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Wellbore stability in water-sensitive shales

Qingfeng Tao
University of North Dakota

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WELLOBRE STABILITY IN WATER-SENSITIVE SHALES

by

Qingfeng Tao

Bachelor of Science, China University of Geosciences, 2000

A Thesis
Submitted to the Graduate Faculty
of the
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Master of Science

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2006
This thesis, submitted by Qingfeng Tao in partial fulfillment of the requirements for the Degree of Master of Science from the University of North Dakota, has been read by the Faculty Advisory Committee under whom the work has been done and is hereby approved.

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Chairperson

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This thesis meets the standards for appearance, conforms to the style and format requirements of the Graduate School of the University of North Dakota, and is hereby approved.

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Department                        Geological Engineering
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ABSTRACT

Wellbore instability is a widespread problem, especially when drilling in deep, low strength shale formations at high temperature and pressure. The instability can be caused by the high compressive effective stress or tensile stress due to the stress concentration and pore pressure increase while drilling. In addition to the in-situ stress and shale strength, shale instability is affected by drilling mud properties including mud weight, temperature and salinity. To assess wellbore stability in these situations, Diek and Ghassemi (2004) developed a non-linear coupled chemo-poro-thermoelasticity theory. The linearized version of the theory allows one to analytically investigate the coupled impacts of mud properties including mud weight, temperature and salinity.

The theory views shale as an isotropic and homogeneous imperfect membrane, and considers the flow of matter across it. A difference of chemical potential between the drilling mud and shale will cause water flow (chemical osmosis), and the difference of solute concentration causes solute flux by diffusion. Heat flow also exists when there is a temperature difference between drilling mud and shale. Due to the low permeability of shale, the conductive fluxes are dominant in the system space, so the convective fluxes are neglected. The three fluxes are driven directly by the gradient of pore pressure, solute concentration and temperature. The three fluxes directly and indirectly impact on the stress distribution around the wellbore and the shale stability.

The field equations of the coupled chemo-poro-thermoelasticity were solved using Laplace transform and implemented in wellbore stability model to analyze the impacts of
mud properties on the wellbore stability and optimize mud properties including mud weight, salinity and temperature to maintain wellbore stable. The results suggest that cooling tends to prevent shear failure, radial spalling and hydraulic fracturing, while heating enhances them. The thermal impact is different from that predicted for chemically inert rock by porothermoelasticity which indicates cooling increases hydraulic fracturing and heating prevents it. Drilling with a higher salinity mud reduces the swelling pressure, thereby enhancing the wellbore stability, and drilling with lower salinity reduces the wellbore stability. Furthermore, the interaction between thermal and chemical phenomena can be used to maintain a wellbore stable while drilling namely, lowering salinity when the mud is cooler than the formation and increasing salinity if the mud is warmer.

The coupled chemo-poro-thermoelasticity solutions were also implemented to analyze the chemical and thermal impacts on the mud weight window. The results suggest that cooling reduces the critical low mud pressure and increases the critical high mud pressure, thereby expanding mud weight window. On the other hand, heating increases the critical low mud pressure and reduces the critical high mud pressure, thereby contracting mud weight window. Drilling with a higher salinity mud tends to reduce the critical low mud pressure and increase the critical high mud pressure, therefore expanding the mud weight window. Drilling with a lower salinity mud contracts the mud weight window by increasing the critical low mud pressure and reducing the critical high mud pressure.
CHAPTER I
INTRODUCTION

Wellbore instability is a problem affecting exploration and development drilling, with a global annual cost estimate about $1.3 billion (Tare and Mody, 2002) resulting from the collapsed holes, sidetrack holes, lost tools and abandonment of wells prior to reaching targets. The problem is encountered in some form in all of the major petroleum basins worldwide, from the North West Shelf (Australia) to the Middle East, the North Sea and the Gulf of Mexico. Shales make up over 75 percent of drilled formations and causes over 90 percent of wellbore instability problems (Tare and Mody, 2002).

Shale is a fine-grained, low permeability and low strength sedimentary rock whose constituents are clay or mud and accessory mineral (quartz, feldspar, zeolite, calcite, sulfides etc.). The low strength and sensitivity to water, makes it one of the weakest rocks encountered in drilling and therefore wellbore instability most often occurs in shale. Due to the low permeability, the convective fluid flow in shale is negligible and the diffusion dominates. When shale is exposed to the water-bearing drilling fluids, the complex physical and chemical interactions between them can affect the shale strength and stability (Oort, 2003). In addition, the temperature gradient between drilling fluids and shales may affect the shale stability. So, the influence of coupled chemical, thermal and hydro-mechanical processes on wellbore stability need to be investigated to help maintain a wellbore stable. In this thesis, a coupled chemo-poro-thermoelastic theory is used to compute stress and pore pressure distributions around a wellbore for assessing
wellbore stability and optimizing mud properties including mud weight, temperature and salinity.

Overview of Oil/Gas Drilling

Rotary drilling rigs are used for almost all drilling done today (Bourgoyne et al., 1986). A sketch illustrating the rotary drilling process is shown in Figure 1. The hole is drilled by rotating a bit to which a downward force is applied. Generally, the bit is turned by rotating the entire drillstring, using a rotary table at the surface, and the downward force is applied to the bit by using sections of heavy thick-walled pipe, called drill collars, in the drillstring above the bit. The cuttings are lifted to the surface by circulating a fluid down the drillstring, through the bit, and up the annular space between the hole and the drillstring. The cuttings are separated from the drilling fluid at the surface.

Figure 1. The Rotary Drilling Process (Bourgoyne et al., 1986)
Drilling fluid is the component of rotary drilling (Bourgoyne et al., 1986) that serves to (1) clean the rock fragments from beneath the bit and carry them to the surface, (2) exert sufficient hydrostatic pressure against subsurface formations to prevent formation fluids from flowing into the well, (3) keep the newly drilled borehole open until steel casing can be cemented in the hole, and (4) cool and lubricate the rotating drillstring and bit. To realize the above functions and drill successfully, the drilling fluid needs to be carefully designed, especially for complex wells and in challenging/hazardous environments.

The drilling fluid design is determined by the following factors (Bourgoyne et al., 1986): (1) the types of formations to be drilled, (2) the range of temperature, strength, permeability, and pore fluid pressure exhibited by the formations, (3) the formation evaluation procedure used, (4) the water quality available, and (5) ecological and environmental considerations. There are three major types of drilling fluid: water-based muds (WBM), oil-based muds (OBM), and synthetic-based muds (SBM). In the past, OBM has been chosen for difficult drilling (water sensitive shales), because it provides excellent wellbore stability, good lubrication, thermal stability, etc. But OBM can cause severe environment pollution. To minimize impacts on the environment, SBM has been used instead of OBM during last decade. But SBM still may cause environment pollution and increase the drilling cost. With the increase of environmental concern, WBM is becoming more attractive as WB muds provide the least impacts on the environment and reduces the drilling cost (Tare and Mody, 2002). But the complex interactions between the shale and water-based drilling fluid make it very difficult to prevent wellbore instabilities. The problem of shale/water interactions have been the subject of numerous
studies in geoscience and engineering (Hanshaw and Zen, 1965; Chenevert, 1970; Carminati et al., 1997; Mody and Hale, 1993; Onaisi et al., 1993; Sherwood and Bailey, 1994; van Oort et al., 1996, 2003; Ghassemi et al., 2002, 2003, 2005), which provide the theories for the optimization of WBM including mud weight, temperature, salinity etc.

Wellbore Instabilities

When a hole is drilled, the surrounding rock will carry the load which was carried by the removed rock. The stress concentration may lead to failure of the borehole. Drilling mud is used to support the rock in order to prevent failure and possible mud loss. It is generally believed, however, that when water-based drilling mud (WBM) contacts the formation (shale) around a borehole, shale tends to absorb the invading water onto grain boundaries or incorporate it into the crystal structure, which therefore causes hydration swelling phenomena (Heidug and Wong, 1996) and deterioration. Hydration swelling weakens the shale and increases the tensile stress in the formation, thereby increasing the wellbore instability. The invasion of the drilling mud also causes the pore pressure increase in the formation, which contributes to wellbore instability by decreasing the effective stresses. It is believed that salts added into the drilling mud efficiently inhibit or reduce hydration swelling and increase the wellbore stability (Oort, 2003). In addition, due to the osmotic characteristics of shale (semi-permeable membrane), saline drilling mud causes osmotic phenomena and drives the fluid flow from the low salinity side to the high salinity one (Young and Low, 1965; Kemper and Rollins, 1966; Fritz and Marine, 1983, Fritz, 1986). If the water is drawn from the formation into the wellbore, it reduces the pore pressure and contributes the wellbore stability. Therefore, water transport
mechanisms and physico-chemical processes between shale/drilling fluid are critical to maintain wellbore stability.

Mud temperature is also an important factor to influence wellbore stability. When a temperature difference exists between the shale and drilling fluid, thermal, solute, and water transport occur in the shale formation. The thermal transport changes the temperature distribution in the formation, causing expansion or contraction of shale. For fluid-saturated rocks, the thermal effect induces pore pressure variations and causes fluid flow and deformation of solid matrix (McTigue, 1986; Kurashige, 1989).

As a result, in addition to in-situ stress, pore pressure, and rock strength, wellbore stability should be assessed by considering mud chemistry and temperature.

In early studies only a linear elastic model was applied to analyze the wellbore stability based on the solutions of stress distribution around the well derived by Kirsch (Jaeger and Cook, 1979). After obtaining the stress distribution and rock strength, rock failure is judged by applying an appropriate failure criterion. The elastic analysis provides a good and fast way to assess the wellbore stability. But, it fails to incorporate the influences of hydraulic, chemical, and thermal processes.

Hydro-mechanical factors are accounted for in the theory of poroelasticity (Biot, 1941; Rice and Cleary, 1976). In this theory, rock deformation is coupled to transient fluid diffusion described by Darcy’s law and the continuity equation. As a result, stresses and strain become time-dependent. Therefore, the stress distribution around wellbore is also time dependent, and so the wellbore stability also depends on time.

The fully coupled solution for a hole in a poroelastic rock was developed by (Rice and Cleary, 1976; Carter and Booker, 1982). Detournay and Cheng (1988) treated for
problem of a vertical wellbore drilled in a chemically-inert formation subjected to anisotropic in-situ stress. The poroelastic analysis reveals the possibility of delayed tensile failure under anisotropic loading, in contrast to the instantaneous failure predicted by elastic analysis (Detournay and Cheng (1988). The poroelastic analysis is extended by Cui et al. (1997) for the general case of an inclined wellbore and was used to analyze for tensile and shear failure by Cui et al. (1999).

The uncoupled porothermoelastic analysis was applied to wellbore stability by Wang and Papamichos (1994). In this analysis, conductive heat flow was considered, but the temperature balance was assumed before the poroelastic processes begin. Li et al. (1998) applied a coupled porothermoelastic analysis to wellbore stability and considered the coupled hydraulic-thermo-mechanical influences.

Chemical osmosis was considered for borehole stability by Mody and Hale (1993). They considered the shale as an ideal semi-permeable membrane and used a material constant – reflection coefficient $\mathcal{R}$ to represent the ideality of an osmotic system. For ideal membrane systems, $\mathcal{R}$ approaches 1 and ions cannot pass the membrane; for porous media having no membrane properties, $\mathcal{R} = 0$, and both ions and water can pass it without selectivity. For nonideal membrane systems, $\mathcal{R}$ varies between 0 and 1. When there is a chemical potential difference between the drilling mud and the shale formation, a pore pressure will be induced by chemical osmosis (Eqn. (1)).

$$\Delta p = \frac{\mathcal{R}RT}{V} \ln \left( \frac{(A_w)_m}{(A_w)_{sh}} \right) \tag{1}$$

Where $R$ is the gas constant, $V$ is the partial molar volume of water, $T$ is absolute temperature, $(A_w)_m$ is the water activity of the drilling mud, and $(A_w)_{sh}$ is the water
activity of the fluid in the shale formation. When considering the wellbore stability, the pore pressure difference induced by chemical osmosis was added to the stress distribution predicted by poroelastic model. But the coupled effects between the chemical osmosis and hydraulic-mechanical processes were not considered.

Sherwood (1993, 1994) introduced chemical osmosis into the Biot’s theory of poroelasticity and developed a coupled chemoporoelasticity theory. But shale was assumed as an ideal membrane system and ions could not pass through the membrane. Ghassemi and Diek (2003) considered the solute transfer in the shale system and developed a coupled chemoporoelasticity theory. But this theory did not include thermal effects and was not applied to wellbore stability analysis.

Ghassemi and Diek (2002) developed an uncoupled chemo-poro-thermoelastic model which was implemented into a wellbore stability analysis design code (Wolfe, 2002). The analysis covered many factors, but it did not consider solute transfer. So, none of the above analyses of wellbore stability cover all of these factors: in-situ stress, pore pressure, rock strength, mud chemistry, mud weight, mud temperature.

Objectives

As the above literature review suggests many factors contribute to the wellbore instabilities with complicated couplings among them. The objective of this thesis is to apply a coupled chemo-poro-thermoelastic model that considers solute transfer and thermal filtration to analyze the stress/strain, pore pressure distribution around the wellbore, and to optimize mud properties for maintaining wellbore stability.
CHAPTER II
POROELASTICITY

Interest in the coupled hydro-mechanical process was initially motivated by the problem of soil consolidation. Terzaghi (1923) proposed the one-dimensional consolidation theory which accounts for the influence of pore fluid on the quasi-static deformation of soils. Biot (1941) presented a linear theory of poroelasticity, which considered the coupled mechanisms between solid and fluid including (i) solid-to-fluid coupling: a change in applied stress produces a change in fluid pressure or fluid mass, and (ii) fluid-to-solid coupling: a change in fluid pressure or fluid mass produces a change in the volume of the porous material. Rice and Cleary (1976) reformulated Biot’s linear constitutive equations and chose constitutive parameters which emphasized two limiting behaviors of a fluid-filled porous material namely, drained and undrained. The linear poroelasticity theory of Biot was later extended to a non-isothermal (McTigue, 1986; Kurashige, 1989; Li etc. 1998) and a chemically-active (Heidug and Wong, 1996; Ghassemi and Diek, 2003) environment. A linear theory of poroelasticity including thermal and chemical effects, which is called chemo-poro-thermoelasticity, has been developed by Ghassemi and Diek (2004) and will be described in Chapter 3.

