end. But who needs a week-end.

This entry allowed the student to share his feelings about having so little time to relax and for me to recall those same feelings and adjust the pace of the course somewhat.

Other students reported times of great joy when working out certain mathematical exercises seemed fun or algebraic concepts became clear:

> I pulled my books out and did the homework problems for today. These didn't prove too difficult so I decided to do the practice problems. They were actually fun and I was bragging to my roommate about my victory. I finally understand . . . I came to class with such a great attitude.

> I've discovered that for the first time I am actually enjoyir g taking math. I'm taking more time to study and understand what I'm doing instead of rushing through and just trying to get the work done.

The journals became a forum for student disclosure where the writing appeared to dispel the negative and intensify the positive. The writers began to exhibit a sense of what Buerk (1992) referred to as empowerment in the mathematics classroom.

Nonetheless, the most explored topic throughout the journals was "test anxiety." The students completed four examinations over the semester. Prior to each exam, several students would write about their struggles or fears regarding the anticipation of the test:

> This is the worst case of test anxiety I've ever had. I think it is because I have failed before and I'm fearing that if I do poorly on the exam, I will lose faith for the rest of the semester.

> Hopefully once I begin writing the exam my anxiety will pass over. I even told my mom to pray for me tomorrow . . .

> Well, Friday is the big day . . . I think the first test with a new teacher is very important. I really don't know what to expect . . .

After taking the examinations, the students displayed a mixture of disappointment and satisfaction:

> The test this past week just reinforced my phobia for math.
>
> I was nervous before taking the test . . . When I got back the test I found out that I knew more than I thought I did! This was a good feeling . . . I was pleased with my grade . . . I can still improve.
>
> A strange thing has begun to take place in math classes. I have never had to battle with "test anxiety." Suddenly I find myself turning into a knotted bundle of nerves before a test I know I'm ready for . . . I walked out of the exam knowing several mistakes that I made simply because I was too tense to think clearly.

One student's entry about the tests was equally humorous and revealing:

> The countdown is already on for the next test. Only 14 days until we get to have some more major fun . . .

This entry seemed to imply the study of mathematics for this student is defined and delineated by examinations.

Moreover, some students described the act of writing as helpful in easing those times of anxiety or confusion:

> Frustrating because I would be able to do something in class and go home . . . and go back over my notes 700 times to complete the homework assignment. It is frustrating . . . but I write . . . I am a writer . . . for me it was kind of like, putting things again into context, making it more appealing.

The journal writing in these cases seemed quite therapeutic:

> At first I [thought] this is weird, but then it helped to write down your feelings and what you are frustrated in.

Writing seemed to become part of what Buerk (1992) referred to as the students' "survival strategies" (p. 78). Writing also allowed students to reflect on the highs and lows of being a learner of mathematics.

Students' Assessment of Individual Progress

Over the semester most of the students became quite willing and able to assess their own daily, weekly or overall progress in the course; yet, their early writings about their successes often assumed a tentative quality. For example, one student wrote she *felt* successful, not that she *was* successful. Others wrote similarly:

> I like what we've done in chapter 4 so far. I feel like I have a pretty good grip on how to graph polynomial functions.
>
> Well, I think I've got down f(x), but still don't quite have a firm hold on adding functions and compositions. I think with a little more practice that they'll fall into place.
>
> As to my progress in this class, it is better than I expected. It seems that I'm more motivated than all of the years before . . . I'm setting my goals pretty high.

The entries confirmed MacGregor's (1993) suggestion that initially self assessment is a challenge for the students because they are unfamiliar "with looking inward" and they "lack confidence in describing their own learning" (p. 10). However, toward the end of the semester the language in the journals began to take on a slightly different tone. Students began to write that they *were* (not felt) confident, that they *were* (not seemed) successful. Success in college algebra for these students seemed less fleeting:

> I was proud of myself this week when my husband (who is currently in Intermediate Algebra) asked me to work out a problem for him and it was a piece of cake!
>
> Right now I am confident, probably for the first time, in my abilities to do logarithms.
>
> Oh well, I have faith in my learning skills . . . I'll be able to keep it up . . . I am confident.

On occasion I would read a telling explanation of how certain students saw themselves as learners of mathematics:

> I feel I have no strengths in any math class what so ever. Everything I learn comes with a struggle. I don't know if it is all in my head or if I really am ignorant when it comes to numbers. Maybe it's because the minute I step into a math class, I feel inferior. I feel inferior to the math book, the teacher, and even the other students. I'm always afraid I'll give some stupid answer, and everyone will end up laughing at me.

> I don't study at all for math classes and I never have. I guess you could say that I am a natural at math. I never did a practice problem or anything for the last test. If I would do some practice problems I could probably get 100's on the tests.

> I guess really, for me, math has always been not hard. I make it hard on myself because I never study . . . so I tend to get lower grades.

The journals contained both reflection and self reflection as students expressed what has been described as "what they have learned in relation to themselves" (MacGregor, 1993, p. 9).

During the exit interviews, two students who planned to be mathematics teachers spoke about their progress as learners of mathematics in relation to their future careers:

> I started thinking about I have to take math and be a teacher of math for kids . . . But before that I have so many years of learning . . . I want to learn different ways of teaching.

> Now that I have done well, I have more confidence and it will help in the future when I have to teach algebra.

Writing provided an avenue for the students to learn and practice self assessment. In 1993, Eaton and Pougiales wrote that students who successfully learn to assess their own progress become autonomous and responsible learners.

Students' Assessment of the Course and Instruction

As students began writing their first journal entries, they exhibited a certain amount of indecision about what to write or what they perceived I wanted them to write. One early entry included the following:

> Writing about class is somewhat more difficult than I had anticipated. Perhaps it will come more naturally with time and practice. As the material gets more complicated and I'm in class more, I should be able to do better than just fill the page.

Every time I would encounter statements of this kind, I would take the opportunity to encourage and praise the writer's efforts. I would also ask questions to which the writer could respond in the next entry, and I would refer the writer back to the list of possible topics I had given each student at the onset of the term. The problem of what to write in the journal seemed to lessen as the students wrote more often. Another early journal entry demonstrated that one student saw great possibilities for his future journal writings:

> I feel that writing a journal entry in math will have some real benefits, not only for the student but also for the instructor . . . One great benefit is if you have a problem or a question about the class you can easily write in your journal or if it is difficult to meet up with the instructor's office hours . . . Another thing that may be helpful is when you write down step by step how to solve a problem. This will give you the chance to actually know why you do each step instead of just doing it. One of the disadvantages to writing in a math class is that it seems like an English class.

Another interesting theme appeared throughout many of the later journal entries. Students began to suggest ways to change the structure and pacing of the course and my instruction:

> Journals are becoming a bit harder to write without going into something you have already talked about. I believe it would be better to do them every other week, so you have a little more to write about.

> This week went by too fast. We covered a lot of material in so little time. Staying caught up has been a challenge. I think in the following chapters we should go slower.
>
> I feel compelled to let you know that the way you present material is clear and makes understanding it relatively simple. It felt, however, like you were cramming in the information on graphing equations so that you could just "get" through it before the exam. (Must keep on schedule?)
>
> I've been trying to do the practice problems from 3.5 and I am totally lost. I think we covered this material way too fast. We should do a review.
>
> It feels as though you are going really slow so far . . . I hope things will start going a little faster soon.

Of course, adjusting the course to meet the needs of each student was impossible. I was encouraged, however, by the fact that students began to exhibit an openness and freedom to speak their minds regarding their assessment of the course and my instruction. The journal writing promoted "student ownership" in the course, as well as the coursework, a fact supported by Kenney in 1992 (p. 17). Moreover, not all assessments were negative, and some compared this class to other mathematics classes:

> I like how you did lots of examples on the board today. At least we know you're human and make mistakes too. My high school math teacher never did and it was really annoying. He was a genius.
>
> I am finding this class to be more interesting than my first year of college math. This writing is also good for my writing skills. I am not the world's greatest writer and this will provide me with more opportunity to write.

At times students would use the journal to assess mathematics education either in general or at this institution:

> You've noted that there is a certain amount of information that we need to go on to calculus or other math classes. I would really like to see a Math 103 class that is geared toward students who will not be continuing on in math.

> I think there should be an algebra course for students who need to go at a slower pace. Instead of this course finishing in Dec. it would be finished in May.

> I can see no reason for the study of mathematics except to force students to use a part of their brain that otherwise would remain untouched. Mathematics was invented primarily during an age of reason when individuals were challenging themselves to think. As a form of rebellion against social and political reform. These people had nothing better to do than to devise an innumerous amount of ways to solve one minute problem. Why? To create stress for future generations!

As students became more comfortable with their journal writing, other themes began to emerge. The word "challenge" began to appear repeatedly in the weekly entries. A few students wrote often about their need for challenge:

> I thought the test was good, but I don't understand why you don't ask challenging problems.

> I think this course will be a good warm up, but I am not expecting to be challenged at all. I've looked ahead in the book and nothing seems to be much more advanced than my previous high school algebra courses. I will honestly be surprised if anything this semester will be any kind of problem.

This student dropped the course before I had a chance to meet his challenge. However, he probably did belong in a higher level course.

More often students wrote about the challenge of the coursework and mathematics in general and about their desire to meet that challenge:

> Algebra is the only class I have to try really hard . . . have to put forth extra effort . . . I like it because it is a challenge for me. If Algebra was easy and I didn't really have to try, I guess it wouldn't be very fun for me.

> Even though we are only working with the basics right now it still seems possible to find a few challenging problems.

