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# OPTIMIZATION OF THE GARCH MODEL PARAMETERS USING A GENETIC ALGORITHM

by

Jonathon Patrick Cummings

A Thesis

Submitted to the Graduate

Faculty of the University of

North Dakota in partial

fulfillment of the

requirements

for the degree of

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Science

Grand Forks, North Dakota
August
2013

This thesis, submitted by Jonathon P. Cummings in partial fulfillment of the requirements for the Degree of Master of Science in Applied Economics from the University of North Dakota, has been read by the Faculty Advisory Committee under whom the work has been done and is hereby approved.

Dr. David T. Flynn, Chairperson

Dr. Pradosh Simlai

Dr. Daniel Biederman

This thesis is being submitted by the appointed advisory committee as having met all of the requirements of the Graduate School at the University of North Dakota and is hereby approved.

Dr. Wayne Swisher

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Jonathon P. Cummings
August 2013

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## ABSTRACT

Financial time series are often characterized by nonlinearity and volatility bunching. Standard regression analysis models cannot capture changing volatilities, potentially leading to erroneous results. The need to more completely model the characteristic volatilities inherent to financial time series eventually led to the creation of the GARCH model. Typical GARCH parameters are (1,1) incorporating a 1-period lag of the regression residual as well as a 1-period lag of the regression volatility. The primary question investigated in this paper is whether the typical GARCH(1,1) parameters are in fact optimal over all time periods and attempts to improve on the typical parameters by minimizing a modified AIC value using a genetic algorithm.

# CHAPTER I

# INTRODUCTION

The desire to understand the nature and characteristics of time series data has led to the development of a variety of models. Models that focus on the character of the data, or some derivative of the data, range from the basic moving average (MA) to the more complicated autoregressive moving average (ARMA). These models attempt to understand the nature of the data itself versus some other characteristic of the data, such as volatility. Other models, such as the autoregressive conditional heteroskedasticity (ARCH) created by Robert Engle, do indeed focus on the underlying volatility by examining the lag structure of the squared residuals.[1] However, ARCH remained unsatisfactory because it neglected to incorporate any additional regression term that allowed for the persistence of economic shocks.

Finally in 1986, Tim Peter Bollerslev presented the GARCH model for Generalized Autoregressive Conditional Heteroskedasticity.[2] GARCH represented an improvement over the ARCH model in that where ARCH was only conditioned on lagged square residuals, GARCH added the additional component of lagged conditioned variances. Additionally, the GARCH model allowed for corrections related to time

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[1] Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," Robert F. Engle, *Econometrica* , Vol. 50, No. 4 (Jul., 1982), pp. 987-1007.

[2] "Generalized Autoregressive Conditional Heteroskedasticity," Bollerslev, Tim, *Journal of Econometrics* 31 (3): 307–327, 1986.

series that exhibited thick tail distributions and volatility clustering, both of which are common, especially in nonlinear financial time series. For example, the following describes the ARCH process in which the current period's volatility is conditioned on the regressive sum of the lagged residuals up to period *t-q*:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$$

Conversely, the GARCH model is represented as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

Note the additional conditioning terms representing lagged period volatilities up to period *t-p*. These additional conditioning factors allow for the periodic economic shocks to reverberate longer in the data than simply one period.

One of the concerns of the GARCH model, or any time series model that uses lagged terms, is the value for q and p. In other words, how many lags should be included in the model to best reflect the underlying character of the data? Typically this is a choice made by the modeler and will depend on the model's forecasting ability as a GARCH(1,1) process, or perhaps a GARCH(4,4) if working with quarterly data. Once the choice is made, it is then applied to the entire data series without concern for potential inherent changes in the data over time.

Therefore, the purpose of this paper is to present a possible solution to the question of the optimal number of lagged periods for (p,q) in the GARCH model. Through the use of an optimization tool called a Genetic Algorithm, or GA, it will be shown that the GARCH(1,1) model is not always the best solution when searching for the optimal values for (p,q). Furthermore, it will be shown that the optimal values for (p,q) in fact change over time, reflecting the dynamic, effervescent nature of financial time series. The resulting model is hereafter termed as the D-GARCH with the “D” representing “dynamic.”