Poroelastical Model (Biot Model)

The Biot model of a fluid-filled material is constructed on the conceptual model of a deformable solid skeleton and a freely moving pore fluid (in other words both solid and fluid phases are fully connected). Two kinetic quantities namely, a solid displacement...
vector, \( u_i \), and a specific discharge vector, \( q_i \), are used to describe the movement of the pore solid and the motion of the fluid relative to the solid separately. Two ‘strain’ quantities are introduced to describe the deformation and the change of fluid content of the porous solid with respect to an initial state: the usual small strain tensor, \( \varepsilon_{ij} \), and the variation of fluid content, \( \zeta \) (defined as the variation of fluid volume per unit reference volume of porous material due to diffusive mass transport). The dynamic variables are the stress tensor, \( \sigma_{ij} \), and pore pressure, \( p \) (Rice & Clerary, 1976; Detournay and Cheng, 1993).

The following assumptions were made by Biot (1941) in constructing the constitutive equations:

1. Isotropy of the rock,
2. Reversibility of stress-strain relations under final equilibrium conditions,
3. Linearity of stress-strain relations,
4. Small strains,
5. The fluid flows through the porous skeleton according to Darcy’s law,
6. Incompressible fluid.

The assumption of incompressible fluid was relaxed later by Biot.

**Constitutive Equations**

The constitutive equations describes the relation between the solid and fluid strain (\( \varepsilon_{ij} \) and \( \zeta \)) and the stress and pore pressure (\( \sigma_{ij} \) and \( p \)):

\[
\varepsilon_{ij} = \sigma_{ij} \frac{1}{2G} - \left( \frac{1}{6G} - \frac{1}{9K} \right) \delta_{ij} \sigma_{kk} \frac{1}{3H} \delta_{ij} p \quad (i = 1,2,3 \quad j = 1,2,3)
\] (2)
\[ \zeta = \frac{\sigma_{kk}}{3H} + \frac{P}{R'} \]  

(3)

\[ \sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33} \]  

(4)

Where the parameters \( K \) and \( G \) are the bulk and shear modulus of the drained elastic solid; the constitutive constants \( H', H'' \) and \( R' \) characterize the coupling between the solid and fluid stress and strain. According to the reversibility assumption, the strain of solid skeleton caused by the increase of pore pressure is equal to the fluid content change caused by the increase of volumetric stress (refer to appendix A)

\[ \frac{\partial \varepsilon_{ij}}{\partial \sigma_{ij}} = \frac{\partial \zeta}{\partial \sigma_{ij}} \]  

(5)

So the relation \( H' = H'' \) (refer to appendix A) can be derived by combining Eqns. (2), (3) and (5).

In contrast to the elasticity theory, one can choose 4 parameters – elastic modulus \( E \), drained Poisson’s ratio, \( v \), undrained Poisson’s ratio, \( v_u \), and Biot’s coefficient, \( \alpha \), as the basic independent constitutive parameters (Rice and Cleary, 1976; Detournay and Cheng, 1993). As will be seen below, permeability coefficient, \( \kappa = \text{permeability/fluid viscosity} \), is the other necessary parameter. Then, other parameters and constants can be expressed in terms of by the four basic parameters:

\[ G = \frac{E}{2(1 + v)} \]

\[ K = \frac{2(1 + v)}{3(1 - 2v)} G \]

\[ R' = \frac{2G(1 + v)(v_u - v)}{\alpha^2(1 - 2v)^2(1 + v_u)} \]
\[ H' = H'' = \frac{2G(1 + \nu)}{3\alpha(1 - 2\nu)} \]  

Put Eqn. (6) into Eqns. (2) and (3)

\[ \varepsilon_{ij} = \frac{\sigma_{ij}}{2G} - \frac{\nu}{2G(1 + \nu)} \delta_{ij} \sigma_{kk} + \frac{\alpha(1 - 2\nu)}{2G(1 + \nu)} \delta_{ij} p \]  

\[ \zeta = \frac{\alpha(1 - 2\nu)}{2G(1 + \nu)} \sigma_{kk} + \frac{\alpha^2(1 - 2\nu)^2(1 + \nu_0)}{2G(1 + \nu)(\nu_0 - \nu)} p \]  

The constitutive equations (7) and (8) also can be rewritten as the dependence of the stress \( \sigma_{ij} \) and pore pressure \( p \) on the strain \( \varepsilon_{ij} \) and fluid content, \( \zeta \) :

\[ \sigma_{ij} = 2Ge_{ij} + \frac{2G\nu}{1 - 2\nu} \delta_{ij} e - \alpha \delta_{ij} p \]  

\[ p = -\frac{2GB(1 + \nu_0)}{3(1 - 2\nu)} e + \frac{2GB^2(1 - 2\nu)(1 + \nu_0)^2}{9(\nu_0 - \nu)(1 - 2\nu)} \zeta \]  

Where \( B = \frac{3(\nu_0 - \nu)}{\alpha(1 - 2\nu)(1 + \nu_0)} \) is the Skempton pore pressure coefficient.

**Field Equations**

Beside the constitutive equations, the following governing equations are also necessary to describe the poroelastic behavior.

The equilibrium equations

\[ \sigma_{ij,j} = 0 \]  

Darcy’s law

\[ q_i = -\frac{k}{\mu} p_i \]  

And the continuity equation for the fluid

\[ \zeta + q_{i,i} = 0 \]
Where \( q_i \) is the specific discharge vector; \( k \) is the intrinsic permeability and \( \mu \) the fluid viscosity; dot denotes the derivative by time.

Putting Eqn. (9) into the equilibrium equation, the Navier-type equation is obtained

\[
G\nabla^2 u_i + \frac{\mu}{1-2\nu_u} u_{k,ki} - \alpha p_j = 0
\]

(14)

Where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \) is the Laplacian.

A diffusion equation can be derived by inserting the constitutive pore pressure relation (Eqns. (8) or (9)) and the expression for Darcy’s law (Eqn. (11)) into the continuity equation (Eqn. (12)). Making the appropriate substitutions, one obtains the following equation

\[
\dot{\zeta} - c_f \nabla^2 \zeta = 0
\]

(15)

Or

\[
\dot{\rho} - k M \nabla^2 \rho = -M \alpha \dot{\varepsilon}
\]

(16)

Where

\[
c_f = \frac{2kB^2G(1-v)(1+\nu_u)^2}{9\mu(1-\nu_u)(\nu_u-v)}
\]

is fluid diffusion coefficient and

\[
M = \frac{2G(\nu_u-v)}{\alpha^2(1-2\nu_u)(1-2\nu)}
\]

the Biot modulus.

Using appropriate boundary conditions, the field equations (Eqns. (14) and (15) or (16)) might be solved for some special problems (refer to Chapter III).
Porothermoelastic Model

Porelastic model assumes the fluid-filled material is isothermal, but non-isothermal conditions are more often regarding to the petroleum and geothermal energy development. Heat transfer in the nonisothermal poroelastic systems causes the expansion/contraction of both solid skeleton and fluid, thereby inducing stress and pore pressure. McTigue (1986) developed complete governing equations for a fluid-saturated, porous, thermoelastic medium which is so called porothermoelastic medium.

Constitutive Equations

The constitutive equations describing the relations of the strain $\varepsilon_{ij}$ and fluid content $\zeta$ to the stress $\sigma_{ij}$, pore pressure $p$ and temperature change $\Delta T$ were developed by combining the linear thermoelasticity with the linear poroelasticity (McTigue 1986).

\[
\varepsilon_{ij} = \frac{1}{2G} \left[ \sigma_{ij} - \frac{v}{1 + v} \sigma_{kk} \delta_{ij} \right] + \frac{\alpha(1-2v)}{2G(1+v)} \delta_{ij} p + \frac{\alpha_s}{3} \delta_{ij} \Delta T \tag{17}
\]

\[
\zeta = \frac{\alpha(1-2v)}{2G(1+v)} \sigma_{kk} + \frac{\alpha^2(1-2v)^2}{2G(1+v)(1+\nu_u)} p - \phi(\alpha_f - \alpha_s) \Delta T \tag{18}
\]

or

\[
\sigma_{ij} = 2Ge_{ij} + \frac{2Gv}{1-2v} \delta_{ij} e_{kk} - \alpha \delta_{ij} p - K \alpha_s \delta_{ij} \Delta T \tag{19}
\]

\[
p = M[\zeta - \alpha \varepsilon_{kk} + (\alpha_f + \alpha_s) \Delta T] \tag{20}
\]

Where $\phi$ is the porosity, and $\alpha_f$ and $\alpha_s$ are the volumetric thermal expansion coefficient for fluid and solid respectively.
**Transport Equations**

Both fluid and heat flow exist in the system. For low permeability rocks, conductive transports are dominant for both fluxes and the convection can be neglected. So the fluid flow is governed by Darcy’s law and the heat flow by Fourier’s law.

Fourier’s law:

\[ \dot{Q}_i = -k^T T_i \]  

(21)

Where \( k^T \) is the thermal conductivity, and \( \dot{Q}_i \) is the heat flux.

**Conservation Equations**

For the system, the fluid mass balance equation is the same as that for poroelastic model (Eqn. (13)). The energy balance equation is

\[ \rho_m c_m \dot{T} + \dot{Q}_{i,v} = 0 \]  

(22)

Where \( \rho_m \), and \( c_m \) denote the total mass density and specific heat respectively. Both the convective energy transport and energy caused by the friction between the fluid and the solid are neglected.

**Field Equations**

By substituting Eqn. (19) into the equilibrium equation (Eqn. (11)), the Navier-type equation for porothermoelastic model is derived:

\[ G \nabla^2 u_i + \frac{G}{1-2\nu_n} u_{k,k} - \alpha p_{,i} + K \alpha_s \delta_{ij} \Delta T_i = 0 \]  

(23)

The diffusion equation is then derived by inserting Darcy’s law (Eqn. (12)) and the constitutive pore pressure relation (Eqn. (20)) into the continuity equation (Eqn. (13)) to get:

\[ \dot{\rho} - k M \nabla^2 p = -M \alpha \dot{u}_{k,k} + (\alpha \alpha_f + \alpha_t) \Delta T \]  

(24)
The heat diffusion equation is obtained by inserting Fourier’s law (Eqn. (21)) into the energy balance equation (Eqn. (22)):

\[ \dot{T} = c_r \nabla^2 T \]  

(25)

Where \( c_r = \frac{k}{\rho c_m} \) is the thermal diffusivity of the rock.

Chemoporoelastic Model

Shale is a non-ideal membrane, so the chemical potential also has direct impact on the pore pressure change. The solute transfer might change the chemical potential distribution, thereby changing the pore pressure and the status of stress and strain of a solution filled solid system. For the chemical active shale, the chemical swelling is also an important factor to control the stress, strain and fluid flow in the system. Sherwood and Baily (1994) assumed an ideal membrane system and neglected the solute transport, developed a constitutive model for the system. Using arguments of irreversible thermodynamics, Heidug and Wong (1996) developed a fully coupled Biot-like model that considers ion transfer. But the non-linear relationship between stress and the solute mass fraction in the model makes it difficult to apply to the real problems. Base on the coupled chemoporoelastic model, Ghassemi and Diek (2003) provided a coupled linear poroelastic model.

Constitutive Equations

The constitutive equations describe the relations of the stress \( \sigma_{ij} \) and fluid content \( \zeta \) to the strain \( \varepsilon_{ij} \), pore pressure \( p \) and solute mass fraction \( C^S \).
\[
\sigma_{ij} = 2G\dot{e}_{ij} + \frac{2Gv}{1-2v} \delta_{ij}\dot{e}_{kk} - \alpha' \delta_{ij}\dot{p} + \frac{\omega_0 x \delta_{ij}}{C^S} \frac{\dot{C}^S}{C^S} \tag{26}
\]

\[
\dot{\zeta} = \alpha\dot{e}_{kk} + \left(\frac{1}{M} + \frac{M^S \omega_0 (\alpha - 1)}{KC^DRT_0 \bar{\rho}_f}\right)\dot{p} + \frac{\chi \omega_0 (\alpha - 1)}{K} \frac{\dot{C}^S}{C^S} \tag{27}
\]

Where
\[
\alpha' = \left(\alpha - \frac{M^S \omega_0}{C^DRT_0 \bar{\rho}_f}\right)
\]

\[
\chi = 1 - \frac{C^S}{C^D}
\]

The dot denotes the derivative by time. \(\bar{C}^S\) and \(\bar{C}^D\) denote the mean values of the solute and diluent mass fraction, respectively, \(T_0\) the average absolute temperature of the system, \(\bar{\rho}_f\) the average fluid density, \(R\) the universal gas constant, \(M^S\) the molar mass of the solute.