During the exit interviews, the students were asked directly about their thoughts and feelings regarding the writing activities in this course. When asked about their initial reactions

to writing in an algebra course, many students made comments like "I thought it was stupid" and "I didn't want to do it." The literature on writing in mathematics alluded to this initial resistance exhibited by the student writers (Gopen & Smith, 1990). Yet, when asked later about the specific writing activities, the responses were varied. All of the students had much to share regarding the journal writing:

> Sometimes it is tough having one on one conversation with your instructor, the schedules don't meet, and deadlines have to be met, and that way, every week you know you are able to, if you have a problem with the class or a question or a comment, it is easy to do it and you know you will get a response back right away . . . and it was very beneficial.
>
> I guess it was helpful for the teacher, the journals, you could see what you were doing, but for me it didn't help me in math class at all.
>
> They helped if you gave us a topic, but sometimes it was hard to try to find something to write about . . . I really liked the extra points. I don't like how a lot of these classes it is just tests and you don't get any points from assignments or anything.
>
> For the journals, I didn't like those because there were too many of them to do. It just kind of got to be busy work. It was really repetitious.
>
> . . . but the journals, the way I look at it is, if you talk about it you are bound to, you are going to be learning about it, I guess, the more you talk about it, the more you keep your mind on it . . . you are learning not only from the numbers but from a different perspective of writing about it and talking about it and in your journals you have to talk to her about things you don't like or problems you were having . . . and she always responded to it. She would write on there and tell you to come in and talk to her . . . I enjoyed it all.

The writing to learn activity to which students responded most positively was the construction of crib sheets. Recalling Britton's (1992) warning that when students do not understand the audience they leave out parts they assume the instructor already knows, I asked that the crib sheets be constructed for a fictional absent student. Each crib sheet was

to consist of written explanations of algebraic concepts and examples of problems from particular lessons. Several students referred to the benefits they received from this exercise:

> The crib sheet was good because . . . it was like you were going to teach somebody else and it helped me actually learn the material. It would stick in my memory better.
>
> . . . that was good practice for rewriting your notes and also when we got them back I kept them so I could look over them for studying.

Other students commented on the positive effects of the various other writing exercises. Regarding the muddiest point exercises where students would write the most confusing topic for the day on a 3 by 5 index card, one student wrote, ". . . you saw that the whole class was having problems with it, not just you." Sipka (1992) described this use of writing as providing "an accurate assessment of a student's level of understanding/confusion" (p. 13).

Another student commented on the usefulness of writing algebraic steps out in words:

> It is helpful because sometimes you go through and you are doing the algebra . . . you are just doing the work and you really don't understand why you are doing the work and when you go through and write it out you actually have to know why you do each step and how each step works.

This writer discovered Sipka's (1992) premise that writing and learning are "umbilically linked" (p. 11) and that being able merely to do the computations does not necessarily imply understanding or thinking.

When asked which writing assignments students found useless, two students responded:

> . . . the math autobiography. Useful for a tutor . . . working one on one with the person, but for a class with 20 students, I really don't see how that could be effective.

> Creating the word problem . . . I try so hard at word problems and it wasn't on the test so I didn't know why we did it.

During the interviews, students were also asked if they would recommend writing in a mathematics course to another student. Two students explained:

> Yes, I would now, I probably wouldn't have when I started but I would now because it has really helped . . . I never realized how much writing actually helped me see things clearer.

> . . . if they were a person who can basically cram before the test, go take a test, get done with it and leave, I would say no. I would say take a regular one but if they really couldn't study well or they just didn't have people to help them study, I would say to take it then...It just kind of reinforces things, putting it down in English.

Only three students said they did not feel that writing helped them in the course at all. One of the three summarized her struggles:

> I just think that the most difficulty was trying to get the mathematical jargon into English terms.

All of the other students said they would consider taking another mathematics course which contained a similar writing component. Two students commented in a more philosophical manner about the usefulness of writing in general:

> . . . we have a society that doesn't know how to write. I work for a newspaper and you should see what comes in and so even though you are writing about math or art or music or something that you wouldn't necessarily consider writing in . . . if nothing else, they are strengthening their writing skills. Because you have to put things in order in your mind in order to put them on paper.

> When you have to write something out, you have to understand what you are doing: you can't bluff.

At the end of the course, all students completed a survey asking for their opinions on the various writing exercises assigned. On this survey, students ranked each exercise

Table 2

Writing Exercises	Mean Rating
Crib sheets	4.1
Muddiest points	3.8
Concept definitions	3.6
Word problem creations	3.5
Weekly journal entries	3.5
Writing of processes	3.4
False statement explanations	3.1
Logarithm qualities	2.9
Writing of solutions	2.7
Mathematics autobiographies	2.6
Graph descriptions	2.3
One-minute summaries	1.4

Note. 1 = not valuable and 5 = very valuable.

according to a five-point Likert Scale, where 5 = very valuable and 1 = not valuable. The resulting means are listed in Table 2. Students rated the crib sheets and the muddiest points of the most value to their learning of college algebra. The lowest rated exercise was the one-minute summaries which many students could not remember doing.

The student participants of this study were continually asked to assess the course and the instruction of the course. Through their written and verbal assessments, it was apparent that these students had become quite proficient at educational assessment. They also demonstrated a unique sense of freedom and candidness in their evaluations. The process allowed them to be part of the construction of a quality mathematics education effort.

Teacher Effects

The writing exercises in this course contained a large affective, as well as a cognitive component. The writings allowed students to use expressive freewriting to describe their thoughts, feelings, goals, expectations, and assessments regarding this course and mathematics in general. These writings were not graded for mechanical or spelling errors or for content. They became valuable feedback for my awareness of the students' epistemological differences, understanding of the effects of past teaching practices on student learning, and authentic assessment of students' academic and mathematical progress. A detailed exploration of these three areas follows.

Teacher's Awareness of Students' Epistemological Differences

The ways in which students understand and construct knowledge are varied and complex. Reading through the students' expressive writings allowed for a glimpse into the various ways in which students learn and understand mathematics. In one journal assignment, students described themselves as learners of mathematics. The writings were a rich mixture of simple to complex, immature to mature self portraits. Although each student interpreted the topic in a different manner, the replies were illuminating:

> In general I consider myself a slow learner of mathematics . . . It is much easier for me to memorize words and facts. I realize that math is not my strong subject.
>
> . . . as a math learner, I am now a math panicker.
>
> I used to feel like I was a brick wall and that information would just bounce off me and that everyone else "got it" except for me. Now I feel like I'm finally "getting it" and it feels good.

This student's description reminded me of MacGregor's (1993) premise that "learning succeeds to the degree that it gradually assists the learner to take control of his or her own learning process" (p. 55). This student seemed in control. More students wrote about themselves as learners of math:

> I know I am more of an algebra person than geometry . . . I like having a right answer. Yet there are many options on how to go about solving a problem, which gives the variety I like.
>
> I have always had a difficult time grasping math. It is a task that must be repeated time and time again. . . While I have the capability to learn mathematics, I need to take things slow.
>
> A slow one! Give me a short story to read or a poem to explicate and I can do it in a snap. But when it comes to math, it's a totally different story...I have a theory: either the average person is left-brained or right-brained. Either good in math type things or English/art type things.
>
> I feel that I have a very strong grasp on math . . . I am a fast learner.

Over the semester, the class worked in small groups regularly. Most of the students responded well to this cooperative methodology. Some comments supporting this positive response include the following:

> I always work better in groups, when I do math problems anyway.
>
> I enjoy working with other students. I think it is helpful to get others' ideas and sometimes we can answer each others' questions.

For many of the students, the combination of writing and small group work fostered and supported their confidence in communicating mathematics.

One student wrote about her "way of learning" in the classroom:

> I like to be the one to answer questions because it was that way in high school . . . It will probably drive you insane but it is my way of feeling, of learning something.

This student did respond quite often in class so I chose this opportunity to assure her that I liked having bright students in the class. I wrote back that she did not drive me insane and that I was very much like her as a student. I also added that I often wondered if my peers had ever wished I would answer less often. After this dialogue, she responded less often in class; yet, she was far from silent. The journal had provided a forum for disclosure for both the writer and the reader and helped what could have become a common classroom complaint: one noisy, bright student smothering and silencing the other voices in the classroom.

Other writings provided snapshots of how students studied for mathematics exams:

> I think I have devised a plan . . . I write down the formula and the name of it and an example of each. I have them typed out and use it as a reference guide.

> I usually just read over things, read over my notes a lot of times and if there is practice problems . . . I will go over them and see if I can answer them, if not, I will do them a few more times.

Students also wrote about their individual preferences regarding learning methods:

> . . . example, example, example, that is how I learn best.

> I don't like learning through action . . . I would rather learn through observation, just like reading, reading textbooks, and taking notes.

> I am a student who can comprehend concepts rather well when they are thoroughly explained to me. Then I must review them and they stick with me.

I was amazed at the number of students who wrote they would rather read or watch a demonstration of an algebraic problem. Yet, the majority of the students wrote they needed a lot of practice to learn most mathematical processes.

Many students wrote that writing reinforced their understanding of the content:

> . . . overall I understand the stuff better because I had to learn it not just memorize it.
>
> I would take and write down some practice problems and theorems . . . before each test . . . if I see it written out, then it is a lot clearer . . .
>
> I find the more I write things, the more I tend to remember them.

These students saw writing as promoting mathematical comprehension through conceptualization and memorization.

Prior to each of the four exams, we would spend one class day reviewing the previous material. Because class had to be cancelled one day, I decided to forego one review session. Almost every one of the subsequent journal entries contained some comment about how necessary each student felt these review sessions were to their success on exams. I realized how dependent the students had become on this review process. Yet, I surmised that what was actually missing was some sort of summary of what we had previously learned. From that point on, I put great efforts into summarizing new and old material on a daily basis. Repeatedly, I found that the journal writing served to guide the direction of instruction.