# CHAPTER II

## LITERATURE REVIEW

A great deal of literature has examined the GARCH model and its parameters as well as methods for optimizing those parameters, including the use of genetic algorithms and neural networks. These techniques are referred to as "fuzzy," or artificial intelligence methods for finding optimal solutions to problems which are typically difficult to solve using classical methods.

The literature is divided into essentially two categories: the need for better modeling of nonlinear systems such as the financial markets and possible tools and methods with which the models might be improved. The literature devoted to these questions is plentiful. A few noteworthy and relevant examples are given in this section.

Regarding the very concept of volatility itself, Nwogugu noted that volatility "can be modeled as the sum of all preferences of market participants over time," indicating that volatility in market prices is derived from investor utility. As such, models such as the traditional GARCH are "structurally deficient and static."[3] Nonlinear, adaptive, fuzzy modeling was needed to adequately capture the dynamics of financial time series. Doing so required a search for optimal solutions in multi-dimensional, nonlinear solution spaces. Adanu noted that in any search for optimization, it is important to keep in mind that a

[3] Nwogugu, Michael. 2006. "Volatility, risk modeling and utility." *Applied Mathematics & Computation* 182, no. 2: 1749-1754.

global optimum must be sought and to take care not to use methods that may be drawn to only local optima. In other words, the researcher should utilize methods that are not contingent on a defined starting point, such as Newton's algorithm, the simplex method, or the conjugate gradient methods.[4] Specifically addressing the GARCH model, Altay-Salih, *et al*, supported the notion that fuzzy programming, i.e. nonlinear programming without artificial restraints imposed by the modeler, produced better results than the traditional GARCH model, especially when bivariate and trivariate cases are considered.[5]

Given the need for fuzzy programming to analyze the dynamics of nonlinear time series, other researchers applied techniques such as genetic algorithms. Nair, *et al*, suggested the use of a genetic algorithm to optimize a decision tree based on 28 popular technical indicators. It was noted that a genetic algorithm is a "parallel search algorithm" which in this case was used to minimize trend prediction error.[6] Li, *et al*, employed genetic algorithm methods to study the scaling properties of wavelet-based indicators for the Dow Jones Industrial Average, allowing for the study of price data at multiple time scales.[7] Havandi, *et al*, explore integrating genetic algorithms and neural networks for stock price prediction in the IT and Airline sectors with the goal of capitalizing on the

[4] Adanu, Kwami. 2006. "Optimizing the GARCH Model – An Application of Two Global and Two Local Search Methods." Computational Economics, no.28: 277-290.

[5] Altay-Salih, Aslihan, Mustafa C. Pinar, and Sven Leyffer. 2003. "Constrained Nonlinear Programming for Volatility Estimation with GARCH Models." *SIAM Review* 45, no. 3: 485-503.

[6] Nair, Binoy B., V. P. Mohandas, and N. R. Sakthivel. 2010. "A Genetic Algorithm Optimized Decision Tree- SVM based Stock Market Trend Prediction System." *International Journal On Computer Science & Engineering* 2981-2988.

[7] Jin, Li, Shi Zhu, and Li Xiaoli. 2006. "Genetic programming with wavelet-based indicators for financial forecasting." *Transactions Of The Institute Of Measurement & Control* 28, no. 3: 285-297.

strengths of each method.[8]

Other researchers turned their attention to the GARCH model specifically. Roh compared the performance of various time series forecasting models such as GARCH, EGARCH and Exponentially Weighted Moving Average when optimized using adaptive neural networks. Roh's focus was not on stock price movement, but rather on the direction and deviation of the stock's volatility.[9] Hung also examined the idea of optimizing the GARCH model as well as more recent innovations of the model (EGARCH, GJR-GARCH, and Fuzzy GARCH). Hung's approach was to use a "particle swarm optimization" (PSO) model which imitates the movement of a swarm of gnats or a flock of birds. The PSO method, like the genetic algorithm, evaluates multiple possible solutions in the solution space to quickly arrive at a global maximum (or minimum depending on the objective).[10] Finally, Luna and Ballini studied the use of an adaptive fuzzy interface system (AdaFIS) to directly evaluate volatility and value-at-risk (VAR) in an effort to improve on traditional measures like GARCH.[11]

The literature summarized here is a merely a smattering of the research done on the application of fuzzy programming to nonlinear models. The remainder of this paper will summarize another possible method with which to optimize the GARCH model.