**Transport Equations**

For isothermal chemically-active porous solid saturated with a binary electrolyte solution, the coupled hydraulic and solute flux might exist due to the gradient of chemical potential and pore pressure.

\[
J^f = -\frac{k}{\mu} \bar{\rho}_f \left(p_i - \frac{\Re \bar{\rho}_f RT_0}{M^S \bar{C}^S \bar{C}^T} \right) C^{S,i}_j \tag{28}
\]

\[
J^s = -\bar{\rho}_f D^S \nabla C^{S,i}_j \tag{29}
\]

Where \(\Re\) is the reflection coefficient, and \(J^f\) and \(J^s\) are fluid flow and solute flux respectively, \(D^S\) the solute diffusivity.
**Conservation Equations**

The conservation equations of fluid and solute mass are

\[ \rho_f \dot{\zeta} + \nabla \cdot J^f = 0 \]  
(30)

\[ \phi \rho_f \dot{C}^s + J^f \cdot \nabla C^s + \nabla \cdot J^s = 0 \]  
(31)

For low permeability rocks, the convective solute transport can be neglected, so Eqn. (30) can be reduced to be:

\[ \phi \rho_f \dot{C}^s + \nabla \cdot J^s = 0 \]  
(32)

**Field Equations**

By substituting the constitutive equation (Eqn. (26)) into the force equilibrium equation (Eqn. (11)), the Navier-type equation for the coupled chemoporoelastic model is derived

\[ G \nabla^2 u_i + \frac{G}{1 - 2v_w} u_{k,k} - \alpha' p_i - \alpha_w x_a C_i^s = 0 \]  
(33)

The solute diffusion equation is derived by combining the solute transport equation (Eqn. (29)) with the solute balance equation (Eqn. (32)):

\[ \dot{C}^s = \frac{D^s}{\phi} \nabla^2 C^s \]  
(34)

The coupled fluid diffusion equation is derived by combining the fluid content constitutive equation (Eqn. (27)) with the fluid transport and conservation equations:

\[ c' \nabla^2 \hat{p} - \dot{\hat{p}} = -c'' \dot{C}^s \]  
(35)

where

\[ c' = \frac{c' \cdot c - c_0}{1 - c_0} \]
\[ c'' = \frac{RT \rho_f}{M^s} \left[ \frac{C^D c_p^x}{C^s (1 - c_0)} - \frac{\Re c^f \phi}{(1 - c_0) C^s C^D D^s} \right] \]

\[ c_0 = \frac{\mu c^f}{k} \left[ 1 + \frac{2\eta(1 - 2\nu)}{G(1 + \nu)} \right] \frac{\alpha_0 M^s}{C^D RT_0 \rho_f} \]

\[ c^f = \frac{2kG(1 - \nu)(\nu_u - \nu)}{\alpha^2 (1 - \nu_u)(1 - \nu)^2} \] (36)
CHAPTER III

ANALYTICAL SOLUTIONS FOR DISTRIBUTIONS OF STRESS, PORE PRESSURE, DISPLACEMENTS, AND STRAINS AROUND A BOREHOLE IN SHALE

The shale formation at depth is assumed, as a system, to be in static equilibrium, including mechanical equilibrium, chemical equilibrium and thermal equilibrium before drilling. Drilling disturbs the equilibrium, causes a redistribution of stresses and induces fluid, solute and heat flux plus swelling or contraction. The solutions of stress, pore pressure, displacement and strain around the borehole are presented below for the case of an instantaneously drilled circular borehole.

Problem Definition

The problem to be considered is that of drilling an inclined wellbore in a poroelastic rock saturated by an ideal solution consisting of a solute and a diluent (a binary electrolyte). The drilling mud has a constant pressure of $p^m$, a temperature of $T^m$ and a solute concentration/solute mass fraction of $C^m$. The formation is assumed to have a pore pressure of $p^{sh}$, a temperature of $T^{sh}$, and a solute mass fraction of $C^{sh}$ at initial state. The in-situ stresses are given by the three principal stresses namely, the vertical stress, $\sigma_v$, maximum horizontal stress, $\sigma_{H}$, and the minimum horizontal stress, $\sigma_{h}$.

The trajectory of the wellbore can be defined by referring to a Cartesian coordinate system with three principal axes $x'$, $y'$, $z'$ that coincides with the direction of the in-situ stresses $\sigma_{H}$, $\sigma_{h}$, $\sigma_v$, respectively. The zenith angle $\beta$ measures the tilt of the wellbore axis from the vertical direction. The azimuth angle, $\Phi$, is measured counterclockwise from the $x'$ axis to the projection of the axis of the wellbore on the plane $x'-y'$. To
conveniently analyze stresses around the wellbore, we introduce another Cartesian coordinate system: $xyz$. The $z$-axis is the axis of the wellbore and is positive in the upward direction. The $x$-axis is the projection of the wellbore axis on the $x'-y'$ plane, and the $y$-axis is the line perpendicular to the $x$-axis on the $x'-y'$ plane (Figure 2). The in-situ stresses in the new coordinate system can be obtained from those in the original coordinate by coordinate transformation (e.g., Fjaer et al. 1992).

\[
\begin{align*}
\sigma_{xx} &= (\cos \beta \cos \phi)^2 \sigma_H + (\cos \beta \sin \phi)^2 \sigma_h + (\sin \beta)^2 \sigma_v \\
\sigma_{yy} &= (\sin \phi)^2 \sigma_H + (\cos \phi)^2 \sigma_h \\
\sigma_{zz} &= (\sin \beta \cos \phi)^2 \sigma_H + (\sin \beta \sin \phi)^2 \sigma_h + (\cos \beta)^2 \sigma_v \\
\sigma_{xy} &= (-\cos \beta \cos \phi \sin \phi) \sigma_H + (\cos \beta \cos \phi \sin \phi) \sigma_h \\
\sigma_{xz} &= \cos \beta \sin \beta (\cos \phi)^2 \sigma_H + \cos \beta \sin \beta (\sin \phi)^2 \sigma_h - (\cos \beta \sin \beta) \sigma_v \\
\sigma_{yz} &= (-\sin \beta \cos \phi \sin \phi) \sigma_H + (\sin \beta \cos \phi \sin \phi) \sigma_h
\end{align*}
\] (37)

Figure 2. Wellbore trajectory in 3-D space.
The shale formation is assumed to be isotropic and homogeneous. The length of the wellbore is much larger than its diameter and the considered region around the wellbore. As a result, the three-dimensional problem can be simplified to two-dimensions using the plane strain assumption (figure 3). That is, by assuming that all out of the plane strains to be zero (i.e., \( e_{zz} = e_{xz} = e_{yz} = 0 \)) with \( \sigma_{zz} \neq 0 \). After the stress, pore pressure, displacement and strain in the plane \( x-y \) are solved, the stress parallel to the \( z \)-axis \( \sigma_{zz} \) can also be obtained by inserting \( e_{zz} = e_{xz} = e_{yz} = 0 \) into the constitutive equation (refer to Appendix D). The plane strain assumption ignores the deformation out of the \( x-y \) plane, so the analysis in this study is only suitable for deep sections of a wellbore.

Figure 3. A Wellbore Cross Section and the Applied Loads.

The stress and pore pressure in the \( x-y \) plane are \( \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \) and \( p^{sh} \). For convenience, we use the mean stress \( P_0 \) and the deviatoric stress \( S_0 \), and \( \theta_r \) instead of \( \sigma_{xx}, \sigma_{yy}, \sigma_{xy} \).
\[ P_0 = \frac{\sigma_{xx} + \sigma_{yy}}{2} \]

\[ S_0 = \frac{1}{2} \sqrt{\left( \sigma_{xx} - \sigma_{yy} \right)^2 + 4\sigma_{xy}^2} \]

\[ \theta_r = \frac{1}{2} \tan^{-1} \left( \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} \right) \]  \hspace{1cm} (38)

where \( \theta_r \) is the angle between the larger principal stress in x-y plane and the x-axis.

Cylindrical coordinates are more convenient than Cartesian coordinates to analyze the stresses and strains around the wellbore. Therefore, cylindrical coordinates are used with theta measured counterclockwise from the direction of the maximum in-situ stress (figure 4).

![Figure 4. Cylindrical Stresses Acting in the Plane Perpendicular to the Wellbore.](image)

Drilling causes the mechanical, thermal and chemical loading on the wall of wellbore. Carter and Booker (1982) and Detouranay and Cheng (1988) provided a method to decompose the loading into 3 different fundamental modes for a pure poroelastic model. The same method is used herein to solve this coupled problem.

(i) Mode I: mean stress loading
\[ \sigma_{rr}^I = p^m - P_0 \]
\[ \sigma_{r\theta}^I = 0 \]
\[ p^I = 0 \]
\[ T^I = 0 \]
\[ C^I = 0 \]  \hspace{1cm} (39) 

(ii) mode II: loadings of pore pressure, temperature and solute.

\[ \sigma_{rr}^{II} = 0 \]
\[ \sigma_{r\theta}^{II} = 0 \]
\[ p^{II} = p^m - p^{sh} \]
\[ T^{II} = T^m - T^{sh} \]
\[ C^{II} = C^m - C^{sh} \]  \hspace{1cm} (40) 

(iii) Mode III: deviatoric loading.

\[ \sigma_{rr}^{III} = -S_0 \cos 2\theta \]
\[ \sigma_{r\theta}^{III} = S_0 \sin 2\theta \]
\[ p^{III} = 0 \]
\[ T^{III} = 0 \]
\[ C^{III} = 0 \]  \hspace{1cm} (41) 

Where \( \sigma_{rr}, \sigma_{r\theta}, p, T \) and \( C \) are change of radial stress, shear stress, pore pressure, temperature, and solute mass fraction on the wall of the wellbore. Note that the changes vanish at infinity.

Governing Constitutive and Field Equations

Based on the first and second law of thermodynamics and mass balance law, the constitutive equations for the shale formation saturated by a binary electrolyte are
developed by Diek and Ghassemi (2004). The transport equations for fluid, solute and heat are also developed.

**Rock Constitutive Equations**

Using the tension positive convention, the linear responses of stress $\sigma_{ij}$ and pore volume $\zeta$ to the strain components, $\varepsilon_{ij}$, pore pressure, $p$, solute mass fraction, $C_s$, temperature, $T$ are as follows (Diek and Ghassemi, 2004):

$$\sigma_{ij} = 2G\dot{\varepsilon}_{ij} + (K - \frac{2G}{3})\ddot{\varepsilon}_{ij} - \alpha' \dot{p}\delta_{ij} + \chi\dot{C}_s\delta_{ij} - \gamma_1\dot{T}\delta_{ij}$$  \hspace{1cm} (42)

$$\zeta = \alpha\dot{\varepsilon} + \beta\dot{p} + \chi\dot{C}_s - \gamma_2\dot{T}$$  \hspace{1cm} (43)

Where the constants:

$$\alpha' = \left(\alpha - \frac{\alpha'^3 c}{\rho_f}\right)$$

$$\beta' = Q + \frac{\phi}{K_f} + \frac{B^D c}{\rho_f}$$

$$Q = \frac{\alpha - \phi}{K_s}$$

$$B^D = \frac{(\alpha - 1)\alpha'^3}{K}$$

$$\chi' = \frac{(\alpha - 1)}{K}\chi$$

$$\chi = (\alpha'^3 - \alpha'^3 c)\frac{R T_0}{M s}$$

$$c = \frac{1}{C^D}$$

$$\gamma_1 = K\alpha_m + s f\alpha'^3 c$$

$$\gamma_2 = \alpha\alpha_m + (\alpha_f - \alpha_m)\phi + s f\alpha'^3 c$$  \hspace{1cm} (44)
Where $\overline{C^S}$ and $\overline{C^D}$ denote the mean values of the solute and diluent mass fraction, respectively. $\overline{s_f}$ is the mean value of the specific fluid entropy, $T_0$ is the absolute temperature of the undistributed or reference state of the system, and $\omega^S$ and $\omega^D$ are the chemical swelling parameters associated with the solute and the diluent in a binary electrolyte solution, respectively. The above constitutive equations (42) and (43) specify the total stresses and variation of fluid content in terms of six independent variables, namely the three displacements $u_i$ related to the solid strains $\varepsilon_{ij}$, the pore pressure $p$, the solute mass fraction $C^S$, and the absolute temperature $T$.