Students often wrote about particular problems they had with concepts such as functions or logarithms. Because of these entries, I was able to identify patterns of confusion and spend more class time demonstrating and observing students working with these types

of problems. Other entries included statements referring to particular methods I would use, such as the following:

> I don't like the way you solve problems like $x^4 - 8x^2 + 16 = 0$ using the u [substitution]. Personally, I understand it better the other way.

In cases such as this, I would first observe the preferred method and if it was mathematically sound and it worked, I would assure the student that my way is not always the only way.

Several students wrote about this course as being an experience of relearning rather than learning. As Dewey (1910) said, education should be the "continuous reconstruction" of knowledge (p. 189). I encouraged these students to use their past experiences to help some of the other struggling students during group work activities. In most cases, this proved a positive experience for all.

Some less tangible problems that students were having in the course came to my attention through the journal entries. Several students wrote of their poor study habits, so I encouraged them to construct and write a new approach or plan for studying as their next entry. I cannot honestly say that I saw much improvement in the grades of these students after this entry although the writing did cause them to analyze temporarily their study habits. One student wrote of a particular obstacle to her learning math:

> When I'm looking at a problem, I just don't know where to start. Once I get started the rest is easy for me. It's just that initial step that I have problems with.

The best response I could come up with for this problem was that we are all pretty much alike in that aspect: the first step is always the most difficult. I also suggested she come and visit me so we could look more closely at her initial thinking/problem solving process.

Reading through these journal entries each week provided me with insightful glimpses into how students see themselves as learners. Every so often I would read of a student's dreams and talents and see a vivid picture of a colorful life outside of the classroom:

> I draw and I don't think on . . . a conscious level. I sometimes think about the colors I use, but mostly I feel it. And, the thing about my drawings is that they are unorganized completely . . . everything blends together to form a visual whole. I use curves and it feels to me like I'm honing the edges of curves to create a steady flow without disruption, gradually increasing or decreasing into itself.

Overall, writing allowed for both student reflection and teacher analysis regarding the teaching and learning of mathematics.

Teacher's Understanding of the Effects of Past Teaching Practices on Student Learning

The first journal entry each student composed was an autobiography which included any or all of the student's mathematical learning experiences. Rose (1992) described this process as the vehicle by which students "recognized feelings and experiences and that the written product could become a record for referral and reflexivity" (p.64). Many of the autobiographical entries gave some reference to a negative episode regarding the learning of mathematics. More often this experience was described as having lasting effects on the student's mathematical progress. One student recalled an incident from grammar school:

> I believe my first math memory comes from the second grade. My favorite math teacher yelled at the class for the very first time because we couldn't understand some math concept. From that point on, it was all downhill for me.

Several other students wrote about "bad" teachers from their junior and senior high school years:

> Our junior high math instructor was horrible. He taught classes and coached football . . . he would call on people and if they didn't know the answer he would kind of make

> fun of you. A lot of the time he just told us how to do the problem, but never showed us how to do it.
>
> . . . the teacher liked to talk about sports more than algebra.
>
> The teacher was sick of me and my questions . . . would get mad at me and say I wasn't really trying and I became afraid of him to the point that Algebra I class meant a stomach ache everyday.
>
> It seemed our teacher just loved to see us squirm when we got to the last page of the test because she'd always include at least two story problems. When I see a story problem on a test, my resting heart rate probably increases to a pace which would be equivalent to the pace of someone running a billion miles. I hate story problems.

Another student's experience was more recent and had to do with a college professor:

> My instructor was hard to understand . . . he had a doctorate in math and everything was "so simple, it's so easy, this shouldn't be that difficult for you." He had that "I have a doctorate in math and I couldn't care less if you pass or fail this 100 level class" attitude.

Whether or not these students' memories or perceptions were entirely accurate, these episodes seem to have had lasting effects on the students' psyches. In the past, mathematics classes often were thought of as filters to screen out the poor students (Gopen & Smith, 1990), and these recollections seemed to support that notion. However, reading of the negative repercussions of some teaching practices served to reinforce and guide my own actions and reactions regarding all of my mathematics students.

Nonetheless, some students wrote about great mathematics teachers and positive learning experiences in their classrooms. These experiences seemed to have made a powerful impact on these students:

> I remember the 4th grade and my teacher . . . she used to yell out a multiplication problem and you would have to say the answer before you caught the stuffed animal she would throw at you . . . Now it's the sense of accomplishing a problem that when

> you first looked at it, you thought there is no way this problem can possibly have an answer . . . numbers have been good to me.
>
> . . . when I was in kindergarten our teacher really liked math and stressed math in the classroom. She made working with numbers and doing problems a game. This started me off on the right foot and sparked my interest in math.
>
> Mr. _____, my high school teacher, has probably pushed me the most through school. He was a real hard teacher but he taught us so much. He would push us and tell us to do our best in everything, not just math and science, but in life too . . . pushed me to believe in myself and in the others who surround me.
>
> Being the daughter of a mathematics instructor, I have experienced many different situations with the subject of math. I enjoy the class and always have . . . Having my dad as an instructor during my eighth grade, tenth grade, and senior year was a great advantage to me. It was nice to have someone there to help you with questions at all times anytime. I believe having a parent as a teacher helped me with my success in the subject.

One student compared me to a teacher who obviously had a strong influence in her mathematical progress:

> You have a different style of teaching than I am used to. My math instructor was all business and his class must be too. He would teach in a solid way. I liked him, although most people didn't. He was a strict teacher but I enjoyed learning from him. He seemed to like me and encouraged me often.

A few of the autobiographies contained humorous anecdotes regarding mathematics:

> I remember the teacher trying not to let us count on our fingers . . . she threatened to make us wear mittens.
>
> . . . I had my first algebra class and I was having troubles so I asked my brother, who is a math whiz. He sat beside me with a wooden spoon and everytime I would make a mistake he would crack me with it. It didn't take me long to understand that section of the book.
>
> . . . in 9th grade . . . Algebra I. I had a very smart boy in my class, who I also had a crush on, and we competed to see who could get the best test scores. Well, needless to say, I got an A+ in Algebra I.

I am unsure of what lessons these comical recollections hold for teachers of mathematics, but I do know that our students' memories shape a large part of how they see themselves today.

Two students wrote about the unfairness of the tracking system of mathematics education where a test determines which level of mathematics a student may study in college. Both students admitted that they were placed in the intermediate level and, thus, not personally affected by this process. Another student wrote about transferring from a private school which offered him " a tougher challenge" into a public school where his "attitude and work ethic declined greatly."

Overall, the mathematics autobiographies highlighted the similarities and the differences among the students' mathematical histories. Over two-thirds of these students came directly from high schools in rural communities to this semi-urban university. Yet, their mathematical successes were seldom dependent upon the schools from which they came and more often upon the positive and negative experiences they encountered in their past mathematics classes. This awareness should serve as a wake up call to all teachers of mathematics; we must strive to promote a positive atmosphere for our students to be successful and confident learners of mathematics.

Teacher's Authentic Assessment of Student's Academic and Mathematical Progress

Over the semester, students earned points from homework assignments, journal entries, and examinations. The points were then converted to a letter grade for each student at the end of the semester. Thus, the students were assessed based on their understanding and retention of the algebraic subject matter. However, a more subjective component was added

into this assessment process. I looked for writings which demonstrated in an authentic manner that each student had progressed in their mathematical abilities over the course.

Several of these authentic writings came under the area that I have labeled "moments of genius." These writings demonstrated that the writer was able to analyze and synthesize certain mathematical or algebraic constructions. For example, after a homework assignment in which students were to write a word problem involving the solving of a linear equation, one student wrote in his journal:

> Here's one I was tempted to hand in: Frank plans on running backwards from Grand Forks to Fargo. Fargo is about 60 miles south of Grand Forks. Frank faces south and starts running at an average of 4 MPH. How long until he arrives in Fargo? Trick question = he runs north because he is facing south and couldn't run around the world unless he can run on water.

This student had already turned his assigned word problem in to me and clearly was thinking and working above and beyond the basic expectations of the assignment.

Another student's progress was depicted in her write-up of how a certain graphing concept allowed her to move beyond a point where she had been stuck for some time:

> When I found out about the vertical line test . . . this made functions easy to "see." This was the first time I really "saw" a function . . . [it] helped me understand what it meant by each y has only one x . . . it helped me to literally see and understand how functions work.

Without these journal writings I might have never known that these students had "got it!" The writing process had assisted these students to become what Sipka (1992) called "autonomous learners" (p. 13).

My assessment of student progress often took the form of reading and comparing later to earlier writings, as in the case of the graph descriptions. In this exercise, half of the class

wrote out descriptions of a graph I had sketched on the board while the other half left the room. Upon returning to the classroom, the latter half then sketched the graphs based on their interpretations of their peers' writings. The first graph exercise was conducted prior to any exploration of this topic during class. This exercise (with a different graph) was then repeated after the graphing material was covered in class. The same writers and interpreters were used so as to allow for the assessment of progress regarding the use of correct mathematical terms. The first graph is depicted in Figure 2 below. Some of the written descriptions follow with the writers identified as A, B, and C.