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[8] Hadavandi, Esmaeil, Hassan Shavandi, and Arash Ghanbari. 2010. "Integration of genetic fuzzy systems and artificial neural networks for stock price forecasting." *Knowledge-Based Systems* 23, no. 8: 800-808.

[9] Hyup Roh, Tae. 2007. "Forecasting the volatility of stock price index." *Expert Systems With Applications* 33, no. 4: 916-922.

[10] Hung, Jui-Chung. 2011. "Adaptive Fuzzy-GARCH model applied to forecasting the volatility of stock markets using particle swarm optimization." *Information Sciences* 181, no. 20: 4673-4683.

[11] Luna, Ivette, and Rosangela Ballini. 2012. "Adaptive fuzzy system to forecast financial time series volatility." *Journal Of Intelligent & Fuzzy Systems* 23, no. 1: 27-38.

# CHAPTER III

# METHODOLOGY

This study utilized the widely available daily data for the S&P 500 Index from 1/2/2003 to 12/26/2012, for a total of 2517 observations. This index was chosen for both its liquidity as well as its popularity as a test series. In order to construct the D-GARCH model, the first step was to determine if the daily price movements were stochastic, i.e., was there a unit root present in the raw data. The results of a Dickey-Fuller test at the 10% level on all observations failed to reject the null hypothesis that no unit root was present.

| Test Statistic | 1% Critical Value | 5% Critical Value | 10% Critical Value |
|---|---|---|---|
| **-1.846** | -3.430 | -2.860 | -2.570 |

*Table 1: Interpolated Dickey-Fuller – raw data*

To eliminate the unit root issues, the percentage change in daily Index closing prices was calculated and the Dickey-Fuller test was run again with the following results, rejecting the null hypothesis that a unit root is present:

| Test Statistic | 1% Critical Value | 5% Critical Value | 10% Critical Value |
|---|---|---|---|
| **-56.164** | -3.430 | -2.860 | -2.570 |

*Table 2: Interpolated Dickey-Fuller – daily percentage change*

Once the unit root issue had been addressed, an initial regression was run on a 1-period lag of the percentage change in closing price. The results indicate that the previous period's lagged percentage change is highly significant at the 1% level (t-value = -5.70).

| | Coefficient | t-value | P>\|t\| |
|---|---|---|---|
| One-period lag, % change | -0.1128574 | -5.70 | 0.000 |

*Table 3: One-period regression on lagged percentage change.*

The residuals from this regression were collected and squared. Then a new regression was run on the lagged squared residuals with the following highly significant results (t-value = 10.98):

| | Coefficient | t-value | P>\|t\| |
|---|---|---|---|
| One-period lag, squared residuals | 0.2139396 | 10.98 | 0.000 |

*Table 4: One-period regression on lagged squared residuals*

A Dickey-Fuller test was run on the squared residuals resulting in a strong rejection of the null hypothesis that a unit root was present (z-value = -40.330). The test for homoskedasticity of the squared residuals indicated that the null hypothesis (that the squared residuals are homoskedastic) can be safely rejected with an F-value of 120.49. Therefore, the squared residuals exhibit heteroskedasticity. This result is important because heteroskedasticity indicates a nonlinear bunching of volatilities in the data. Without the facility to account for inconsistent volatilities, other analyses will produce incorrect results. The solution is to run the GARCH model which accounts for varying lagged volatility as well as lagged squared residuals.

| | Coefficient | z-value | P>\|z\| |
|---|---|---|---|
| One period lag, ARCH component | 0.1638675 | 34.38 | 0.000 |
| One period lag, GARCH component | 0.8870074 | 437.30 | 0.000 |

*Table 5: GARCH(1,1) regression on one-period lagged residuals*

Note that the both the lag of the squared residual (the ARCH component) and the lag of the variance (the GARCH component) are highly significant (z-values = 34.38 and 437.30 respectively).

As the next step in the process, the GARCH residuals were squared and collected along with the predicted variances. The squared residuals and the variances from the GARCH regression were used to create a modified Akaike Information Criteria (AIC) for each according to the following formula:

$$AIC=\ln [(s^2) + 2m/T]$$

Where $s^2$ represents the average sum of the residuals squared, m is the number of parameters in the regression (to be determined), and T is the number of observations (also to be determined). A composite $AIC_c$ was created using the sum of the AIC for the squared residual component as well as the variance component. Finally, an average composite $AIC_{ac}$ was computed of a fuzzy number of periods for which the AIC would be minimized. A constant 20-day period was examined throughout the total sample size to allow for the values of (p,q) to vary with time.