If we define the chemo-mechanical parameters $\omega^S$ and $\omega^D$ for a binary electrolyte solution by:

$$\omega^S = \omega^D = \omega_0 \frac{M^S}{RT_0} \quad (45)$$

The coefficients in Eqn. (42) and (43) can be rewritten as:

$$\dot{\alpha} = \left( \alpha - \frac{M^S \omega_0}{C^D RT_0 \rho_f} \right)$$

$$\beta = Q + \frac{\phi_0}{K_f} + \frac{\omega_0 (\alpha - 1) M^S}{K C^D RT_0 \rho_f}$$

$$\chi = \frac{\omega_0}{C^S} \left( 1 - \frac{\overline{C^S}}{\overline{C^D}} \right)$$

$$\dot{\chi} = \frac{\alpha - 1}{K} \chi$$

$$\gamma_1 = K \alpha_m + \frac{\overline{s_f} a^D \omega_0 M^S}{RT_0}$$
\[
\gamma_2 = \alpha \alpha_m + (\alpha_f - \alpha_m) \phi_0 + \frac{\bar{s}_f \alpha D \omega_0 (\alpha - 1) M^S}{K R T_0} \tag{46}
\]

**Transport Equations**

In the case of no inertia, the dissipation function can be represented by (Diek and Ghassemi 2004):

\[
2D = -\omega T \nabla p - J^S \cdot \nabla (\mu^S - \mu^D) - J^q \cdot \frac{\nabla T}{T} \tag{47}
\]

Note that \(J^f\), \(J^S\), and \(J^q\) are the independent fluxes produced by the independent forces:

\[
X^f = -(\nabla p / \bar{\rho_f}) \, , \quad X^S = -\nabla (\mu^S - \mu^D) \, , \quad \text{and} \quad X^q = -(\nabla T / T) \, , \quad \text{respectively.}
\]

Accordingly, the phenomenological equations may be written as:

\[
J^f = -L^{11} \frac{\nabla p}{\bar{\rho_f}} - L^{21} \nabla (\mu^S - \mu^D) - \frac{L^{13}}{T} \nabla T \tag{48}
\]

\[
J^S = -L^{21} \frac{\nabla p}{\bar{\rho_f}} - L^{22} \nabla (\mu^S - \mu^D) - \frac{L^{23}}{T} \nabla T \tag{49}
\]

\[
J^q = -L^{31} \frac{\nabla p}{\bar{\rho_f}} - L^{32} \nabla (\mu^S - \mu^D) - \frac{L^{33}}{T} \nabla T \tag{50}
\]

In other words, the driving forces of the bulk fluid, solute, and heat flows are purely physico-chemical in nature (gradients of hydraulic pressure, chemical potential, and absolute temperature). The phenomenological coefficients \(L^{\alpha \beta}\) are independent of the driving forces but may depend on the pore pressure, temperature, and the solute concentration. As a first approximation, one assumes the following phenomenological coefficients for isotropic flows:

\[
L^{11} = \frac{\bar{\rho_f} k}{\eta} ; \quad L^{12} = L^{11} \mathcal{R} ; \quad \frac{L^{13}}{T} = -K^T
\]
\[ L^{31} = \rho_f^2 \xi \rho \; ; \; L^{32} = \frac{\tilde{\rho}_f^2 \tilde{D}^S}{A} ; \; L^{33} = \rho_f^T \tilde{D}^T \]

\[ L^{31} = \tilde{\rho}_f \xi \tilde{\rho} \; ; \; L^{32} = \frac{\tilde{\rho}_f \tilde{\rho}^T}{A} ; \; L^{33} = k^T \]  \hspace{1cm} (51)

Where \( A = \frac{\tilde{\rho}_f^T RT_0}{M^S \tilde{C}^S \tilde{C}^D} \) \hspace{1cm} (52)

For dilute solutions, one may use the following approximation:

\[ \nabla (\mu^S - \mu^D) \approx \frac{A}{\tilde{\rho}_f} \nabla C^S \]  \hspace{1cm} (53)

Next, it is assumed that Onsager’s reciprocity theorem is not valid (\( L^{\alpha\beta} \neq L^{\beta\alpha} \)) for shales, such that the components of the fluxes satisfy the following order of magnitudes:

\[ |L^{21} X^f| \ll |L^{22} X^S| \]

\[ |L^{31} X^f| \sim |L^{32} X^S| \ll |L^{33} X^q| \]

Alternatively, one can assume the vanishing of some of the phenomenological coefficients associated with the insignificant components of fluxes:

\[ L^{21} \sim L^{31} \sim L^{32} \sim 0 \]

The resulting significant fluxes are:

\[ J^f = -\frac{\tilde{\rho}_f k}{\eta} (\nabla p - \Re A \nabla C^S) + K^T \nabla T \]

\[ J^S = -\tilde{\rho}_f D^S \nabla C^S - \tilde{\rho}_f D^T \nabla T \]

\[ J^q = -k^T \nabla T \]  \hspace{1cm} (54)

The bulk fluid flows due to the coupling of three processes: hydraulic conduction, chemical osmosis, and thermal convection. The chemical flux is due to the coupling
of chemical solute diffusion and thermal osmosis. The diffusion of heat is only due to the gradient of the gradient of the absolute temperature.

**Field Equations**

The field equations are developed by substituting the suggested phenomenological and constitutive equations into the momentum (stress equilibrium), mass, and energy balance equations. The momentum balance states the mechanical equilibrium condition:

$$\sigma_{ij,ij} = 0 \quad (55)$$

The mass balances state the conservation of fluid and solute masses, respectively (Diek and Ghassemi, 2004):

$$\zeta + \nabla \cdot \dot{w} = 0 \quad (56)$$

$$\bar{\rho} \dot{C}^s + \nabla \cdot J^s = 0 \quad (57)$$

And finally, the energy balance is given by (Diek and Ghassemi, 2004):

$$\rho_T c_T \dot{\bar{T}} + \nabla \cdot \dot{J}^s = 0 \quad (58)$$

Where $\rho_T$ and $c_T$ denote the total mass density and specific heat, and where we ignored: (1) the terms proportional to $\varepsilon_\theta$, $\dot{p}$, and $\mu^\theta$ representing the inter-convertibility of thermal, mechanical, and chemical energy; (2) the heat transfer due to the fluid and solute flows.

**Navier-type Equations**

Substituting (37) into (54), one gets the first three or coupled Navier-type field equations:

$$\left( K + \frac{G}{3} \right) \nabla (\nabla \cdot u) + G \nabla^2 u - \alpha \nabla p + \chi \nabla C^s - A \nabla T = 0 \quad (59)$$
Fluid Diffusion Equation

Conservation of a weakly compressible and weakly thermally expansible fluid mass in the absence of a volume source yields the local continuity equation (55) which has been linearized by neglecting the effect of the spatial fluid density variation. Substituting (38) and (51) into (55), one gets the fourth field equation or the coupled fluid diffusion equation:

\[
\alpha(\nabla \cdot \dot{u}) + \beta \dot{p} + \chi \dot{C}^S + A_z \dot{T} - \frac{k}{\eta} [\nabla^2 p - \Re A \nabla^2 C^S] + K^T \nabla^2 T = 0 \quad (60)
\]

Solute Diffusion Equation and Thermal Diffusion Equation

Conservation of the solute mass in rocks yields Eqn. (56). The fifth field equation or solute diffusion equation can be obtained by substituting (52) into (56):

\[
\phi \dot{C}^S - D^S \nabla^2 C^S - \overline{C}^S D^T \nabla^2 T = 0 \quad (61)
\]

Substituting (17) into (21), one gets the thermal diffusion or sixth field equation:

\[
\rho \tau_c \dot{T} - k^T \nabla^2 T = 0 \quad (62)
\]

Solutions of Distributions of Solute, Stress/Pore Pressure, Strain and Displacement

Combined the field equation and the boundary conditions, the solutions are obtained (refer to Appendix B and C for the derivations). The tension positive convention is used in the following solutions.

All of the following solutions are expressed in s-space (Laplace transform space). Numerical inversion is needed to apply the solutions into real problems. But, this inversion routine can not be used for very short times so that Detournay and Cheng (1988) derived short time asymptotic solutions. Certainly the chemo-poro- thermoelastic asymptotic solutions at small time also can be derived. However, the drawback of such an approach is that one needs to judge when to use the short term
solution or the long term solutions. To solve the problem, a more general solution in the Laplace or s-space is developed which can be numerically inverted for any time.

It is easy to note that all of the solutions in s-space (mode II and mode III) include terms which involve the ratios of the modified second kind Bessel functions, \( \frac{K_m(\lambda r)}{K_n(\lambda a)} \), where \( m \) and \( n \) are the order number, \( \lambda \) is a \( s \) dependent coefficient, \( r \) is the radial distance and \( a \) is the well radius of wellbore. When the solutions are applied to small time, both \( K_m(\lambda r) \) and \( K_n(\lambda a) \) approach zero. Thus, the numerical solution for the ratio \( \frac{K_m(\lambda r)}{K_n(\lambda a)} \) is not available, and the solutions can not be numerically inverted for small time. However, If we use the function \( K'_m(\lambda r) = K_m(\lambda r)e^{\lambda r} \) instead of \( K_m(\lambda r) \) then, \( K'_m(\lambda r) \) has a finite value no matter how small the time. The ratio \( \frac{K_m(\lambda r)}{K_n(\lambda a)} \) can be rewritten as \( \frac{K_m(\lambda r)}{K'_n(\lambda a)} e^{-\lambda(r-a)} \). Therefore, the general solutions for any time can be obtained by substituting \( \frac{K'_m(\lambda r)}{K'_n(\lambda a)} e^{-\lambda(r-a)} \) for \( \frac{K_m(\lambda r)}{K'_n(\lambda a)} \) in the numerical inversion part of the program codes.

Mode I

\[
2GU_{rr}^I = -(P^h - P^m) \frac{a^2}{r} \quad (63)
\]

\[
\sigma_{rr}^I = (P^h - P^m) \frac{a^2}{r^2} \quad (64)
\]

\[
\sigma_{\theta\theta}^I = -(P^h - P^m) \frac{a^2}{r^2} \quad (65)
\]

\[
e_{rr}^I = (P^h - P^m) \frac{a^2}{r^2 2G} \quad (66)
\]

\[
e_{\theta\theta}^I = -(P^h - P^m) \frac{a^2}{r^2 2G} \quad (67)
\]

mode II
\[ T = \frac{T_M - T_{sh}}{sK_0(\beta_T)} K_0(\xi_T) \]  

(68)

\[ C^S = \left[ (C^S_S - C^S_{sh}) + \frac{C^S D^T (T_M - T_{sh})}{C_T \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} \right] K_0(\xi_{ch}) - \frac{C^S D^T (T_M - T_{sh})}{sK_0(\beta_{ch})} K_0(\xi_T) \]  

(69)

\[ p = \left[ p_M - p_0 + \frac{Y_{ch}}{c \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} - \frac{Y_T}{c \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} \right] \frac{K_0(\xi)}{sK_0(\beta)} - \frac{Y_{ch}}{c \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} \]  

(70)

\[ U_r = \left[ p_M - p_0 + \frac{Y_{ch}}{c \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} - \frac{Y_T}{c \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} \right] a\eta\alpha \left[ a K_1(\beta) \frac{K_1(\xi)}{\beta K_0(\beta)} \right] + \]  

\[ - \frac{Y_{ch}}{c \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} \frac{C^S_S - C^S_{sh}}{C_T \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} \frac{a\eta\chi}{G\alpha s} \left[ a K_1(\beta_{ch}) \frac{K_1(\xi_{ch})}{\beta_{ch} K_0(\beta_{ch})} \right] \]  

(71)
\[ E_{rr}^{\prime\prime} = \left[ p_M - p_0 + \frac{Y_{ch}}{c \phi (D^3 - 1)} \right] \eta \alpha \left[ -\frac{a^2}{r^2} K_1(\beta) + a K_1(\xi) + K_0(\xi) \right] + \left( \begin{array}{c}
\frac{Y_{ch}}{c \phi} \eta \alpha \left[ (C_M - C_{sh}) + \frac{C^S D^T (T_M - T_{sh})}{\beta} \right] \eta \chi \left[ -\frac{a^2}{r^2} K_1(\beta) + a K_1(\xi) + K_0(\xi) \right] \\
\frac{Y_T}{c} \left[ (C_T - C_{sh}) + \frac{C^S D^T (T_M - T_{sh})}{C_T \phi} \right] \eta \chi \left[ -\frac{a^2}{r^2} K_1(\beta) + a K_1(\xi) + K_0(\xi) \right]
\end{array} \right)
\]

(72)

\[ E_{\theta\theta}^{\prime\prime} = \left[ p_M - p_0 + \frac{Y_{ch}}{c \phi (D^3 - 1)} \right] \eta \alpha \left[ -\frac{a^2}{r^2} K_1(\beta) + a K_1(\xi) \right] + \left( \begin{array}{c}
\frac{Y_{ch}}{c \phi} \eta \alpha \left[ (C_M - C_{sh}) + \frac{C^S D^T (T_M - T_{sh})}{\beta} \right] \eta \chi \left[ -\frac{a^2}{r^2} K_1(\beta) + a K_1(\xi) + K_0(\xi) \right] \\
\frac{Y_T}{c} \left[ (C_T - C_{sh}) + \frac{C^S D^T (T_M - T_{sh})}{C_T \phi} \right] \eta \chi \left[ -\frac{a^2}{r^2} K_1(\beta) + a K_1(\xi) + K_0(\xi) \right]
\end{array} \right)
\]

(73)
\[
\tilde{S}_{rr}^{ii} = \left[ p_M - p_0 + \frac{Y_{ch}}{c \phi \left( \frac{D^5}{D^3 - 1} \right)} - \frac{Y_T}{c \overline{C_T} \left( \frac{D^5}{D^3 - 1} \right)} \right] \frac{2 \eta \alpha}{\alpha s} \left[ \frac{a^2}{r^2} K_1(\beta) - \frac{a}{r} K_1(\xi) \right] + \\
\left\{ \frac{Y_{ch}}{c \overline{C_T} \left( \frac{D^5}{D^3 - 1} \right)} \frac{2 \eta \alpha}{\alpha s} \left[ \left( C_M \overline{C_{sh}} \right) + \frac{C^5 D^7 (T_M - T_{sh})}{C_T \phi \left( \frac{D^5}{D^3 - 1} \right)} \right] \right\} \frac{2 \eta \chi}{\alpha s} \left[ \frac{a^2}{r^2} K_1(\beta_{ch}) - \frac{a}{r} K_1(\xi_{ch}) \right]
\]

\[
\tilde{S}_{00}^{ii} = \left[ p_M - p_0 + \frac{Y_{ch}}{c \phi \left( \frac{D^5}{D^3 - 1} \right)} - \frac{Y_T}{c \overline{C_T} \left( \frac{D^5}{D^3 - 1} \right)} \right] \frac{2 \eta \alpha}{\alpha s} \left[ \frac{a^2}{r^2} K_1(\beta) - \frac{a}{r} K_1(\xi) \right] - \\
\left\{ \frac{Y_{ch}}{c \overline{C_T} \left( \frac{D^5}{D^3 - 1} \right)} \frac{2 \eta \alpha}{\alpha s} \left[ \left( C_M \overline{C_{sh}} \right) + \frac{C^5 D^7 (T_M - T_{sh})}{C_T \phi \left( \frac{D^5}{D^3 - 1} \right)} \right] \right\} \frac{2 \eta \chi}{\alpha s} \left[ \frac{a^2}{r^2} K_1(\beta_{ch}) - \frac{a}{r} K_1(\xi_{ch}) \right]
\]

(74)