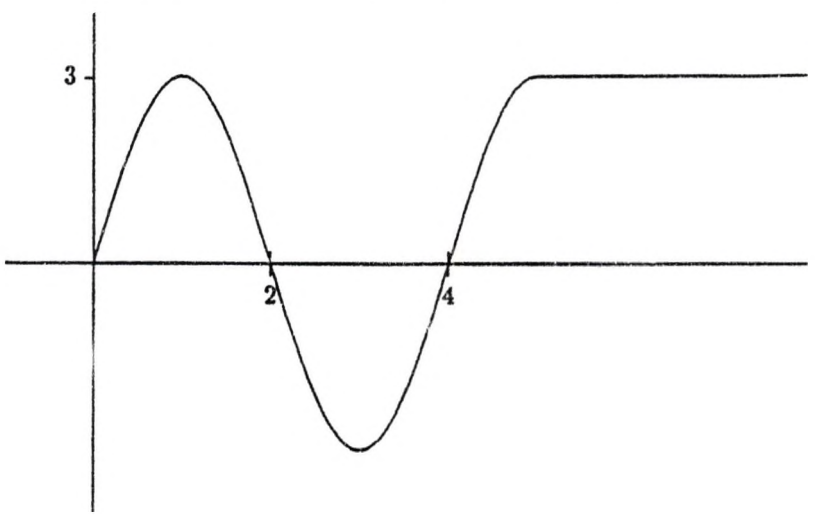


Figure 2. The first graph description exercise.

A: The wavy line starts at the point zero, zero and rises (going horizontally) to a +3 on the y axis, then falling on the x axis to a positive two on the x axis, then back up to a positive four and suddenly goes straight up to a positive four and suddenly goes straight (horizontally) when it hits a 3.

B: Draw a snakelike line (on the right side of the graph) starting at zero, with the top of the first curve up to 3 on the y axis, back down to 2 on the x axis, continuing down to make another curve. Bring it back up to 4 on the same line, continue up to 3 again (on y) and straight out to the right with an arrow.

C: The graph starts at 0,0 and curves up to 1,3. It then curves down crossing at 2,0 and continues down to 3,-3. It then curves back up crossing at 4,0 and continues up to 4,3. This should look similar to a sine wave. It then continues to run parallel to the [illegible] height of 3.

The latter graph descriptions by the same writers illustrate the progress the students had made regarding the language and conceptual understanding of graphs. The second graph is illustrated in Figure 3 with the descriptions following.

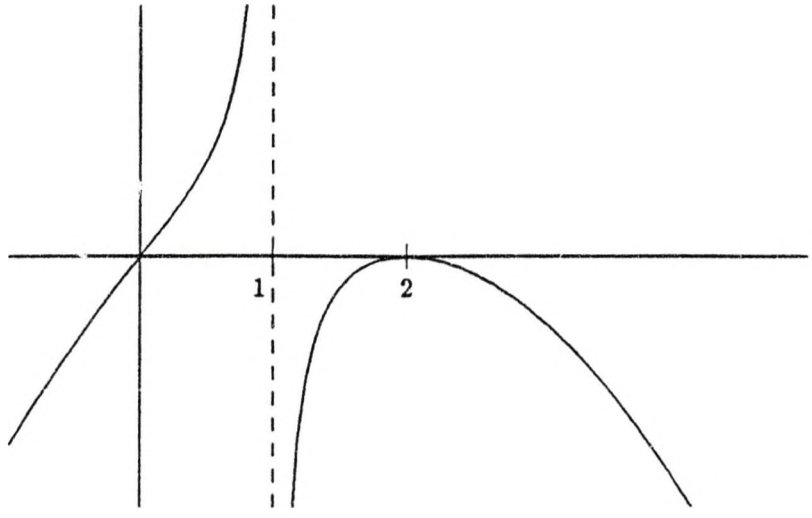


Figure 3. The second graph description exercise.

A: The Vertical Asymptote goes through the point (1,0). With the line that goes through the point (0,0) that curves up to the asymptote but never touching it and does the same on the negative side. There is also a parabola that goes downward from the point (2,0).

B: Draw the graph of a line that crosses the xy axis at (0,0) and is opposite $y = x^3$. Draw the graph of a parabola with the origin at (2,0) and use the equation $x < 0$ to determine the direction of the parabola.

C: You have 2 separate graphs to construct on the same xy plane. The first one is a parabola with a vertex of (2,0) and it goes down. The next one has an asymptote at (0,0) and shoots up towards the asymptote. The graph also shoots down from (0,0).

Although the written graph descriptions depict a certain awkwardness at writing mathematically, the students' descriptions had progressed to the point of using more appropriate mathematical terms. This exercise allowed me to assess in an authentic manner the progression of the students' understanding of graphing concepts.

Through the comparison of each student's earlier and later journal entries, I also was able to assess other areas of mathematical comprehension and learning. The following vignette charts one student's growth in understanding algebra over the semester. Reading through her journal entries made assessing Marci's progress in the course easier.

Marci's Algebra Story

Marci was born in the United States but moved to France when she was very young. A year ago she moved back to the United States and enrolled at this college, taking Intermediate Algebra last Spring. In class she spoke English very well but often had trouble understanding mathematical words and concepts. She wrote about these problems in her autobiography:

> My first memory of math is that I liked it in kindergarten . . . I moved to a different country . . . my new teacher was a good teacher but I was afraid of her . . . I began to have . . . problems with math . . . I never hated math . . . I think that one reason why I had a hard time understanding some problems was that I did not know how to relate them to the "real" world.

As the semester unfolded, she wrote in her journal:

> This was the first week we had problems to do. It scared me . . . and I wondered if I had forgotten everything I learned last semester . . . I have discovered that the

> language confuses me sometimes. They will use a "math-word" that I might not understand and I am not able to "see" the problem.
>
> Last week we had our first test . . . I was not happy about it . . . about the problems I left unfinished . . . I noticed that I needed time to read the problems, and that stole a lot of time.
>
> Last week was a confusing week . . . it seemed like it was so many new formulas to use and sometimes it was hard to know how to use them.
>
> By using functions you can make a graph but the tough part is to find out how to find the right points.
>
> So the line could cross the H.A., but would it be a point . . . or an open circle?
>
> I have been thinking a lot about what or how to study for my next test . . . I will try and start going through the material a few days earlier and make sure I have someone nearby that knows the material . . . I do think my notes are fairly clear and they have helped a lot more than the book.
>
> When I started working on these last word problems I was a little nervous, but that disappeared when I understood that the word problems were actually easy to visualize.
>
> This is the last week of class, and so far it seems like I am getting it . . . I am actually kind of amazed that this semester has gone by so fast, and I have enjoyed it too.

Then, during her exit interview, Marci told the interviewer the following:

> . . . I have improved with algebra and math. I feel more comfortable doing it and I am not as scared any more as I was last semester to try at least . . . I think it has been an experience because it has been a new way to learn math and most often it has made math more fun.

Although Marci's story is unique, as are all of the other student participants' stories, her progress in the course is fairly typical. Writing assisted Marci in the visualization of and creation of relevance for certain mathematical concepts and allowed her to relax and have fun learning algebra.

Student and Teacher Effects

In every classroom, student and teacher relationships take on different forms. At times, connections are made which foster significant learning experiences for both parties. Yet, at other times, disconnections occur where student and teacher relationships become much more complex. Over the semester, several of both types of experiences occurred.

One of the more powerful assessment experiences for both the students and myself was the Small Group Instructional Diagnosis. A member of the SGID team met with my students (without me) and oversaw a small group assessment of the course. Many students wrote positively about this experience:

> I was really quite pleased to see that there was a midterm evaluation, so to speak . . . this type of evaluation was much more effective than the end of the course evaluations . . . you pick up things during the course you like and you don't like.
>
> I had never seen some neutral person come in to the class and evaluate the instructor. It was nice to be able to express what we liked and disliked about your class.
>
> I thought that the session we had with the lady critiquing the class was a very good idea. I think it's good to get the student's perspective . . . in their own words. It also let you know where the students are at and how your teaching methods are affecting the class. If you take those things (I know you will) and put them to use in the class, the class will only get better.

Several students were dismayed at the poor turnout for this session; only 13 of the 27 students were present. I had announced the SGID date to the class and several students chose not to attend. Overall, the students who attended the session seemed pleased to be allowed to offer input regarding the curriculum and instruction of this algebra course. After reviewing the strengths and weaknesses of the course that the students had given (see Appendix C), I was able to make a few improvements in my teaching methods.

Another significant connection being made between the students and myself took the form of sharing. Students would write about personal experiences in their journals, sharing sad or happy events with me. One student wrote:

> My sister helped me study for this test. She is 16 . . . I think she should be a teacher . . . she clearly explained everything . . . and she pays close attention when she watches me . . .

I responded:

> Have you told her this? You should. She sounds great!

Another student shared this news:

> I decided to get married . . . set the date for Oct. 29 of this year. I didn't know that I would be so busy with all the plans . . .

Another student who had cancer wrote:

> Although it isn't the issue it has been in the past, sporadic health problems are still interfering in my attendance. This seems to be getting better, and I see no reason that the improvement should not continue.

Once in awhile, students would share pieces of their lives that were similar to my past and present experiences. I would then disclose relevant facts about myself and my life outside the classroom. One young mother, describing her family, stated that she had two kids, the oldest being seven and she only 25. I shared with her that I also had my son when I was very young. Another student stated that he did not want to be a "suit and tie guy." I, in turn, shared with him that my son had the very same attitude and today is a moderately successful chef. I remember incidents when I, too, realized that my instructors were not so different from me and the way my attitude toward them changed for the better. Journal writing provided an informal forum for disclosure or sharing by both the student and teacher.

The journals were used in many ways by the students. Probably the most significant was that of two-way communication. One student wrote about this dialogue specifically:

> So many teachers these days are so withdrawn from their students. By having these journals, I feel you have opened up a communication between you and your students.