As mentioned previously (see Introduction above), one of the key components in the D-GARCH process is the optimization of (p,q) using a genetic algorithm (GA).

In the search for the optimal values of (p,q), the value of the $AIC_{ac}$ was minimized by allowing the values for (p,q) to vary independently between 0 and 1000. Therefore, there are one million potential combinations for (p,q) that must be evaluated. The GA is uniquely suited to search for the optimal combination of (p,q) such that the value of $AIC_{ac}$ is minimized.

**Genetic Algorithms:**

Created by John Holland of the University of Michigan in 1975 and defined in his landmark work "Adaptation and Natural Selection," a genetic algorithm mimics the evolutionary process of strands of DNA by treating data as "chromosome strings," evaluating the "fitness" of the string compared to a pool of its competitors, then either kills off the contender or allows it to live for another generation.[12] The strings are also allowed to "crossover," meaning they divide and exchange "genetic material" in an effort to increase fitness. This creates child strings which are also evaluated for fitness against both the parent strings as well as the other competitors in the pool. An initial pool of competitors is created, mutation of the chromosomes is allowed with a defined probability, and crossover occurs with a defined probability. As the GA progresses, the program may be thought of as searching the fitness landscape for an optimal solution. For this study, the GA used is an MS Excel Add-in called *Genehunter*.[13] The parameters were as follows:

| | |
|---|---|
| Value to be optimized (minimized): | $AIC_{ac}$ |
| Adjustable values for optimization: | (p,q) |

[12] Holland, J. H. (1975/1992). Adaptation in Natural and Artificial Systems. Cambridge, MA: MIT Press. Second edition (1992). (First edition, University of Michigan Press, 1975).

[13] Available from Ward Systems Group at http://www.wardsystems.com/genehunter.asp.

| | |
|---|---|
| Range of possible values for (p,q): | 0-1000 |
| Initial population: | 100 |
| Probability of crossover: | 90% |
| Probability of mutation: | 1% |
| Evolution cutoff: | 100 generations |

The GA was run over non-overlapping 20-day periods. The initial period was (t, t-20), then (t-21, t-40), etc… throughout the 2517 observation sample. In total, 76 20-period blocks were evaluated by the GA. The optimal (minimized) $AIC_{ac}$, p, and q were recorded as well as the $AIC_{ac}$ with (p,q) = (1,1), or a basic GARCH(1,1) model. Optimized pairs of (p,q) which minimize $AIC_{ac}$, and thus define the optimal GARCH parameters, could range from (0,0), or no lag terms, to (1000,1000).

Once the optimized pairs were determined by the GA, the volatility was calculated according to the GARCH equation in Chapter 1 above. For comparison purposes, the volatility implied by the GARCH(1,1) model was also calculated. The logs of both were calculated and summarized for further analysis.

# CHAPTER IV

# RESULTS

Of the 76 evaluations of the data by the GA, over 39% of the time the optimal (p,q) pair which minimized the $AIC_{ac}$ was not the standard (1,1).[14] Volatilities were computed using (p,q) = (1,1) and the (p,q) determined by the algorithm. The discrepancy between the GARCH(1,1) model and the D- GARCH optimized model is illustrated in the graph below. For comparison purposes the log values of the D-GARCH volatilities were subtracted from the log values of the standard GARCH(1,1) volatilities.