(75)
Fluid flow

\[
\tilde{J}^f = \tilde{\rho} f k \left[ p_M - p_0 + \frac{Y_{ch}}{(c' \phi - 1)} - \frac{Y_T}{(C_T - 1)} \right] \left[ \beta \frac{K_1(\xi)}{sa K_0(\beta)} + \rho \frac{RT_0}{M^S C^S C^D} \right] \frac{K_1(\xi_{ch})}{K_0(\beta_{ch})}
\]

\[
\tilde{\rho} f k \beta_{ch} \left[ \frac{Y_{ch}}{c' \phi - 1} \left( (C_M - C_{ch}) + \frac{C^S D^T (T_M - T_{ch})}{C_T \phi (D^S C_T - 1)} \right) \right] \frac{K_1(\xi_{ch})}{K_0(\beta_{ch})}
\]

\[
\tilde{\rho} f k \beta_T \left[ \frac{Y_T}{c' \phi - 1} \left( \frac{C^S D^T (T_M - T_{ch})}{C_T \phi (D^S C_T - 1)} \right) \right] \frac{K_1(\xi_T)}{K_0(\beta_T)}
\]

(76)

Solute flux

\[
\tilde{J}^S = \tilde{\rho} f \left[ (C_M - C_{ch}) + \frac{C^S D^T (T_M - T_{ch})}{C_T \phi (D^S C_T - 1)} \right] \frac{D^S \beta_{ch} K_1(\xi_{ch})}{sa K_0(\beta_{ch})} + \rho f \beta_T \left[ D^T (T_M - T_{ch}) - \frac{C^S D^T D^S (T_M - T_{ch})}{s C_T \phi (D^S C_T - 1)} \right] \frac{K_1(\xi_T)}{K_0(\beta_T)}
\]

(77)

Heat flow

\[
\tilde{J}^q = -k^T \beta_T (T_M - T_{ch}) \frac{K_1(\xi_T)}{sa K_0(\beta_T)}
\]

(78)
\[ Y_{ch} = c' \left[ (C_M^s - C_{sh}^s) + \frac{C_s^s D^\prime (T_M^s - T_{sh}^s)}{C_s^s \phi (D^s / C_s^s \phi - 1)} \right] \]

\[ Y_f = (T_M^s - T_{sh}) \left[ \frac{C_s^s D^\prime}{C_s^s \phi (D^s / C_s^s \phi - 1)} - c_f \right] \quad (79) \]

\[ c' = \frac{k}{\left( \frac{\eta}{G} \alpha' + Q + B \right)} = \frac{c_f}{1 - c_0} \quad (80) \]

\[ c_0 = \frac{c_f}{k} \left[ \frac{1 - \alpha(1 - 2v)^2}{G(1 - v^2)} \right] \omega_0 M^s \rho_f = \frac{c_f}{k} \left[ \frac{1 - 2\eta(1 - 2v)}{G(1 + v)} \right] \omega_0 M^s \frac{C^D}{C^D T_0 \rho_f} \]

\[ c'_f = \frac{2kG(1 - v)(v_u - v)}{\alpha^2 (1 - v_u)(1 - v)^2} \quad (82) \]

\[ c' = -\frac{c'_f}{(1 - c_f)k} \left[ \frac{kR \phi \rho_f T_0}{D^s M^s C^s C^D} + \frac{(\alpha - 1)}{K} - \frac{\eta}{G} \right] \chi' \quad (83) \]

\[ c'_{\phi} = \frac{c'_0}{1 - c_0} \quad (84) \]

\[ c'_0 = \frac{c'_f}{k} \left[ \frac{2(1 - 2v) \alpha \alpha_m + (\alpha_f - \alpha_m) \phi \phi_0 + \frac{\eta_0 M^s}{C^D T_0}}{K} \omega_0 (\alpha - 1) \right] \quad (85) \]

Mode III

\[ s_{p \, III} \]

\[ S_0 \cos 2\theta = \frac{2(1 - v)C_s K_2(\xi) / K_2(\beta) + 2(1 - 2v)(1 - 2v) / (1 - v_u) - 3(1 - v)/A(1 + v)}{r^2} \]

\[ (1 - 2v)(1 - v_u)(1 - 2v) \alpha^2 + [2\alpha(1 - 2v) - 3(1 - v)/A(1 + v) \]

\[ \times \left[ K_2(\xi) / \xi + (1 + 6/\xi^2)K_2(\xi) \right] \quad (86) \]

\[ s_{s \, r_{\, III}} \]

\[ \frac{2(v_u - v)(\alpha - A)C_i}{(1 - v_u)(1 - 2v) \alpha^2 + [2\alpha(1 - 2v) + (v - 3)(v_u - v)/A(1 + v) \]

\[ - (1 + v)(1 - 2v) \alpha^2 + (1 - 3 - 2\alpha \nu)(v_u - v)/A \frac{2C_s a^2}{r^2} - \frac{6C_s a^4}{r^4} \]

\[ (1 - v_u)(1 + v)(1 - 2v) \alpha^2 + [2\alpha(1 - 2v) - 3(1 - v)(v_u - v)/A(1 + v) \]

\[ \times \left[ K_2(\xi) / \xi + (1 + 6/\xi^2)K_2(\xi) \right] \quad (87) \]

35
\[
\begin{align*}
\mathcal{S}_9^m &= - \frac{2(v_u - v)(\alpha - A)C_1}{S_0 \cos 2\theta \left( (1-v_u)(1-2v)\alpha^2 + [2\alpha(1-2v)+(v-3)](v_u-v)A/(1+v) \right) \times \left[ \frac{K_1(\xi)/\xi + 6K_2(\xi)/\xi^2}{K_2(\beta)} + 6C_3a^4 \right]} \\
\mathcal{S}_{r\theta}^m &= \frac{4(v_u - v)(\alpha - A)C_1}{S_0 \sin 2\theta \left( (1-v_u)(1-2v)\alpha^2 + [2\alpha(1-2v)+(v-3)](v_u-v)A/(1+v) \right) \times \left[ \frac{K_1(\xi)/\xi + 6K_2(\xi)/\xi^2}{K_2(\beta)} \right] - \frac{(1+v)(1-2v)\alpha^2 + (\alpha - 3 - 2\alpha\nu)(v_u - v)A}{(1-v_u)(1+v)(1-2v)\alpha^2 + [2\alpha(1-2v)-3(1-v)](v_u-v)A} \times \frac{C_2a^2 - 6C_3a^4}{r^2} } \\
\mathcal{S}_{\theta\theta}^m &= \frac{(v_u - v)(\alpha - A)C_1}{S_0 \cos 2\theta \left( (1-v_u)(1-2v)\alpha^2 + [2\alpha(1-2v)+(v-3)](v_u-v)A/(1+v) \right) \times \left[ \frac{K_1(\xi)/\xi + (1+6/\xi^2)K_2(\xi)}{K_2(\beta)} \right] - \frac{2\nu/(1-2v) + \alpha Q_2}{2(1-v)/(1-2v) + \alpha Q_2} \times \frac{C_2a^2 - 3C_3a^4}{r^2}} \\
\mathcal{S}_{\nu\nu}^m &= \frac{(v_u - v)(\alpha - A)C_1}{S_0 \sin 2\theta \left( (1-v_u)(1-2v)\alpha^2 + [2\alpha(1-2v)+(v-3)](v_u-v)A/(1+v) \right) \times \left[ \frac{K_1(\xi)/\xi + (1+6/\xi^2)K_2(\xi)}{K_2(\beta)} \right] - \frac{1/(1-2v) + \alpha Q_2}{2(1-v)/(1-2v) + \alpha Q_2} \times \frac{C_2a^2 - 3C_3a^4}{r^2}} \\
\mathcal{S}_{\nu\theta}^m &= \frac{C_2a^2}{r^2} \\
\mathcal{S}_{\nu\nu}^m &= \frac{(v_u - v)(\alpha - A)C_1}{S_0 \cos 2\theta \left( (1-v_u)(1-2v)\alpha^2 + [2\alpha(1-2v)+(v-3)](v_u-v)A/(1+v) \right) \times \left[ \frac{K_1(\xi)/\xi + 2K_2(\xi)/\xi^2}{K_2(\beta)} + \frac{C_2a^2}{r} + \frac{C_3a^4}{r^4} \right]}
\end{align*}
\]
\[
\frac{sG U_{\theta}}{aS_{0} \cos 2\theta} = \frac{\sigma_{zz}}{2(1-v)/(1-2v) + \alpha Q_{2}} = -\frac{(v_{u} - v)(\alpha - A)C_{1}}{\frac{2K_{2}(\xi)}{\xi^{2}}} - \frac{C_{2} a^{2}}{r} + C_{4} a^{4} \frac{r}{r^{3}}
\]

(95)

Where \( S_{0} = (\sigma_{ii} - \sigma_{zz}) / 2 \)

\[
A = \frac{M^{S} \omega_{0}}{C^{D}RT_{0} \rho_{f}}
\]

\[
Q_{2} = \frac{(\alpha - A)/G}{1/M + (\alpha - 1)A/K}
\]

\[
C_{1} = -\frac{2(1-v)/(1-2v) + \alpha Q_{2}}{(1-v)\alpha(1-2v)^{2} + (1-v)Q_{2}} - \frac{Q_{2}K_{1}(\beta)}{\beta K_{2}(\beta)}
\]

\[
C_{2} = \frac{(1-v)[2(1-v)/(1-2v) + \alpha Q_{2}]}{(1-v)/(1-2v) + \alpha(1-v)Q_{2}} - \frac{\alpha(1-2v)Q_{2}K_{1}(\beta)}{\beta K_{2}(\beta)}
\]

\[
C_{3} = -\frac{1}{6} \frac{(1-v)(5-4v)}{(1-2v)^{2}} + 3\alpha(1-v)Q_{2} + \frac{3\alpha(1-2v)Q_{2}K_{1}(\beta)}{\beta K_{2}(\beta)} + \frac{12\alpha(1-2v)Q_{2}}{\beta^{2}}
\]

(96)

Stress along z-axis

\[
\sigma_{zz} = \sigma_{rr} + \sigma_{rr} \frac{\cdot}{} + \sigma_{rr} \frac{\cdot}{} + \sigma_{\theta\theta} + \sigma_{\theta\theta} \frac{\cdot}{} + \sigma_{\theta\theta} \frac{\cdot}{} + \sigma_{\phi\phi} + \sigma_{\phi\phi} \frac{\cdot}{} + \sigma_{\phi\phi} \frac{\cdot}{}
\]

\[
- (1-2v)\left( \alpha^{\cdot} p + \alpha^{\cdot} p + \chi C^{S} + \gamma_{T} \right)
\]

(97)
CHAPTER IV

ANALYSIS OF THE PLANE STRAIN SOLUTIONS IN TIME DOMAIN

The solutions for stress, pore pressure, strain and distances and other induced results are in the Laplace transform space. It is not possible to analytically invert all of these to the time domain so that only numerical values by numerical inversion of Laplace transformation are obtained. This is accomplished using the method of Stehfest (1970).

Experimental data on the full range of chemo-poro-thermoelastic properties of shale are not commonly available. The data set (Table 1) used here has been extracted from Cui et al. (1997, 1998); van Oort et al. (1994), Tanner (1953) and Lide (2001). All the results presented in the paper use the geomechanics sign convention, i.e., compression positive.

Axi-symmetric Loading and Responses

The rock is considered to be homogeneous and isotropic so that the perturbations caused by mud pressure, solute mass fraction and temperature are isotropic, therefore the induced stress, pore pressure, strain and displacements are independent on the direction. For the axi-symmetric loadings, the stress, strain and pore pressure in the shale formation may change directly by the following processes:

(i) Hydro-mechanical process (poroelastic effect): the fluid flow caused by the pore pressure gradient (mud pressure loading) results in the change of pore pressure and volume in the shale formation, and induces stress and strains in the shale;
(ii) Thermal expansion/contraction (thermoelastic effect): the
temperature change (thermal loading) directly causes the
expansion/contraction of the solid, and the fluid in the pores which induces
stress, strain and pore pressure of the shale;

(iii) Chemical osmosis: the chemical potential gradient (chemical
loading) drives the diluent flow which results in the change of pore
pressure, stress and strain of the shale;

(iv) Physico-chemical interactions: hydration swelling and
shrinkage related to invasion of shale by diluent and solute.

\[ \frac{\dot{\sigma}_{kk}}{3} = \frac{K\dot{C}}{3} - (\alpha - A)\dot{p} + \chi\frac{C^s}{2}(\beta_1 + \beta_2)\dot{T} \quad (98) \]

\[ A = \frac{M^s \omega_0}{C^0 RT_0 \rho_f} \quad \beta_1 = K\alpha_m \quad \beta_2 = \frac{s_o \omega_0 M^s}{C^0 RT_0} \]

Eqn. (98) (tension positive) derived from the constitutive equation Eqn. (42) in
Chapter III describes the linear responses of total stress \( \sigma_{kk} \) to volumetric strain \( \varepsilon_{kk} \),
pore pressure \( p \), solute mass fraction \( C^s \) and temperature \( T \). If chemical osmosis is
ignored and a drained condition ( \( \dot{p} = 0 \) ) with \( \sigma_{kk} = 0 \) is considered, Eqn. (98) yields:

\[ \frac{K\dot{C}}{3} = -\chi\frac{C^s}{2}(\beta_1 + \beta_2)\dot{T} \quad (99) \]

This equation indicates that the increase of solute concentration causes a positive
volumetric strain i.e., contraction of the shale matrix, and the increase of the
temperature causes a negative volumetric strain i.e., expansion.