Yet, this form of communication also caused me to read some things which were somewhat uncomfortable. One male student who responded quite often in class and constantly wrote about needing a challenge, chose to use one journal entry as a place to vent his frustration with me:

> Well, I'm really sick of writing these . . . the only thing I have to say is that you need to pay better attention to what you work wrong. On all my assignments there is always something wrong, like you took a point off because I wrote ÷ and not divide. Math is full of symbols, and I don't understand what's wrong with using them in our assignments. Also, I don't like it when you say, "Don't worry if you can't do that, that fast, he's just smart." I don't think I am that smart, I think this is college and we should expect a challenge.

After thinking over my response for one day, I wrote back:

> I apologize if I offended you. The fact does remain that much of the class already feels challenged (at least they say so in their journals.) That was inappropriate of me to say that and I will try not to be condescending.

I also wrote that he should see me about the particular homework mistakes. After this incident, this student seemed to change remarkably. He was less aggressive in class and stopped writing about his need for a challenge. I can only surmise that using the journal to vent was what he needed to move beyond our "disconnection." After this exchange, I did watch what I said in reference to this student in class. During his exit interview, he told the interviewer:

> Pat and I had our moments in class. Always fun that way.

The interviewer asked if he had fun or disagreements with the instructor. He responded that disagreements was "a better way to put it." I realized from this experience that the journal writing was an excellent forum for allowing students to communicate openly, but along with that aspect came the opening up of myself to criticism and painful awarenesses of some of my poor teaching practices.

Nonetheless, many of the students wrote entries describing a feeling of closeness they were experiencing in the classroom:

> We got to know each other and became friends with each other.
>
> Having people in class help you with a question helps so much.
>
> . . . you can communicate with classmates and get helpful hints from them.
>
> I sensed a little of the comraderie that I remember from ninth grade here. Maybe its the algebra or maybe the small classroom.

A sense of community seemed to evolve over the term as students formed study groups and developed close connections with each other. Two females told me they had become best friends since the class. The small group writing exercises helped bring about the friendly atmosphere of the classroom, although other factors such as the students' personalities certainly contributed to the development of this community of learners. The act of communicating on paper served to encourage and foster more classroom communication and discourse, a fact that Rose (1992) also discovered in her study on writing in the mathematics classroom. Moreover, because students became part of the construction of the instructional format of the course, they had a vested interest in the course, much the same way people feel about their communities.

Summary

The purpose of this study was to explore how writing in this college algebra course affected the learning experiences of this group of students. The effects of this intensive writing program upon the individual participants were many and varied. However, there were several major outcomes that held true for most of the participants of the study:

1. Writing promoted student comprehension of mathematics.
2. Writing facilitated students in making personal meaning of mathematical concepts.
3. Writing became a vehicle for dialogue between student and teacher.
4. Writing allowed for student reflection regarding the learning of mathematics.
5. Writing fostered teacher analysis of the rhythms of learning mathematics.
6. Writing brought moments of clarity and of genius.
7. Writing permitted authentic and alternative assessment.
8. Writing provided a forum for disclosure for both student and teacher.
9. Writing assisted in the development of a community of learners by encouraging classroom discourse.
10. Writing caused disadvantages for some students and the teacher.

One student perhaps summarized best of all:

> It made me look at math in a different way because I have never had a course where we had to write, but I liked it.

The last chapter, Chapter VI, describes the findings including the advantages and disadvantages of the writing to learn process and presents the conclusions of the study.

Several recommendations also are given for the inclusion of writing in the mathematics classroom, the replication of this study, and further research.

CHAPTER VI

FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

> The goal of education-the goal of
> good teachers everywhere-is to go
> beyond where we have gone.
> Willoughby, 1990, p. 5

The purpose of this study was to explore how writing in a college algebra course affected the learning experiences of a group of undergraduate college students. The methodology employed in the study was phenomenological research, a form of qualitative inquiry which utilizes interactive techniques to discover the meaning of specific phenomena. Data for the study were collected from student and teacher writings, observations, and interviews. The data were then coded and categorized to uncover the patterns salient to the study. Several findings emerged from these patterns.

This chapter begins with a summary and discussion of these findings. The next section presents the conclusions of the study. The chapter concludes by giving several recommendations regarding the inclusion of writing in the mathematics classroom as a form of curricular or pedagogical reform, the replication of this study, and avenues for further research.

Findings

From the data collected for this study, several findings surfaced. Most of the findings regarding the inclusion of writing in a college algebra course were positive; yet, a few disadvantages were noted. Nine benefits are presented and discussed in the following section. The section concludes with a discussion of the disadvantages experienced by some of the participants of the study.

Writing Promoted Student Comprehension of Mathematics

The journal entries and other writing to learn exercises promoted student understanding of certain mathematical concepts by allowing the writer to think on paper. The student writers began to think about mathematics in ways that were often quite different from how they had thought in the past, a fact supported by several students' writings on functions. The writings also forced the students to think more deeply about mathematics. By writing out processes and problem solving steps, the students' attention focused on understanding and conceptualization and, at least temporarily, off memorizing and reciting. Writing also facilitated student retention of mathematical concepts. This finding was supported by students' comments to both myself and the outside interviewer and was important because some memorization is necessary for mathematical learning to take place. Overall, this study reaffirmed the literature citing writing as a means to reinforce and improve the writer's critical thinking skills (Fulwiler, 1986; Sipka, 1992).

Writing Facilitated Students in Making Personal Meaning of Mathematical Concepts

Repeatedly, the students were asked to write out definitions of mathematical concepts such as a circle or a function. Through these writings, the writers began to construct a

personal meaning for abstract mathematical ideas. In 1992 Rose described the writing in mathematics process as students using their own vocabularies to construct for each a "very individualized meaning, like an inside joke or a personal translation" (p. 65). In the study, it became clear through their written constructions that the students began to sense ownership in the content. An example of this ownership was illustrated by the chemistry major's reference to his future use of algebra in measuring medicine when he becomes a doctor. Other support for this finding occurred when students explored mathematical problems of their own making in their journal entries.

Writing Became a Vehicle for Dialogue Between Student and Teacher

In their journal entries, students would address questions to me concerning course content, and I would respond in writing. The reasons for this written exchange included time constraints, students' lack of courage to ask questions in class, and the fact that questions often occurred after class when students were working on homework. This exchange fostered a positive rapport between teacher and students as evidenced by the relaxed and candid nature of many students' journal entries. The journals also became a forum for students to ask questions and give suggestions or comments regarding the course, curriculum, or instruction. Because of this continuous pedagogical feedback and discussion via the journal, I believe I became a better, more aware teacher.

Writing Allowed for Student Reflection Regarding the Learning of Mathematics

The acts of reflection and self-reflection caused most of the students to think more about the learning of mathematics than they ever had before. This form of metacognition helped create more autonomous and self-directed learners, a finding Kenney (1992) also

discovered in his study of students writing in mathematics. By the end of the course, many students displayed a self-awareness and self-knowledge about their abilities to learn and communicate mathematics. Examples of this self-knowledge were found in several compelling journal entries where students described themselves either positively or negatively as learners of mathematics. Students also wrote about their highs and lows regarding learning mathematics which appeared to intensify the positive and dispel the negative.

Writing Fostered Teacher Analysis of the Rhythms of Learning Mathematics

As students wrote of their struggles with and victories in the learning of algebra, I began to appreciate the highs, lows, and plateaus experienced by these learners. Brookfield (1990) described the rhythm of learning as a "transitional mambo" encompassing "two steps forward, one step back" (p. 52). He suggested that an understanding of the effects that outside issues and occurrences have on student learning could help bring about alternative and more effective modes of instruction. In some ways, all students have nontraditional or abnormal obstacles to overcome along their academic paths. Moreover, the rhythms of learning which occur in the mathematics classroom also have substantial effects on the quality of each student's comprehension. Only after I apologized for singling out and embarrassing one of the students did this student become an attentive and less critical learner. The rhythms of learning permeate the students' lives. The phenomenal part of this discovery of rhythmic saturation is that it is different for each student.

Writing Brought Moments of Clarity or Genius

The forum of the journal brought forward dramatic moments of clarity and of genius I would otherwise have missed. One of the highs of teaching is seeing when students finally

"get it!" Through the journal entries, I entered the students' thoughts. I also witnessed moments when the student became both teacher and learner, for example when a student would compose and solve an original mathematical problem.

Writing Permitted Authentic and Alternative Assessment

The students of this course were assigned grades earned from four exams and several homework and writing assignments. Of the 27 students who completed the course, 13 received A's, nine received B's, and five received C's. (Two of the nine who quit coming to class received F's. The other seven had dropped the course before the drop date.) However, nontraditional and ungraded assessment also took place in this classroom. Through the comparison of early and later writings, I was able to assess the students' changing comprehensions of certain mathematical concepts. The graph descriptions provided one example of such assessment; in this exercise, the mathematical vocabulary improved from the first to the second writing of graph descriptions. Also, by reading journal entries, I discovered the depth of the students' understanding of concepts such as functions and logarithms.

Writing Provided a Forum for Disclosure for Both Student and Teacher

John Dewey (1990) wrote that there must be disclosure for learning to take place. The journals in this course became safe places for students to share parts of their lives that they might otherwise have kept to themselves. Moreover, students expressed themselves in more personal ways than classroom discourse allowed. This disclosure helped me to achieve a more holistic understanding of each student. Because I believe that the act of teaching involves the giving away of self, I was also able to share parts of my life that were relevant

to the students' writings in this forum. This two-way sharing helped create an atmosphere of trust and freedom which added positively to the research and to the teaching and learning of algebra.