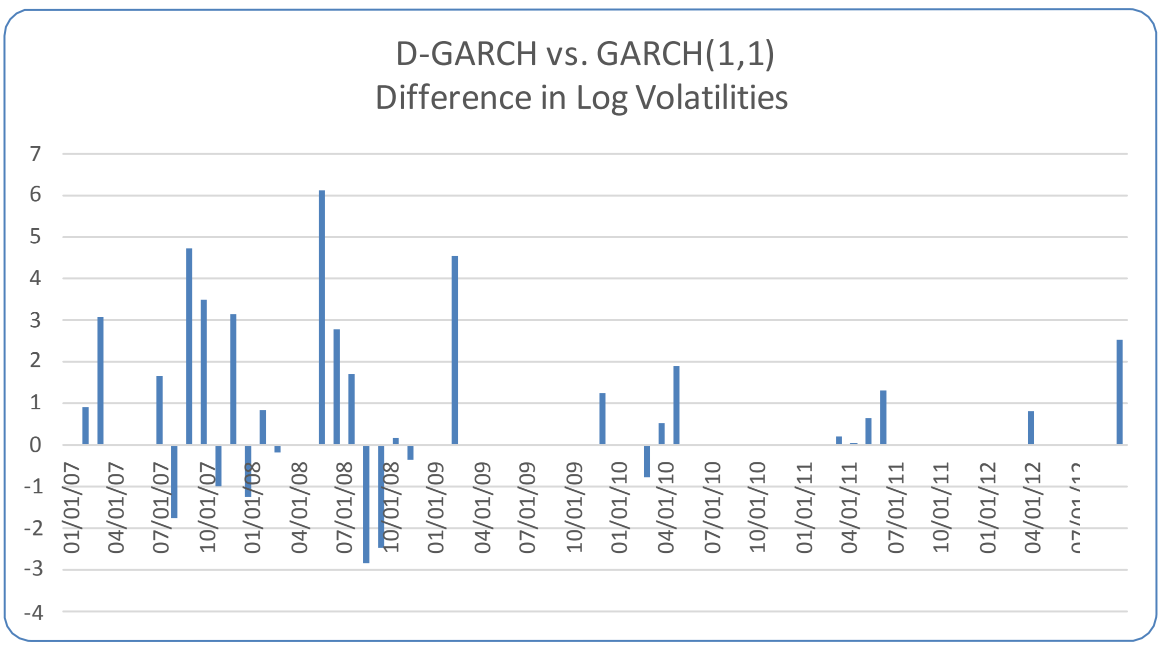


*Figure 1: D-GARCH vs. GARCH(1,1), difference in log volatilities*

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[14] Out of 76 GA runs, 30 times the optimal (p,q) pair was something other than (1,1).

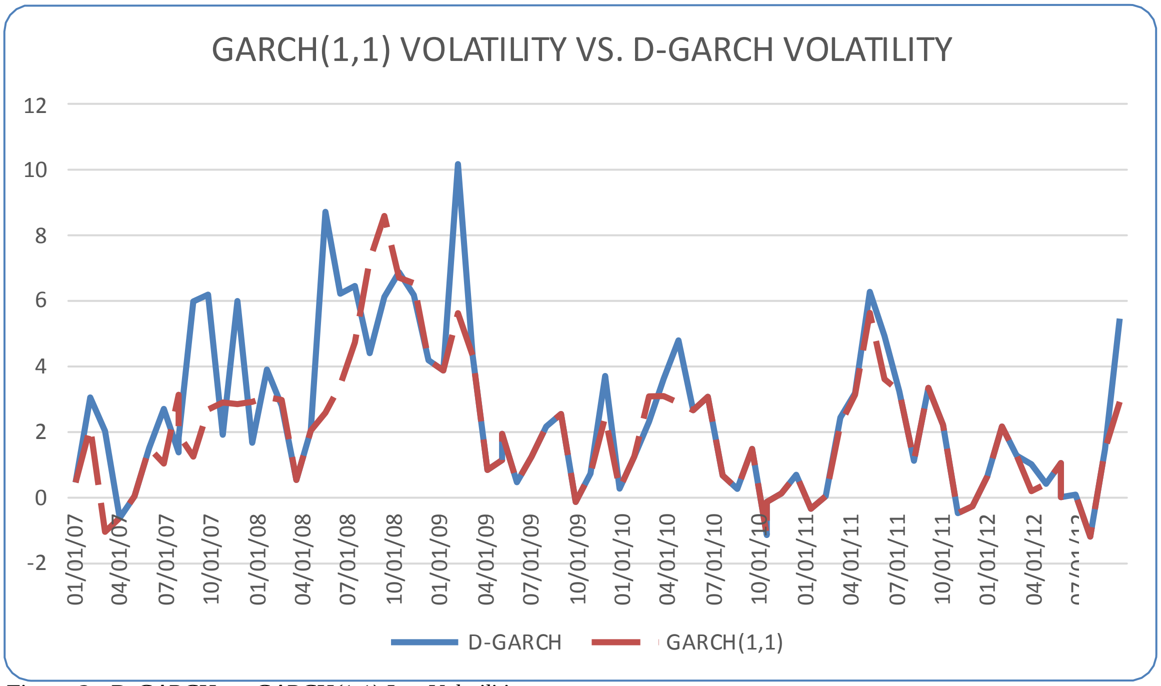


*Figure 2: D-GARCH vs. GARCH(1,1) Log Volatilities*

Note in the graph above that the D-GARCH volatilities tend to emphasize the "frothiness" of the market more than the traditional GARCH(1,1) model, producing higher peaks. This is especially evident during the turbulent downturn and subsequent recovery of the S&P 500 Index as illustrated in the following two graphs:

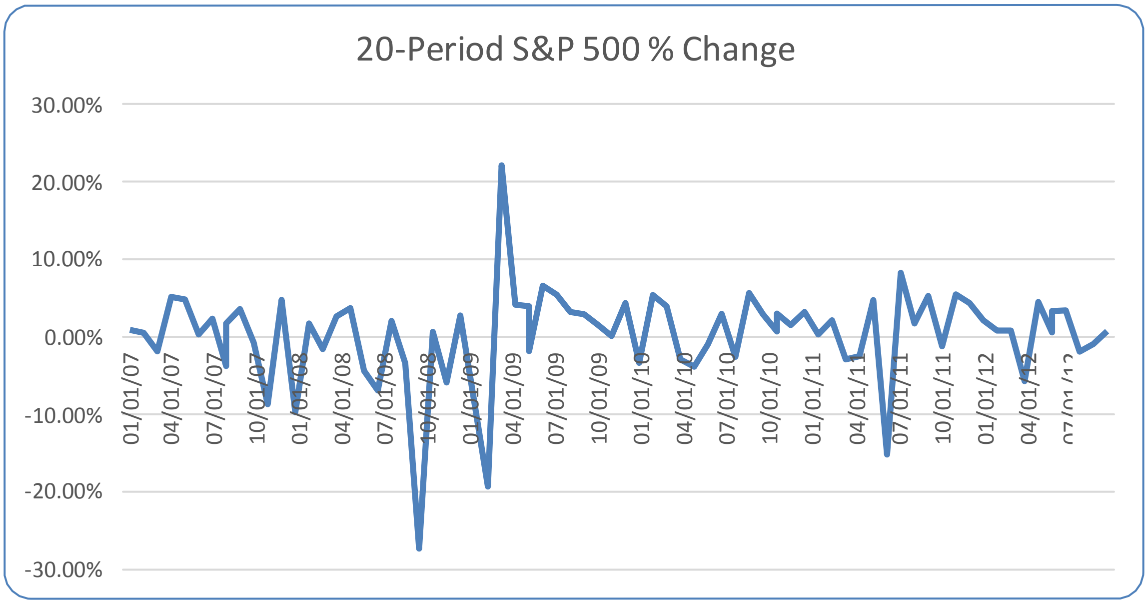


*Figure 3: 20-Period S&P 500 Percentage Change*

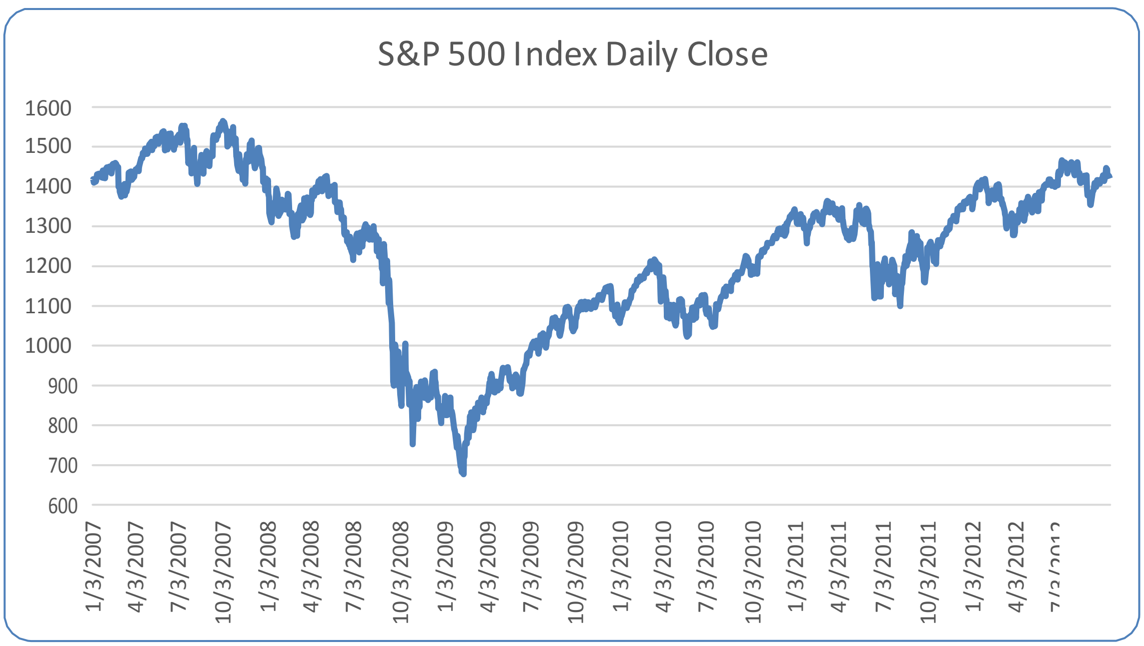


*Figure 4: S&P 500 Daily Close*

The determination of which combination of (p,q) was optimal was based strictly in minimizing the modified AIC as described above. In doing so, this study was able to focus strictly on the information that could be gleaned from the data series. Certainly other variables could be introduced (interest rates, quarterly GDP, etc…), but the univariate GARCH was used to isolate the optimal (p,q) found through the GA. The results from using the GA to determine the optimal (p,q) ranged from (1,1) to (1000,1000). The periods during which the algorithm found the greater values for (p,q) corresponded to periods of higher volatility, especially during the recessionary period from mid-2008 to mid-2009 (see graphs above).

Additionally, it should be noted that other criteria for discriminating among potential (p,q) combinations could be implemented. Other information criteria such as the Bayesian Information Criteria (BIC) could be tested. Another possibility is to generate a test to determine which (p,q) combination minimizes the difference between the calculated volatilities and the historical standard deviation for the same test data. These other possibilities were not addressed in this study.

## CHAPTER V

## FUTURE RESEARCH

The optimization of the GARCH parameters through the use of the GA as described above may have direct application to investment strategy, particularly to option trading. For example, the heralded Black-Scholes option valuation model incorporates a volatility factor according to the following formula for the value of a European call option[15]:

Value of call option: $C = SN(d1) – Ke^{-rt} N(d2)$

Where: $d1 = [\ln(S/K) + (r + v/2)T] / (v*T)^{1/2}$

$d2 = d1 – (v*T)^{1/2}$

S = current stock or index price

K = strike price

N = cumulative standard normal distribution