Considering the fluid response:
\[
\zeta = \alpha \varepsilon_{kk} + \beta \dot{p} - \frac{1-\alpha}{K} \chi C^S - (\alpha_1 - \alpha_2) \dot{T}
\]

(100)

\[
\alpha_1 = (\alpha\alpha_m + (\alpha_f - \alpha_m)\phi) \quad \alpha_2 = \frac{s_m \alpha_h (1-\alpha) M^S}{C^D KRT_0}
\]

\[
\beta \dot{p} = \frac{1-\alpha}{K} \chi C^S + (\alpha_1 - \alpha_2) \dot{T}
\]

(101)

For undrained condition and \(\dot{\varepsilon}_{zz} = 0\), Eqn. (101) can be obtained from Eqn. (100) (tension positive). It can be seen that increasing \(C^S\) tends to increase the pore pressure in the formation (fluid is lost when drained). This is concomitant with the volumetric contraction of the shale described earlier. The thermal expansion of shale formation due to the increase of \(T\) causes the pore pressure to increase; but as mentioned above, the rock swells with an increase of \(T\) which tends to decreases the pore pressure, therefore the magnitude of the coefficient \((\alpha_1 - \alpha_2)\) determines whether the pore pressure increases or decreases with temperature.

The chemo-poro-thermoelastic model are used some cases to explore the impacts of salinity and temperature on the induced stress, strain, pore pressure and strain when drilling a borehole in deep shale formation. For the heating cases \((T^m > T^{sh})\), we set the mud temperature \(T^m = 115 \, ^\circ C\) and the initial temperature of the shale formation \(T^{sh} = 65 \, ^\circ C\). For the cooling case \((T^m < T^{sh})\), \(T^{sh}\) is \(115 \, ^\circ C\) and \(T^m\) is \(65 \, ^\circ C\). the average system temperature is considered to be \(90 \, ^\circ C\) in those cases and \(T^m = T^{sh} = 90 \, ^\circ C\) is for the isothermal cases. The initial solute mass fraction of the mud \((C^m)\) and shale \((C^{sh})\) are set 0.2 and 0.1 for the case \(C^m > C^{sh}\), respectively, and 0.1 and 0.2 for the case \(C^m < C^{sh}\), respectively.
Distribution of Temperature, Solute Mass Fraction and Their Impacts on Pore Pressure

In the phenomenological equations, the gradients of temperature, solute mass fraction and pore pressure are the driving forces and produce flows of heat, solute, and liquid. Heat transfer is assumed to be driven only by the temperature gradient.

Table 1. Parameters Used for Chemo-poro-thermoelastic Responses of Wellbore.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear modulus ( G )</td>
<td>760 MPa</td>
</tr>
<tr>
<td>Biot’s coefficient ( \alpha )</td>
<td>0.966</td>
</tr>
<tr>
<td>Drained Poisson’s ratio ( \nu )</td>
<td>0.219</td>
</tr>
<tr>
<td>Undrained Poisson’s Ratio ( \nu_u )</td>
<td>0.461</td>
</tr>
<tr>
<td>Permeability coefficient ( \kappa )</td>
<td>0.333 x10^{-16} m²/Pa.s</td>
</tr>
<tr>
<td>Porosity ( \phi )</td>
<td>0.2989</td>
</tr>
<tr>
<td>Mean solute fraction ( C^0 )</td>
<td>0.15</td>
</tr>
<tr>
<td>Fluid mass density ( \rho_f )</td>
<td>1111.11 kg/m³</td>
</tr>
<tr>
<td>Fluid bulk modulus ( K_f )</td>
<td>3291 MPa</td>
</tr>
<tr>
<td>Reflection coefficient ( \Re )</td>
<td>0.2</td>
</tr>
<tr>
<td>Molar mass of solute (NaCl) ( M_s )</td>
<td>0.0585 kg/mol</td>
</tr>
<tr>
<td>Swelling coefficient ( \omega )</td>
<td>1.5 MPa</td>
</tr>
<tr>
<td>Solute diffusivity ( D^s )</td>
<td>2.0x10^{-7} m²/s</td>
</tr>
<tr>
<td>Thermal expansion coefficient of solid ( \alpha_m )</td>
<td>1.8x10^{-5} K^{-1}</td>
</tr>
<tr>
<td>Thermal expansion coefficient of fluid ( \alpha_f )</td>
<td>3.0x10^{-4} K^{-1}</td>
</tr>
<tr>
<td>Thermal diffusivity ( c^f )</td>
<td>1.6x10^{-6} m²/s</td>
</tr>
<tr>
<td>Coefficient of thermal diffusion ( D^f )</td>
<td>6.0x10^{-12} m²/(s.K)</td>
</tr>
<tr>
<td>Specific entropy (NaCl, ( C^0 = 0.15 )) ( S_0 )</td>
<td>3686 J/(kg.K)</td>
</tr>
<tr>
<td>Skempton’s coefficient ( B )</td>
<td>0.915</td>
</tr>
<tr>
<td>Fluid diffusivity ( c^f )</td>
<td>6.0 x10^{-8} m²/s</td>
</tr>
<tr>
<td>Fluid bulk modulus ( K_f )</td>
<td>3290 MPa</td>
</tr>
<tr>
<td>Elastic modulus ( E )</td>
<td>1853 MPa</td>
</tr>
<tr>
<td>Bulk modulus ( K )</td>
<td>1099 MPa</td>
</tr>
<tr>
<td>Solid bulk modulus ( K_s )</td>
<td>32600 MPa</td>
</tr>
</tbody>
</table>

For a low permeability shale formation, the thermal diffusion coefficient (order of 10^{-6} m²/s) is larger than the diffusion coefficient for solute and diluent flow (order of 10^{-9} m²/s). So, the impacts of the gradient of solute and pore pressure on the heat flow are neglected. But, the gradients of temperature and solute mass fraction are considered as driving forces for the solute diffusion. The water (diluent) is driven by the gradient of
pore pressure and solute. Convective fluid flow induced by thermal gradient is neglected because the low permeability of the shale formation and conductive flow is dominant. Beside the phenomenological transport, the fluid flow is coupled with the deformation of solid matrix.

Figure 5. Distribution of Temperature around the Wellbore.

Figure 6. Distribution of Solute Mass Fraction without Thermal Osmosis.

Figure 5 shows the distribution of temperature around the wellbore as a function of time and the ratio of distance to the radius of wellbore (r/a) for the case of heating.
The temperature in the rock formation decreases with distance, while increases with time when the distance is fixed. At very long time, the temperature of the formation approaches the temperature of the drilling mud.

Figure 7. Impacts of Thermal Filtration Coefficient on the Distribution of Solute Mass Fraction.

Figure 6 shows the distribution of solute mass fraction for the case of isothermal drilling, and similar result has been presented and explained by Ghassemi and Diek (2003). For non-isothermal conditions, the solute flux is controlled not only by the chemical potential gradient, but also by the temperature gradient. Figure 7 shows the distribution of solute mass fraction due to the thermal filtration. It can be observed that for the assumed thermal filtration coefficient, heating increases the solute mass fraction in the formation and cooling decreases it. The effect of thermal filtration on the solute flux is dependent on the thermal filtration coefficient $D^T$.

Figure 8 shows the induced pore pressure when the drilling mud has a higher salinity than the shale formation. The induced pore pressure is zero on the wall of the wellbore and faraway from it, and negative inside the formation for the given example. The peak value decreases with time and moves into the formation. At a fixed point, the magnitude of induced pore pressure increases with time, reaches a
peak and then decreases with time. The effects disappear at infinite time because the chemical osmosis and excess pore pressure gradually disappear as a result of solute and diluent flow.

Figure 8, Distribution of Induced Pore Pressure by Chemical Loading (C\textsuperscript{m}=0.2, C\textsuperscript{sh}=0.1) under Isothermal Condition.

Figure 9, Impacts of Reflection Coefficient on the Distribution of Induced Pore Pressure around the Wellbore by Chemical Loading One Hour after Drilling.
Figure 10, Impacts of Chemical Swelling Coefficient on the Distribution of Induced Pore Pressure around the Wellbore by Chemical Loading One Hour after Drilling.

The induced pore pressure is a result of the two processes: chemical osmosis and physico-chemical interactions. When the reflection coefficient $\mathcal{R}$ is 1.0, shale acts as an ideal membrane and only diluent (water) can pass through it. When the reflection coefficient $\mathcal{R}$ is 0.0, both solute and diluent can pass it without selectivity, and so there is no chemical osmosis. Therefore, for a given chemical loading, the effect of chemical osmosis on the pore pressure is dependent on $\mathcal{R}$. The decrease/increase of the pore pressure associated with osmosis causes the contraction/expansion of the shale to also very away from the well (Figure 35) when the drilling mud has a higher/lower salinity than that of the shale formation. Figure 9 shows that the magnitude of the induced pore pressure increases with the increasing of $\mathcal{R}$.

The physico-chemical interactions also induce pore pressure, and magnitude of the physico-chemical interactions related to the changes in solute concentration and pore pressure are dependent on the chemical swelling coefficient, $\omega_0$. These interactions cause the shale to shrink/swell (Figure 36), and also increase/decrease the...
pore pressure. Figure 10 shows that the magnitude of the induced pore pressure decreases with the increasing of $\omega_0$, because in this case, the physico-chemical interactions cause the shale to shrink and have a contrary effect to that of chemical osmosis.

![Figure 11. Distribution of Induced Pore Pressure by Thermal Loading for Porothermoelasticity.](image)

If a temperature difference between the drilling mud and the formation is considered, the expansion of the solid matrix and the fluid in the pores induces a pore pressure stress. By a new thermal balance is established, the effect on induced pore pressure gradually disappears. Figure 11 shows that the heating induces a positive pore pressure and cooling a negative pore pressure.
When both thermal loading and chemical loading are applied, the induced pore pressure is directly related to the combined processes of thermal expansion, chemical osmosis, and physico-chemical interactions. Figure 12 shows the induced pore pressure around the wellbore under thermal and chemical loading for Porothermoelasticity, Chemoporoelasticity, and Chemo-poro-thermoelasticity respectively.
pressure for porothermoelasticity, chemoporoelasticity and chemo-poro-thermoelasticity respectively. When swelling is neglected ($\omega_0 = 0 \text{ MPa}$), the induced pore pressure for chemo-poro-thermoelasticity is the sum of the induced pressures corresponding to porothermoelasticity and chemoporoelasticity. However, the induced pore pressure for chemo-poro-thermoelasticity is much smaller than the sum of porothermoelasticity and chemoporoelasticity, when chemical swelling/shrinkage is considered ($\omega_0 = 1.5 \text{ MPa}$).

Figure 13 gives the induced pore pressure distribution under various chemical and thermal loading for chemo-poro-thermoelasticity one hour after drilling. According to Figure 13, mud salinity has a negative effect on the induced pore pressure, a higher salinity mud induces a lower pore pressure. But, temperature has a positive effect on pore pressure and a higher temperature difference between the mud and formation induces a higher pore pressure.

*Chemically- and Thermally-Induced Stresses*

The expansion or contraction of solid and fluid due to the change of temperature causes the change of stress when the boundary is restricted. For the given problem, the rock around the wellbore is partially restricted. So, thermal disturbances would result in loss of stress and pore pressure equilibrium. The swelling or shrinking of the shale due to complex physico-chemical interactions also contributes to the change of stress. The theory used takes these physico-chemical processes into account phenomenologically.
Radial Stress

Figure 14. Temporal Variation of Chemically-Induced Radial Stress around the Wellbore.

Figure 15. Impacts of Reflection Coefficient on the Distribution of Induced Radial Stress around the Wellbore by Chemical Loading One Hour by Drilling.
Figure 16, Impacts of Chemical Swelling Coefficient on Distribution of Induced Total Radial Stress around the Wellbore by Chemical Loading One Hour after Drilling.

As shown in Figure 14, there is no induced total radial stress under symmetrical thermal and chemical loading on the wall of the wellbore because the rock material at the wall can freely expand/shrink in that direction. The induced total radial stress at a fixed point inside the formation is tensile (for $C^{m}>C^{sh}$) and is a function of time, it increases with time to a maximum (due to chemical osmosis) and then decreases due to the solute transfer and dissipation of the induced pore pressure. Theoretically, the induced total radial stress vanishes at a fixed radius at infinite time. This is suggested in Figure 14, but cannot be illustrated explicitly because the numerical integration here is unable to deal with solutions at infinite time; however, this point can be mathematically proven (refer to Appendix E). Figure 14 and Figure 15 show that when the solute concentration in the drilling mud is higher than in the formation, the induced total radial stress is tensile. For a given chemical loading, the magnitude of the induced total radial stress increases with the increasing of the reflection coefficient, $\mathcal{R}$ (Figure 15), as well as with the chemical swelling coefficient, $\omega_0$ (Figure 16).
Figure 17. The Distribution of the Induced Total Radial Stress by Thermal Loading for Porothermoelasticity around the Wellbore One Hour after Drilling.

Figure 17 shows the induced total radial stress by thermal loading for porothermoelasticity; and note that heating causes a compressive total radial stress (positive) and the cooling induces a tensile radial stress (negative) inside the formation. Figure 18 shows the induced radial stress under thermal and chemical loading for chemoporoelasticity, porothermoelasticity, and chemo-poro-thermoelasticity respectively. The induced total radial stress for chemo-poro-thermoelasticity without swelling/shrinkage is the sum of the induced radial stress according to chemoporoelasticity and porothermoelasticity. But, the induced total radial stress for chemo-poro-thermoelasticity is larger than the sum of chemoporoelasticity and porothermoelasticity, when swelling/shrinkage is considered ($\omega_0 = 1.5$ MPa).
For chemically-active poroelastic rock, the effective stress is not only a function of the total stress and the pore pressure, but also the swelling pressure (Eqn. (102)). So, the effective radial stress at the wall (r/a=1.0) is not zero as predicted by a
poroelastic model. Under isothermal conditions, drilling with a mud that has a lower salinity than the formation induces tensile effective radial stress (Figure 22), and can causes radial spalling around the wellbore; and drilling with a higher salinity mud induces compressive effective radial stress (Figure 22). The induced effective stress at the wall is independent of time. However, inside the rock, the magnitude of induced effective radial stress increases with the increasing of the reflection coefficient, $\Re$ (Figure 20) for a given chemical loading. For a given $\omega_0$, the maximum change in the effective radial stress might be inside the formation. Figure 21 shows that the magnitude of the induced effective radial stress is also positive and proportional to the chemical swelling coefficient, $\omega_0$, but its maximum change occurs at the wall.

$$\dot{\sigma}' = \dot{\sigma} - \alpha' p + x C^s - \frac{\omega_0 M^s}{C^M RT_0}$$

(102)

![Figure 20, Impacts of Reflection Coefficient on the Distribution of Induced Effective Radial Stress around the Wellbore by Chemical Loading One Hour after Drilling.](image)
Figure 21. Impacts of Chemical Swelling Coefficient on the Distribution of Induced Effective Radial Stress around the Wellbore by Chemical Loading One Hour after Drilling.