Writing Assisted in the Development of a Community of Learners by Encouraging Classroom Discourse

The writing in the classroom lent itself so neatly to collaboration that often students would work in small groups, both in and out of class, writing, creating, solving, or discussing mathematical problems. The act of communicating on paper also served to encourage and promote classroom communication, paralleling findings of other similar studies (Kenney, 1992; Maher, 1992). This particular classroom was always alive with noisy learning and discourse about mathematics and writing. Moreover, several students formed strong friendships with their peers, a fact supported by students' comments to myself and the outside interviewer.

The writing exercises assigned each class day generated much classroom discussion. As I wrote out processes and methods of operation on the board, students would suggest steps and variations to other steps. When students realized I was interested in their feedback, they became very free with their comments about the course. The sense of being a part of mathematics education reform also helped develop a sense of community in the classroom. Students expressed joint ownership in learning by sharing their thoughts and opinions on the classroom activities.

Writing Caused Disadvantages for Some Students and the Teacher

The benefits to both students and teacher were many; yet, there were also some drawbacks to the inclusion of writing within this college algebra course. The disadvantages included student resistance, grading policy concerns, teacher isolation, and time inefficiently spent by both students as writers and teacher as reader.

There were two students who indicated that they felt the writing was useless to their understanding of mathematical content. One of these students said he did not mind writing in the journal because it gave him "free points." He clearly thought the inclusion of writing in this course was frivolous and did not constitute learning. In his case, the points he earned from the journal writing brought his grade from a much deserved "D" to a "C." The grading of the journals was structured so that points were earned for volume, not content. I realize now that a more efficient use of journals should include a consideration of content.

The other student expressed that she felt we could have covered more material if we had not spent time writing about "stuff." I agreed with her that writing in the mathematics classroom does take time from other activities and expressed my opinion that time is well spent on forcing conceptualization over memorization. However, not all students want to learn that way and not every student reaps benefits from the same instructional format. Different learning styles must be considered in every teaching situation.

I came to the realization that *intensive* writing in the mathematics classroom is not necessarily the most effective avenue to successful learning. Less writing may be better than more. Several different writing activities, but not daily writing activities, would have utilized classroom time more efficiently. More time could then have been spent on the more effective

writing exercises such as crib sheet development and small group explorations and reports. One other disadvantage I discovered was that I often felt isolated in my research because only a few of the mathematics instructors who were readily accessible were including writing in their mathematical instruction.

The amount of time I spent reading and responding to all of the students' writing was substantial. I believe this was time well spent; yet, I would not have time to read and respond in this manner for more than one or two writing intensive mathematics classes per term. The time required of teachers to create and conduct writing intensive mathematics courses is a major drawback to this type of teaching and learning methodology. The solution to this problem would include making judicious choices regarding the types and amount of writing exercises used in mathematics courses.

In summary, several benefits and some disadvantages from the writing to learn activities were realized by the students and the teacher of the course. The data show that the inclusion of writing within the college algebra course generally brought about productive mathematics learning for most of the student participants of the study.

Conclusions

Writing may be used effectively to support mathematics instruction. Writing in mathematics, however, must not be considered a remedy for all of the maladies associated with mathematics education. The curricular and pedagogical reform called for by the National Council of Teachers of Mathematics (1989) and other mathematics educators should include writing to learn, as well as several other instructional techniques, such as short lectures, small group work, and discovery projects. This study suggests that chalk-and-talk should not be

the only medium of instruction in any educator's portfolio. Writing to learn as an active form of mathematics learning is clearly a positive move away from the passivity I have witnessed produced by the overuse of the lecture format of instruction.

A balance of conceptualization and computation is needed for mathematical learning to take place. The student's focus must come off memorization as the primary learning tool in mathematics; yet, some memorization in beginning courses is clearly necessary for the learning and future use of mathematical ideas. As I discovered in this study, writing not only supports conceptualization and practice but also facilitates student retention of mathematical concepts.

The results of this study, while not generalizable, were revealing in terms of the benefits and drawbacks one group of undergraduates saw in a writing intensive college algebra course. Students enter mathematics classrooms with different learning styles and many with deeply ingrained learning habits. Because of these differences, writing to learn should not be used as the primary approach to learning mathematics. Writing to learn exercises must be selected with the needs of a particular group of learners constantly in mind. For example, the participants of this study responded well to the use of journal writing as a forum for self-reflection and question posing.

Including writing in the mathematics curriculum was a rewarding endeavor. The reader and writers both benefitted from the experience: the reader learned more about the student writers, and the writers acquired another forum for mathematical growth and communication.

Writing promotes several critical thinking skills fundamental to the learning of any subject. Although caution must be taken when deciding what and how much writing to include in particular courses and how to assess that writing, writing to learn should be considered an important tool in mathematics education. I will certainly continue to use some writing in all of my mathematics instruction.

Recommendations

This section gives recommendations for the use of writing to learn mathematics exercises as a tool for the teaching and learning of mathematics in both secondary and postsecondary institutions. The section concludes with recommendations for the replication of this study and recommendations for further research regarding writing in mathematics.

Recommendations for Teachers

Based on the findings of this study, several recommendations for the inclusion of writing to learn as an instructional tool are made. When planning and implementing writing within a mathematics course either at the college or secondary level, instructors should do the following:

- plan to teach only one or two courses per term which include extensive writing activities;
- plan extra time for reading and grading writing assignments;
- consider the needs of the particular group of students in the course;
- develop a criteria-based assessment process;
- develop clear objectives for each writing exercise;
- expect resistance from some students;

- use a variety of writing exercises;
- develop small group, and individual writing activities;
- structure writing exercises that focus on the conceptualization rather than the memorization of concepts;
- borrow, share, and discuss ideas with other mathematics educators; and
- anticipate the fact that some writing activities will be successful and others will not.

Writing to learn mathematics should be considered a significant form of mathematics education reform. I caution instructors, however, not to consider this methodology as the only agent for instructional change. I recommend the use of a variety of instructional techniques (small group work, lecture and demonstration, peer teach, and writing to learn) when attempting to enhance the teaching and learning of mathematics.

Recommendations for the Replication of this Study

Several changes are recommended if a similar study were undertaken. Steps could be taken to increase validity by reducing the role of the instructor as sole collector of data. Videotaping or having an unbiased observer view the class might be incorporated as a check on instructor perception. All of the exit interviews could be conducted by someone other than the instructor of the course. The data gleaned from these less subjective efforts could then be used to confirm or negate the findings of the study.

To understand more fully the effects of writing on mathematical learning, a follow up study of the student participants might be undertaken one or two semesters after the course. Several questions could be posed to these students: How do they feel about the experience

of writing in a mathematics course now? Are they using any of the writing exercises in their mathematical studies today? What do they remember about the experience? How would they describe themselves today as learners of mathematics? Is this description different from their initial writing on this subject?

The structure of the mathematics journal could also be changed. I recommend a more structured format where each entry includes specific components, such as a summary of the week's lessons, a write-up of one algebraic problem, and any thoughts or questions about the course or coursework. The assessment process could also be amended based on the successful completion of related criteria.

Other writing assignments might include at least one long paper exploring a mathematical concept or procedure. The writings in the course might then have a greater effect on the assessment outcome. I strongly recommend the use of other mathematics instructors as secondary readers of these writings, allowing for a greater degree of objectivity to the grading.

Although there are several changes I would make if I were to repeat this study, the experience is one I would recommend for any mathematics instructor. The knowledge I gained about how students understand and learn mathematics proved invaluable to my growth as an educator.

Recommendations for Further Research

Since the 1970s, several excellent studies have been conducted regarding writing in mathematics. However, many unanswered questions about writing in mathematics as an

instructional tool and as a form of pedagogical and curricular reform are unanswered. The following research questions are offered as avenues for further exploration:

- What are longitudinal effects on writing in the mathematics classroom?
- Does success in a writing intensive mathematics course assure later success in a traditional mathematics course?
- What intrinsic effects does writing in mathematics have on the students?
- What types of students do better in writing to learn mathematics courses than traditional mathematics courses?
- Does writing in mathematics stimulate visualization of mathematical concepts?

For those mathematics educators interested in effecting change in how students learn mathematics, I recommend exploring any of the questions above. Through the words and voices of the student participants of this study, I discovered that only through continuous inquiry into how students view and understand mathematics will the quality of mathematics education improve.

APPENDIX A

COLLEGE ALGEBRA

Fall 1994

Pat Mower (instructor)
office: Witmer 304A & CTL 101E
phone: 777-3613 or 777-2881
office hours: M & W at 2-3 & F at 12-1 Witmer 303

$$ax^2 + bx + c = 0$$

OVERVIEW OF THE COURSE:

College Algebra is an algebra course which prepares you for using mathematics and algebra in higher level mathematics courses, business or science courses, and life in general This class in particular is an interdisciplinary course which integrates writing throughout the algebra to improve and expand your ability to conceptualize and communicate mathematically.

COURSE PHILOSOPHY:

The mission of this course is to bring meaning and value to your study of algebra through the integration of several critical skills.

GRADING POLICIES:

4 exams (100 pts. each)	= 400
10 Assignments (20 pts. each)	= 200
Journal entries (5 pts. each)	= 50
TOTAL POINTS	= 650

TEXT:

College Algebra, 2nd. ed.
Aufmann, Barker, & Nation

COLLEGE ALGEBRA

TENTATIVE SCHEDULE:

WEEK 1–5	INTRO MATH AUTOBIOGRAPHY CHPTS. 1–3.2 EXAM 1 SEPT. 26
WEEK 5–9	CHPTS. 3.3–4 EXAM 2 OCT. 24
WEEK 9–12	CHPTS. 5–6 EXAM 3 NOV. 16
WEEK 12–16	CHPTS. 7–9 EXAM 4 DEC. 12
FINAL INTERVIEWS	DEC. 15 THURS. 8:00

ASSIGNMENTS: 12 homework and in-class assignments will be given. The lowest 2 scores earned will automatically be dropped. These assignments will include computation and writing to learn problems. Practice problems will also be assigned most classdays. I encourage you attempt all problems and to ask questions when needed.