r = risk-free rate of return

v = volatility

T = time until option expiration

All of the factors of the Black-Scholes option valuation model are explicitly known at any period in time except for the volatility (v). The computed values for

[15] Black, Fischer; Myron Scholes (1973). "The Pricing of Options and Corporate Liabilities". *Journal of Political Economy* 81 (3): 637–654.

ln(h(t)) may be easily used to calculate the theoretical option values and compared with the actual option values to identify discrepancies, and therefore potential trading opportunities. One of the inherent issues with the Black-Scholes model is the assumption of constant volatility over a given period of time (for example annualized). Due to the dynamic nature of the D-GARCH process, it may prove to better calculate true volatility and thus more accurately reflect the current theoretical option value.

This paper addressed the D-GARCH process in discrete 20-day blocks of daily closing data for the S&P 500 Index. Other additional research possibilities include replicating this study using weekly, quarterly, and annual data over other data samples and other financial markets, as well as other criteria for determining optimality of the (p,q) combination.

# CHAPTER VI

## SUMMARY

The D-GARCH process utilizes a modified Akaike Information Criteria (AIC) optimized using a genetic algorithm to identify the optimal parameters for the GARCH model of time series volatility. The D-GARCH model produced a more optimal solution in 39% of the cases sampled while agreeing with the traditional GARCH(1,1) model in 61% of the cases. In particular, when the market under study (S&P 500 Index) becomes more "frothy," the D-GARCH better highlights the extreme volatility than the GARCH(1,1) model. It remains to be seen if the application of the volatilities calculated by the D-GARCH process can better calculate theoretical options prices.

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