If the chemically active system is subjected to a temperature gradient, the induced effective radial stress would also depend on the temperature difference between the drilling fluid and the formation. The induced effective radial stress for chemo-poro-
thermoelasticity without swelling/shrinkage ($\omega_0 = 0$ MPa) is the sum of the induced effective radial stress according to porothermoelasticity and chemoporoelasticity. But, the magnitude of the induced effective radial stress for chemo-poro-thermoelasticity with a swelling ($\omega_0 = 1.5$ MPa) is larger than the magnitude of the sum of porothermoelasticity and chemoporoelasticity. Figure 22 shows that cooling enhances the induced compressive (positive) effective radial stress (due to the physico-chemical interactions associated with cooling), and has its maximum effect at the wall. Figure 23 shows that heating induces a tensile effective radial stress (negative). Thus, heating counteracts the physico-chemical interaction associated with using a higher salinity mud; it might induce a tensile failure (radial spalling) in the radial direction.

Figure 23. Induced Effective Radial Stress as a Function of Radius under Chemical Loading for Chemo-poro-thermoelasticity One Hour after Drilling While the Drilling Mud is Warmer Than the Formation.

**Tangential Stress**

The induced total tangential stress is different from the induced total radial stress, and the induced total tangential stress on the wall is not zero as the rock is not free to expand/shrink. At the wall, the induced tangential stress (Eqn.(103), compression
positive in this equation) by chemical and thermal loading is time independent and is
caused by the difference of fluid pressure, solute mass fraction and temperature
between the drilling fluid and the formation. If all of the constants $\alpha$, $\chi$ and $\gamma$ which
are dependent on the material properties are positive, the induced total tangential
stress on the wall is positive and proportional to the fluid pressure difference and
temperature difference; and it is negatively proportional to the difference of solute
mass fraction:

$$
\sigma_{\theta\theta}^{II} = \frac{2\eta}{\alpha} \left[ \alpha' (p_M - p_0) - \chi(C^m - C^{sk}) + \gamma_1 (T^m - T^{sk}) \right] \quad (103)
$$

$$
\alpha' = \left( \alpha - \frac{M^S \omega_0}{C^D R T_0 \rho_f} \right), \quad \chi = \frac{\omega_0}{C^S} \left( 1 - \frac{\bar{C}^S}{C^D} \right), \quad \gamma_1 = K \alpha_m + \frac{s_M \omega_0 M^S}{C^D R T_0} \quad (104)
$$

Figure 24 shows that drilling with a higher salinity mud induces a tensile total
tangential stress at and in the vicinity of the wall; the maximum values occurs inside
and extends outward with time. The magnitude of the induced total tangential stress
increases with the increasing of the reflection coefficient, $\Re$ (Figure 25), as well as
with the swelling coefficient, $\omega_0$ (Figure 26). The maximum change due to $\Re$ is
inside the formation and the induced tangential stress on the wall is independent of $\Re$,
as it is caused by the physico-chemical interaction and thus is dependent only on $\omega_0$.
However, the maximum change due to $\omega_0$ is on the wall.
Figure 24. Temporal Variation of Induced Total Tangential Stress by Chemical Loading under Isothermal Condition.

Figure 25. Impacts of Reflection Coefficient on the Distribution of Induced Total Tangential Stress around the Wellbore by Chemical Loading One Hour after Drilling.
Figure 26. Impacts of Chemical Swelling Coefficient on the Distribution of Induced Total Tangential Stress around the Wellbore by Chemical Loading One Hour after Drilling.

Figure 27. Induced Total Tangential Stress as a Function of Radius for Porothermoelasticity.
Figure 28. Induced Total Tangential Stress under Chemical Loading for Porothermoelasticity, Chemoporoelasticity and Chemo-poro-thermoelasticity Respectively for Cooling Case.

Figure 27 shows the induced total tangential stress caused only by thermal loading as a function of radius. Heating induces a compressive tangential stress (positive) and cooling induces tensile one (negative). Figure 28 and Figure 29 show the combined total tangential stress induced by the thermal and chemical loadings. The induced tangential stress for chemo-poro-thermoelasticity without a swelling ($\omega_0 = 0$ MPa) is the sum of the induced stress according to porothermoelasticity and chemoporoelasticity. But the magnitude of the induced tangential stress for chemo-poro-thermoelasticity with a swelling ($\omega_0 = 1.5$ MPa) is larger than the magnitude of the sum of porothermoelasticity and chemoporoelasticity.
Figure 29. Induced Total Tangential Stress under Chemical Loading for Porothermoelasticity, Chemoporoelasticity and Chemo-poro-thermoelasticity Respectively for Heating Case.

Effective tangential stress is an important variable in wellbore stability analysis. Due to stress concentration, the effective tangential stress at a given location on the wellbore wall could be large enough to cause compressive failure. Deviatoric loading (mode III) also could cause a tensile effective tangential stress in another direction, leading to tensile failure.

Figure 32 and Figure 33 show that drilling with a higher salinity mud induces compressive effective tangential stress, while drilling with a lower salinity mud causes tensile stress. The magnitude of the induced effective tangential stress increases with the increasing of \( \mathcal{R} \) (Figure 30) and \( \omega_0 \) (Figure 31); however, it is invariant with respect to \( \mathcal{R} \) at the wellbore wall. The induced effective tangential stress for chemo-poro-thermoelasticity without a swelling (\( \omega_0 = 0 \) MPa) is the sum of the induced one according to porothermoelasticity and chemoporoelasticity. But the magnitude of the induced effective tangential stress for chemo-poro-thermoelasticity
with a swelling ($\omega_0 = 1.5$ MPa) is larger than the magnitude of the sum of porothermoelasticity and chemoporoelasticity.

Figure 30. Impacts of the Reflection Coefficient on the Induced Effective Tangential Stress by Chemical Loading.

Figure 31. Impacts of the Swelling Coefficient on the Induced Effective Tangential Stress by Chemical Loading.
Figure 32. Induced Effective Tangential Stress under Chemical Loading for Porothermoelasticity, Chemoporoelasticity and Chemo-poro-thermoelasticity Respectively for Cooling Case.

Figure 33. Induced Effective Tangential Stress under Chemical Loading for Porothermoelasticity, Chemoporoelasticity and Chemo-poro-thermoelasticity Respectively for Heating Case.

*Axial Stress $\sigma_{zz}$*

The induced strain along $z$-axis $e_{zz}$ is constant and zero (plane strain assumption), the effect of chemical and thermal loading on $e_{zz}$ can be reasonably ignored for deep
wellbores (refer to Chapter III). However, the stress along z-axis $\sigma_{zz}$ is not zero, but dependent on the chemical and thermal loadings (refer to Chapter III).

Figure 34 shows that drilling with a higher salinity mud induces a tensile total axial stress at and in the vicinity of the wall; the maximum values occurs inside and extends outward with time. The magnitude of the induced total axial stress increases with the increasing of the reflection coefficient, $\mathcal{R}$ (Figure 35), as well as with the swelling coefficient, $\omega_0$ (Figure 36). The maximum change due to $\mathcal{R}$ is inside the formation and the induced axial stress on the wall is independent of $\mathcal{R}$, as it is caused by the physico-chemical interaction and thus is dependent only on $\omega_0$. However, the maximum change due to $\omega_0$ is on the wall.

Figure 34. Temporal Variation of Induced Total Axial Stress by Chemical Loading under Isothermal Condition.
Figure 35. Impacts of Reflection Coefficient on the Distribution of Induced Total Axial Stress around the Wellbore by Chemical Loading One Hour after Drilling.

Figure 36. Impacts of Chemical Swelling Coefficient on the Distribution of Induced Total Axial Stress around the Wellbore by Chemical Loading One Hour after Drilling.

Figure 37 and 38 show the combined total axial stress induced by the thermal and chemical loadings. The induced axial stress for chemo-poro-thermoelasticity without a swelling ($\omega_0 = 0$ MPa) is the sum of the induced stress according to porothermoelasticity and chemoporoelasticity. But the magnitude of the induced
axial stress for chemo-poro-thermoelasticity with a swelling ($\omega_0 = 1.5 \text{ MPa}$) is larger than the magnitude of the sum of porothermoelasticity and chemoporoelasticity.

Figure 37. Induced Total Axial Stress under Chemical Loading for Porothermoelasticity, Chemoporoelasticity and Chemo-poro-thermoelasticity Respectively for Cooling Case.

Figure 38. Induced Total Axial Stress under Chemical Loading for Porothermoelasticity, Chemoporoelasticity and Chemo-poro-thermoelasticity Respectively for Heating Case.
Effective axial stress is as important as effective radial and axial stress in wellbore stability analysis. Figure 41 and 42 show that drilling with a higher salinity mud induces compressive effective axial stress, while drilling with a lower salinity mud causes tensile stress. The magnitude of the induced effective axial stress increases with the increasing of $\mathcal{R}$ (Figure 39) and $\omega_0$ (Figure 40), however, it is invariant with respect to $\mathcal{R}$ at the wellbore wall. When $\omega_0$ is zero or only porothermoelasticity is considered, cooling induces tensile effective axial stress on the wall and heating induces compressive axial stress on the wall. However, when $\omega_0$ is not zero and chemical swelling is taken account, cooling induces compressive effective axial stress on the wall and heating induces tensile effective axial stress on the wall. The induced effective axial stress for chemo-poro-thermoelasticity without a swelling ($\omega_0 = 0 \text{ MPa}$) is the sum of the induced one according to porothermoelasticity and chemoporoelasticity. But the magnitude of the induced effective axial stress for chemo-poro-thermoelasticity with a swelling ($\omega_0 = 1.5 \text{ MPa}$) is larger than the magnitude of the sum of porothermoelasticity and chemoporoelasticity.

![Induced effective axial stress](image)

Figure 39. Impacts of the Reflection Coefficient on the Induced Effective Axial Stress by Chemical Loading.
Figure 40. Impacts of the Swelling Coefficient on the Induced Effective Axial Stress by Chemical Loading.

Figure 41. Induced Effective Axial Stress under Chemical Loading for Porothermoelasticity, Chemoporoelasticity and Chemo-poro-thermoelasticity Respectively for Cooling Case.
Figure 42. Induced Effective Axial Stress under Chemical Loading for Porothermoelasticity, Chemoporoelasticity and Chemo-poro-thermoelasticity Respectively for Heating Case.

*Induced Strain by Symmetric Loadings*

The effects of temperature and salinity on the stress and pore pressure distribution around the wellbore were discussed above. From the constitutive equations for shale, a change of strain can be caused by the swelling or shrinking due to physico-chemical interactions or thermal expansion or contraction.

*Induced Radial Strain*

The induced radial strain (Eqn. (105), compressive positive in the equation) on the wall resulting from mud pressure, salinity and temperature is independent of time, positively proportional to the gradient of solute mass fraction, and negatively proportional to the gradient of temperature and mud pressure.

\[
\varepsilon_{rr}'' = \frac{2\eta}{G\alpha} \left[ -\alpha' (p_M - p_0) + \chi(C^m - C^{sh}) - \gamma_1 (T^m - T^{sh}) \right] \tag{105}
\]

Drilling with a higher salinity mud causes a compressive radial strain in the vicinity of wall (Figure 43). The maximum compressive radial strain is inside the
formation not on the wall and extends outward with time (Figure 43). The magnitude of the induced radial strain is also dependent on $\Re$ and $\omega_0$. Figure 44 shows that it increases with the increasing of $\Re$ and the maximum change is inside the formation. Figure 45 shows that the radial strain increases with the increasing of $\omega_0$ and the maximum change is on the wall.

Figure 43. Induced Radial Strain as a Function of Radius by Chemical Loading for Chemoporoelasticity.

Figure 46 shows the induced radial strain by thermal loading for porothermoelasticity. Cooling induces a compressive radial strain (positive) and heating induces a tensile radial strain (negative). Figure 47 and Figure 48 show the combined induced radial strain by thermal and chemical loading. The induced radial strain for chemo-poro-thermoelasticity without a swelling ($\omega_0=0$ MPa) is the sum of the induced one according to porothermoelasticity and chemoporoelasticity. However, if a swelling ($\omega_0=1.5$ MPa) is considered, the induced radial strain is larger than the sum of porothermoelasticity and chemoporoelasticity.
Figure 44. Impacts of the Reflection Coefficient on the Induced Radial Strain by Chemical Loading for Chemoporoelasticity.

Figure 45. Impacts of the Swelling Coefficient on the Induced Radial Strain by Chemical Loading for Chemoporoelasticity.
Figure 46. Induced Radial Strain as a Function of Radius by Thermal Loading for Porothermoelasticity.

Figure 47. Induced Radial Strain as a Function of Radius by Chemical Loading for Porothermoelasticity, Chemoporoelasticity, and Chemo-poro-thermoelasticity Respectively for Cooling Case.
Figure 48. Induced Radial Strain as a Function of Radius by Chemical Loading for Porothermoelasticity, Chemoporoelasticity, and Chemo-poro-thermoelasticity Respectively for Heating Case.