JOURNALS: At the end of most weeks, I will ask you to write in a journal about your progress in and thoughts about this course. Each entry should be at least one page, but not more than two pages in length. The style will be freewriting and I will not be grading you on grammar and/or spelling. However, I urge you to be clear as your thoughts are very important to me.
Each entry is worth 5 points. 13 entries will be assigned, but only 10 entries will count for credit.

COMMENTS:

Most of the learning in this course takes place during class hours, so you must be present to learn!

Please feel free to discuss with me any aspect of this course.

COLLEGE ALGEBRA

MATH JOURNAL

DESCRIPTION:

The math journal is a collection of your writings exploring your thoughts regarding this College Algebra course. Your writings will be an important part of the ongoing struggle to improve the teaching of undergraduate mathematics.

FORMAT:

At the end of most weeks throughout the Fall Semester, you will write in a notebook provided solely for this purpose about this course. Each entry should be at least one page, but not more than 2 pages in length. The writing style will be freewriting, grammar and spelling will not be graded. You will be allowed 10-15 minutes of classtime to begin or finish each entry. The journal will be handed in each Monday and returned to you each Wednesday.

I encourage you to use a double-sided format: the left page for spontaneous comments throughout the week and the right page for the actual journal entry.

GRADING:

13 entries will be assigned. Each entry, up to 10 entries, will be worth 5 points. 50 points are possible.

CONTENT:

Write whatever comes to mind regarding your progress, struggles, accomplishments, anxieties, experiences, etc. in this class. Consider this your opportunity to have a say in how mathematics should be taught or presented to students, or consider this a chance to have a one-on-one dialogue with your math teacher, me. Your thoughts on this course and on math in general are invaluable to the process of improving the teaching and learning of College Algebra. So please feel free to express your opinions. You will not be penalized in anyway for your honesty. In fact, I look forward to constructive criticism!

A list of suggested topics and questions follows for the days you need inspiration. Your first journal entry will be your math autobiography and will be the only assigned topic of the journal, details follow.

POSSIBLE JOURNAL ENTRY TOPICS & QUESTIONS

**What problems did you encounter in class this week?

**Write about your strengths and weaknesses in this course or in math in general.

**How do feel about writing in a math course?

**Write about what you learned in class today or this week for an absent student?

**Evaluate your progress in this course so far.

**Describe some concept you have learned in this class to a friend's child.

**Describe your feelings about the study of mathematics.

**Write about what you see as the value of the study of mathematics or about its lack of value.

JOURNAL ENTRY #1 MATH AUTOBIOGRAPHY

Write a 1-2 page description of your past experiences, good, bad or indifferent, regarding mathematics or numbers in general. Include math courses and/or teachers you remember, anxieties or successes in math, family members or friends who have had some influence over your math ability or interest, or any other experience or thought you have had regarding mathematics.

APPENDIX B

College Algebra

Exam # 1 Name_____

Solve the following equations:
6 points each
Show all work!!

1. $3x - (3 - 4x) = x + 2(3 - x)$

2. $\frac{3}{x+2} + \frac{7}{x+2} = \frac{5}{x-2}$

3. $y^2 - 8y - 26 = 0$

4. $2x^2 + 5x = 3$

5. $(2a - 5)^{1/3} = 3$

College Algebra

Exam # 1

The following statements are false. Choose 2 of the 3 and circle the numbers of the statements you select. Then write out thoroughly why each is incorrect or give an example with a written explanation showing why the statement is false. 5 points each.

6. A positive discriminant means there are 2 rational solutions.

7. The x intercept of a graph in the xy plane might be the point (0,3).

8. It is unnecessary to check the solutions for fractional exponential equations.

9. Solve: $\sqrt{x + 1} - 3 = \sqrt{x + 4}$ (6 points)

College Algebra

Exam # 1

10. The perimeter of a 4 sided figure is 108 feet. The shortest side is one-half the longest side. The other two sides are each four more than the shortest side. Find the lengths of all four sides. 8 points

Solve the following inequalities. State the solutions in interval notation. 6 points each

11. $-2 < 5x + 3 \leq 10$

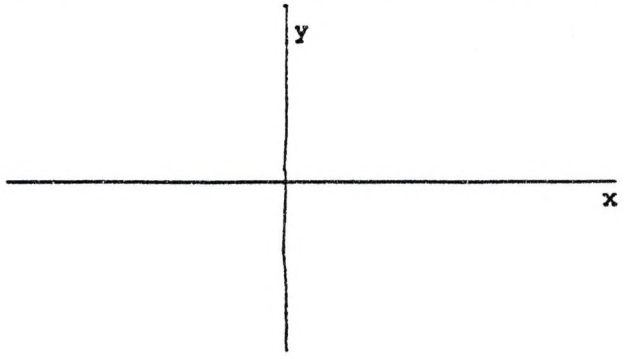
12. $|4 - 2x| > 8$

13. $\frac{x + 3}{x - 2} \leq 0$

College Algebra

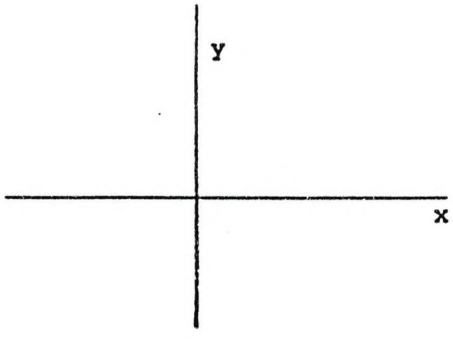
Exam # 1

14. Draw the graph described below in the xy plane: (6 points)
    a) When y = 0, x = 4.
    b) When x = 0, y = 2 and -2.
    c) The shape of the curve is a parabola.
    d) The curve is symmetric with respect to the x axis.

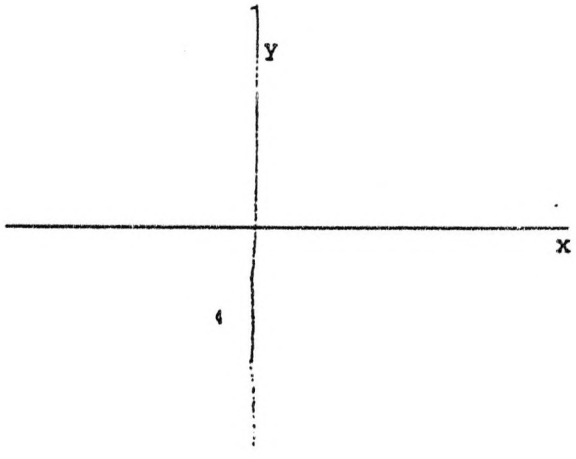


Graph the following, showing all major points. 6 points each

15. $5y = 2x - 5$



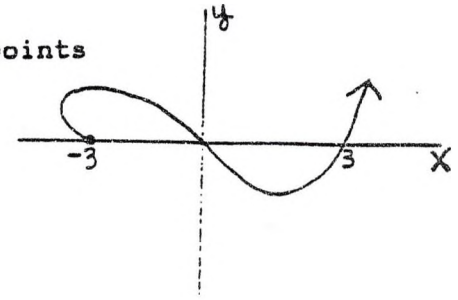
16. $y = |x + 2|$



college algebra

Exam # 2 *Name*_____

1. Is the following a graph of a function? Write out precisely why or why not. 4 points



2. Find the domain for the following functions: 6 points each

a) $f(x) = x^2 - 1$

b) $g(x) = \dfrac{\sqrt{7x - 5}}{x + 2}$

3. Given $f(x) = \sqrt{x} + 1$, $g(x) = x - 1$, and $h(x) = x^3 - 1$, find: 6 points each

a) $(h/g)(x)$

b) $(f \circ g)(x)$

college algebra

Exam # 2 *Name*_____

4. Find an equation for the line that passes through the points (-2,4) and (1,8). Write in slope intercept form. 8 points

5. Find an equation for the line that passes through (3,0) and is perpendicular to the line with equation $2x - 4y = 8$.
8 points

6. Graph the following function, showing all major points.
8 points
$f(x) = 2x^2 - 4x - 16$

college algebra

Exam # 2 *Name*_____

7. Sketch a rough graph for the following polynomial function, showing all major points: 12 points

$$P(x) = (2x - 1)(x - 2)(x + 2)$$

8. Demonstrate 2 ways to show that 3 is NOT a zero of $P(x) = x^3 + 2x^2 - 4x + 1$. 6 points

9. Define clearly in your own words the concept of a zero of a function.
4 points

college algebra

Exam # 2 *Name*_____

10. Using synthetic division, find all the solutions when P(x) = 0 for

$$P(x) = 2x^4 + 2x^3 + 2x^2 + 6x - 12$$

a) List these solutions. 3 points
b) Write P in linear factored form. 8 points

11. Use Descartes rule to construct a chart which shows all possible amounts of positive or negative rational and complex solutions. 10 points

BONUS 3 POINTS Give the equation for the circle with center (0,0) and radius -3.

College Algebra NAME _____

Pat Mower Page 1

No graphing calculators. Hand this page in when finished and pick up the rest of the exam. Show all asymptotes and major points on the graph! 10 points each.

1. $y = \dfrac{x+3}{2x+4}$

2. $y = log_2 |x|$

3. $f(x) = 4^{-x}$

College Algebra NAME _____

Pat Mower Page 2

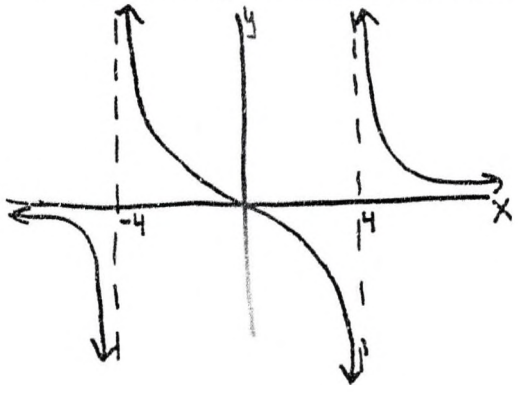
4. **The following are false statements. Rewrite each statement correctly. 3 pts each.**

a) The quotient of 2 logs is equal to the log of a difference.

b) A function can only have 1 or 2 vertical asymptotes.

5. Find log_2 10 using a change of base to base 10. Round to 100th's place. Show all work. 6 pts.

6. List 3 major features of the following graph, using the appropriate math terms. 6pts.



Show all work for credit! 7pts. each.

Solve the following for x:

7. $log_3(x^2+2) = log_3\ 3 + log_3\ x$

8. $log.01 = x$

9. $3^x = 20$ (round to 100th's place)

10. $ln\ e - ln\ 1 = x$

11. $log_x^{25^2} = 2$

12. $log_b(-2-x) = log_b\ x$

BONUS Solve for x: $log_2 5 = log(-x)$ 3pts.

College Algebra

Final Exam Name_____

Solve the following systems of equations completely:

1. $\begin{cases} 4x - 10y = 12 \\ 2x - 5y = 5 \end{cases}$ 8 pts.

2. $\begin{cases} x + 2y - 3z = 0 \\ 5x - y + 9z = 14 \\ 2x + 4y - 6z = 0 \end{cases}$ 10 pts.

3. Choose 2 of the 3 following terms to define: 3 points each
   a) determinant

   b) inconsistent system

   c) cofactor

College Algebra

Final Exam Name_____

4. Solve this system of equations using Cramer's Rule.

$$\begin{cases} 3x - 7y = 1 \\ 5x + 6y = 2 \end{cases}$$

8 pts.

Given $A = \begin{bmatrix} 1 & 2 & -3 \\ -1 & 0 & 5 \\ 0 & 6 & 8 \end{bmatrix}$, find the following:

5. M_{23} 4 pts.

6. C_{12} 6 pts.

7. $|A|$ 8 pts.

College Algebra

Final Exam Name_____

8. Find the first 3 terms and the 10th term of the sequence with 8 pts.

$$a_n = \frac{n + 1}{n}$$

9. Find the first four terms of the recursively defined sequence: 8 pts.

$$a_1 = 1, \quad a_n = na_{n-1}$$

Find the following: 6 pts. each

10. $\frac{9!}{6!3!}$

11. $\sum_{i=1}^{5} (i^2 - 1)$

College Algebra

Final Exam Name_____

12. Rewrite this statement using the same words so that it is true: 4 pts.
A sequence is a sum of an ordered list of numbers.

Choose 2 of the 3 equations to solve: 9 pts. each

13. $x^2 - 4x = 12$

15. $\sqrt{x + 2} = x$

16. $\log x - \log 2x = \log 3x$

BONUS: 3 POINTS Solve for x: $\sum_{i=1}^{x} (i + 5) = 30$

APPENDIX C

To: Pat Mower
From: Victoria Beard
Date: 30 October 1004
Re: SGID

At your request, on Friday, October 28, 1994, I met your College Algebra class, Math 103, in Witmer 117, for a Small Group Instructional Diagnosis. Thirteen students were in attendance. We will meet on Monday, October 31, at 10:00 a.m. to discuss the following results.

STRENGTHS

TEACHER

-very personable
-easy to listen to and talk to outside of class
-makes herself accessible
-gives students individual attention
-strives to make sure everyone knows the material

CLASSROOM MANAGEMENT

-covers everything in class
-gives time to absorb material presented in class
-review days help to prepare for material on test
-good that new material does not infringe on review period
-good teaching speed

CONTENT

-good background for other courses
-goes over material that isn't even on the test

GROUP WORK

-encourages group work
-gives us time in class to get together in groups

GRADING

-grading seems very fair
-gives partial credit for wrong problems on test, homework
-gives chances to earn extra points

HOMEWORK

-not excessive amounts of homework
-assigns helpful homework

JOURNALS

-journals help us communicate with the teacher
-like journals: one on one with teacher!
-another way to ask questions

AREAS FOR IMPROVEMENT

TEXT
- -make better use of text
- -teach us how to learn from the book
- -reference text to class presentation, both for missing class and in general

SOLUTION MANUAL
- -get new solution manual
- -solution manual leaves out some steps

ADJUSTMENTS FOR DIFFERING SKILL LEVELS
- -don't assume that we already know any material
- -students have different backgrounds: some need more depth/explanation
- -assumes that some things are a review but some of us haven't seen it before

EXPLANATIONS
- -only shows one way to do problem
- -simplifies things too quickly; should work problems out completely
- -better explanations of material with more examples

TIME MANAGEMENT
- -not enough time to ask questions in class until after we do the problems
- -allow time (15 minutes) to ask questions at end of each class

PACE
- -slow down the pace a bit
- -she is very knowledgeable, but maybe forgets what it is like to be learning the information for the first time

TESTS
- -not enough time allowed for completion of problems on tests
- -should list formulas on the test; provide a formula sheet
- -leave more time at end of test for problems

JOURNALS
- -the journal entries don't really help us
- -assign the journals perhaps every two weeks, not every week

APPENDIX D

INTERVIEW QUESTIONS

1. Tell me a little about yourself. Who are you? What is your major and/or career plans? What math courses have you had before? Do you like math?

2. How would you rate this math course (103) compared to the other math courses you have taken? Are there any adjectives you can think of to describe this course?

3. Did you find the writing assignments and/or exercises helpful to your learning of algebra? In what way? In particular, did the journal entries help you in learning and or any other way? Were there any writing assignments that you especially liked or did not like?

4. How does this course relate to your life today or in the future? Will you use anything you learned over the semester in your life or future life's work? How?

5. Did your algebraic skills or understanding improve over the semester? How? How does this improvement compare to your earlier efforts in learning math?

6. Tell me one think you remember today that you learned well in this course.

APPENDIX E

FINAL JOURNAL ENTRY

The following is a list of some of the different writing exercises we used over the semester. Please comment on the usefulness or lack of usefulness and/or your reaction to any or all of these activities.

Journal Entries

Math Autobiography

Crib Sheets

Writing out of processes

Writing out of particular solutions

Muddiest points

One-minute summary

Log qualities

Creation of word problems

Defining concepts

Explanation of what is wrong with a false statement

[You may use the back for any final comments.]

College Algebra
Your facts re. the log function

1. The basic logarithmic form is the inverse of the basic exponential function.

2. A log is an exponent.

3. The logarithmic function f with base b is defined by $f(x) = \log_b x$.

4. The base of a log must be greater than zero and can not be equal to one.

5. In the form $\log_b x$, x can't be a negative number.

6. In the form $\log_b x$, x is called the argument, b is the base, and y is the logarithm.

7. The log of one is always equal to zero.

8. If the argument of a log is the same as its base, then the log is equal to one.

9. f [$f(x) = \log_b x$] is a one-to-one function.

10. f has a graph with an x intercept of (1,0).

11. f has the set of real numbers as its range.

12. The log of a product is equal to the sum of the logs.

13. The log of a quotient is equal to the difference of the logs.

14. The log of a number raised to a power is equal to the product of the power and the log.

15. Common logarithms are logarithms with a base of ten.

16. Natural logarithms are logarithms with a base of e.

17. If the bases of 2 logarithms are the same and the logarithms are equal, then the arguments are equal.

18. If the base b is a positive real number and b to x power equals b to y power, then x equals y.

19. y equals $\log_b x$ is logarithmic form of $b^y = x$.

20. If x, a, & b are positive real numbers with a & b not equal to one, then $\log_b x$ is equal to $\frac{\log_a x}{\log_a b}$

APPENDIX G

Writing to Learn College Algebra

Consent Form

The goal of this study is to expand the understanding of how students make meaning of mathematics through writing.

You are invited to participate in a study of writing to learn College Algebra. We hope to learn how writing about mathematics and writing to learn mathematics facilitates learning. The study will take place over Fall Semester 1994 in this section of College Algebra. You will be asked to write weekly in a math journal about your progress in the course and will earn a certain amount of points toward your final grade. The journal entries will be photocopied to be used as data for this study. You will be asked to write only about subjects as they relate to this course and not about other aspects of your life. You will also be asked to write out mathematical processes and concepts in words on a regular basis. Three class sessions will be videotaped over the semester. At midterm you will take part in an evaluation session with someone other than me to determine how the class is going. I will not be able to identify you from the session report. At the end of the semester, after grades are turned in, you will be interviewed for approximately one half hour where again you will assess the course. These interviews will be tape recorded. All writings, audiotapes, and videotapes will be destroyed within one year of the final written results of the study. At all times, your confidentiality will be honored. Your real name will not be used in the research reports or subsequent journal articles.

If you choose not to participate in this study, you will want to drop the course and enroll in another section of College Algebra. If all other sections are full, you will be allowed to sit in a class until a place becomes available. Refusal to participate in this study will not hurt your relationship with the instructor or any other member of the math department.

Pat Mower
Box 7189
Center for Teaching and Learning
University of North Dakota
777-3613

****

All of my questions have been answered and I am encouraged to ask any questions that I may have concerning this study in the future. I have read all of the above and willingly agree to participate in

this study explained to me by_____

_____

Student Signature Date

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