Induced Tangential Strain

Figure 49. Induced Tangential Strain as a Function of Radius by Chemical Loading for Chemoporoelasticity.

The induced tangential strain on the wall is constant and zero. Figure 49 shows that drilling with a higher salinity mud induces a compressive tangential strain, the
peaks of which increases and extends with time. The magnitude of the induced tangential strain is also dependent on $\mathcal{R}$ and $\omega_0$, and increases with the increasing of them (Figure 50 and Figure 51).

Figure 50. Impacts of the Reflection Coefficient on the Induced Tangential Strain by Chemical Loading for Chemoporoelasticity.

Figure 51. Impacts of the Swelling Coefficient on the Induced Tangential Strain by Chemical Loading for Chemoporoelasticity.
Figure 52 shows the induced tangential strain by thermal loading for porothermoelasticity. Cooling induces a volume reduction and thus a positive tangential strain and heating induces an expansion, i.e., a negative tangential strain. Figure 53 and Figure 54 show the combined induced tangential strain by thermal and chemical loading. The induced tangential strain for chemo-poro-thermoelasticity without a swelling ($\omega_0=0$ MPa) is the sum of that for porothermoelasticity and that for chemoporoelasticity. However, if a swelling ($\omega_0=1.5$ MPa) is considered, the induced tangential strain is larger than the sum of that for porothermoelasticity and that for chemoporoelasticity.

![Figure 52. Induced Tangential Strain as a Function of Radius by Thermal Loading for Porothermoelasticity.](image-url)
Deviatoric Loading and Responses

When a borehole is drilled through a formation subjected to non-hydrostatic stress, a deviatoric load will act on the well bore. The deviatoric solutions (mode III) for
pure poroelasticity have been presented (Detournay and Cheng, 1988). The deviatoric responses for chemoporoelastic and chemo-poro-thermoelastic model are also dependent on the chemical properties of the formations that appear in the coefficients of the field equations. Here the temperature and solute mass fraction are the same between mud and shale formation, so there is no influence from mode II loading. The maximum and minimum in-situ horizontal stresses are set as 1 MPa and -1 MPa, respectively and the effects of mode III are explored. All of chemoporoelastic parameters are listed in Table 1.

*Induced Stress and Pore pressure by Deviatoric Loading*

Deviatoric loading induces time-dependent stress and pore pressure variation not only along the radial direction but also along the tangential direction. So a tangential fluid flow is induced by deviatoric loading (Detournay and Cheng, 1988). Stress and pore pressure distribution is also heterogeneous around the wellbore. So heterogeneous failure can be caused and compressive failure and tensile failure can possibly occur around the same wellbore but in different directions.

*Induced Pore Pressure*

Immediately, after a borehole is drilled, the pore pressure changes due to the change of pore volume. Then, the induced pore pressure dissipates with time and vanishes at infinite time. The induced pore pressure in the direction of maximum horizontal stress is negative, so the net pore pressure is reduced (Figure 55). In the direction of minimum horizontal stress, the induced pore pressure is positive, so the pore pressure is increased (Figure 56). For a permeable wall, the induced pore pressure on the wall is zero for a finite time, but the maximum value is in the immediate vicinity of wall at the instant the borehole is drilled. Figure 55 shows that the chemical swelling coefficient $\omega_0$ has an impact on the induced pore pressure. At
short times, the magnitude of the induced pore pressure slightly increases with the increments of $\omega_0$. At large time, however, it slightly decreases with the increments of $\omega_0$.

Figure 55. Induced Pore Pressure as a Function of Radius at $\theta = 0, \pi$ by Deviatoric Loading.

Figure 56. Induced Pore Pressure as a Function of Radius at $\theta = \pi/2, 3\pi/2$ by Deviatoric Loading.
Figure 57. Induced Pore Pressure as a Function of Radius at $\theta = 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.

*Induced Total Stress*

Figure 58. Induced Total Radial Stress as a Function of Radius at $\theta = 0, \pi$ by Deviatoric Loading.

Figure 58 and Figure 59 show the induced radial stress in the direction of maximum horizontal stress and the direction of minimum horizontal stress respectively. The induced radial stress is negative at $\theta = 0, \pi$ and positive at $\theta = \pi/2,$
$3\pi/2$. Figure 60 shows the impacts of $\omega_0$ on the induced radial stress. At short times, the affects are obvious and the magnitude of the induced radial stress decreases with the increasing of $\omega_0$. But the effects rapidly decrease with time and vanish at large time.

![Graph showing induced total radial stress as a function of radius at $\theta = \pi/2, 3\pi/2$ by deviatoric loading.](image1)

Figure 59. Induced Total Radial Stress as a Function of Radius at $\theta = \pi/2, 3\pi/2$ by Deviatoric Loading.

![Graph showing induced radial stress as a function of radius at $\theta = 0, \pi$ for deviatoric loading under various chemical swelling coefficients.](image2)

Figure 60. Induced Radial Stress as a Function of Radius at $\theta = 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.
Figure 61 and Figure 62 show the induced tangential stress in the direction of maximum and minimum horizontal stress, respectively. The induced tangential stress is negative at $\theta = 0, \pi$ and positive at $\theta = \pi/2, 3\pi/2$, and so the wall will expand at $\theta = 0, \pi$ and contract at $\theta = \pi/2, 3\pi/2$. For the short times (1 Sec to 1 Hour), the wellbore wall is drained while the rock inside the formation (next to the wall) is under undrained condition. This creates a stiffness contrast in the rock with the stiffer portions carrying more of the load, therefore the peaks of the induced tangential stresses occur inside the formation and they decrease with time due to the pore pressure dissipation when the situation changes from undrained to drained condition. However, at long times, the peaks occur at the wall and increase with time. Figure 63 shows the impact of $\omega_0$ on the induced total tangential stress. At short times, the magnitude of induced total tangential stress decreases with increasing of $\omega_0$. But the impact vanishes at long times.

![Figure 61. Induced Tangential Stress as a Function of Radius at $\theta = 0, \pi$ by Deviatoric Loading.](image)
Figure 62. Induced Tangential Stress as a Function of Radius at $\theta = \pi/2, 3\pi/2$ by Deviatoric Loading.

Figure 63. Induced Tangential Stress as a Function of Radius at $\theta = 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.

Figure 64 and Figure 65 show the induced axial stress in the direction of maximum and minimum horizontal stress, respectively. The induced axial stress is negative at $\theta = 0, \pi$ and positive at $\theta = \pi/2, 3\pi/2$. Figure 66 shows the impact of $\omega_0$ on the induced
total axial stress. The magnitude of induced total axial stress decreases with increasing of $\omega_0$.

Figure 64. Induced Axial Stress as a Function of Radius at $\theta=0, \pi$ by Deviatoric Loading.

Figure 65. Induced Axial Stress as a Function of Radius at $\theta=\pi/2, 3\pi/2$ by Deviatoric Loading.
Figure 66. Induced Axial Stress as a Function of Radius at θ= 0, π for Deviatoric Loading under Various Chemical Swelling Coefficients.

In addition to the radial, tangential and axial stress, shear stress is induced by deviatoric loading inside the formation. The induced shear stress on the wall is independent of time, but it varies with the polar angle θ and reaches the maximum at θ= π/4, 5π/4 and the minimum at θ= 3π/4, 7π/4 as shown in Figure 67.

Figure 67. Shear Stress at the Wellbore Wall, Induced by Deviatoric Loading, as a Function of θ.
**Induced Effective Stress**

Effective stress is the key parameter in failure analysis. The effective stress induced by mode III can be calculated by combining the induced pore pressure and induced stress. Figure 68 shows the induced effective radial stress at $\theta=0, \pi$ by deviatoric loading. The induced effective radial stress on the wall is independent of time. However, inside the formation it decreases with time and changes from compressive to tensile. This is because the rock carries more stress loading under undrained condition than when drained (as time increases, the rock changes from an undrained condition characterized by a dramatic change of pore pressure and unchanged pore volume, to an approximately drained condition characterized by a small change of pore pressure but a large change in fluid content). Figure 69 shows the induced effective radial stress at $\theta=\pi/2, 3\pi/2$ by deviatoric loading. Note that it is also a constant on the wall and increases with time inside formation and varies from tensile to compressive with time by the same reason as the above.

![Graph showing induced effective radial stress](image)

Figure 68. Induced Effective Radial Stress as a Function of Radius at $\theta=0, \pi$ by Deviatoric Loading.
Figure 69. Induced Effective Radial Stress as a Function of Radius at $\theta = \pi/2, 3\pi/2$ by Deviatoric Loading.

Figure 70. Induced Effective Radial Stress as a Function of Radius at $\theta = 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.

Figure 70 shows the impact of $\omega_0$ on the induced effective radial stress. At short time (1 Sec), the induced effective radial stress increases with the increasing of $\omega_0$ at $\theta = 0, \pi$, but only inside the formation. This is because the pore pressure is induced inside the formation only at short times due to undrained condition which is changed
to drained condition with time; and the induced pore pressure is dependent on \( \omega_0 \) and has a direct effect on the effective radial stress.

Figure 71 shows the induced effective tangential stress at \( \theta = 0, \pi \) by deviatoric loading. The induced tangential stress decreases with time at the wall and its vicinity due to the pore pressure dissipation when the formation changes from undrained condition to the drained condition. This process affects both total stress and pore pressure; the former changes from tensile to compressive with the increasing of radius because inside the formation a negative pore pressure is induced due to undrained condition and changes the effective tangential stress from tensile to compressive.

Figure 71. Induced Effective Tangential Stress as a Function of Radius at \( \theta = 0, \pi \) by Deviatoric Loading.

Figure 72 shows the induced effective tangential stress at \( \theta = \pi/2, 3\pi/2 \) by deviatoric loading. the induced tangential stress increases with time and changes from compressive to tensile with the increasing of radius. Figure 73 shows that the induced effective tangential stress changes by responding to the undrained behavior at short times and the drained response at long times.
Figure 72. Induced Effective Tangential Stress as a Function of Radius at $0=\pi/2, 3\pi/2$ by Deviatoric Loading.

Figure 73. Induced effective tangential stress by mode III as a function of time at the wall.

Figure 74 shows the impacts of $\omega_0$ on the induced effective tangential stress at $0=0, \pi$. At a short time, the induced effective tangential stress increases with the increasing of $\omega_0$ at $0=0, \pi$, but there is no change on the wall.
Figure 74. Induced Effective Tangential Stress as a Function of Radius at $\theta = 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.

Figure 75. Induced Effective Axial Stress as a Function of Radius at $\theta = 0, \pi$ by Deviatoric Loading.

Figure 75 shows the induced effective axial stress at $\theta = 0, \pi$ by deviatoric loading. The induced axial stress decreases with time at the wall and it vicinity. Figure 76 shows the induced effective axial stress at $\theta = \pi/2, 3\pi/2$ by deviatoric loading, the induced axial stress increases with time. Figure 77 shows the impacts of $\omega_0$ on the
induced effective axial stress at $\theta = 0, \pi$. The magnitude of induced effective axial stress increases with the increasing of $\omega_0$ at $\theta = 0, \pi$, but there is no change on the wall.

Figure 76. Induced Effective Axial Stress as a Function of Radius at $\theta = \pi/2, 3\pi/2$ by Deviatoric Loading.

Figure 77. Induced Effective Axial Stress as a Function of Radius at $\theta = 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.
The induced mode III stresses result in strains. The time-dependent induced strain is also dependent on the polar angle $\theta$ in addition to radial distance $r$. Figure 78 shows the radial strain induced by deviatoric loading as a function of radius and time at $\theta = 0$, $\pi$. The induced radial strain on the wall is tensile (expansion) and constant. But inside the formation; it decreases with time, the peaks for 1 Sec and 1 Hr are compressive (positive). The later peaks are tensile (minus) at long times because initially the formation behaves undrained and the fluid is unable to flow out or into the pore, but the induced pore pressure begins to dissipate with time and changes into drained condition at long times (the stiffer rock at undrained condition carries more of the tensile loading than the rock at drain condition).

Figure 78. Induced Radial Strain as a Function of Radius at $\theta = 0$, $\pi$ by Deviatoric Loading.

Figure 79 shows the induced radial strain as a function of radius and time at $\theta = \pi/2$, $3\pi/2$ by deviatoric loading. The induced radial strain on the wall is compressive (consolidation) and constant. Inside the formation, it increases with time and the peaks for 1 Sec and 1 Hr are tensile (minus). Figure 78 and Figure 79 also show that
the induced radial strain changes dramatically with time, for at short times the rock behaves undrained and then it turns into drained condition at long times. Figure 80 shows that the induced radial strain decreases with the increasing of $\omega_0$.

Figure 79. Induced Radial Strain as a Function of Radius at $\theta= \pi/2, 3\pi/2$ by Deviatoric Loading.

Figure 80. Induced Radial Strain as a Function of Radius at $\theta= 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.
Figure 81. Induced Tangential Strain as a Function of Radius at \( \theta = 0, \pi \) by Deviatoric Loading.

Figure 82. Induced Tangential Strain as a Function of Radius at \( \theta = \pi/2, 3\pi/2 \) by Deviatoric Loading.

Figure 81 shows the induced tangential strain at \( \theta = 0, \pi \) by deviatoric loading. It decreases with time and changes from tensile to compressive with the increasing of radius which is due to the transition of effective tangential stress from tensile to compressive (Figure 81).
Figure 82 shows the induced tangential strain at $\theta = \pi/2, 3\pi/2$ by deviatoric loading. It increases with time and changes from compressive to tensile with the increasing of radius because of the effective stress change. Figure 83 shows the induced tangential strain at various $\omega_0$. One cannot observe much difference among the results because the elastic strain is much larger than that due to chemical swelling.

![Graph showing induced tangential strain](image)

**Figure 83.** Induced Tangential Strain as a Function of Radius at $\theta = 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.

Figure 84 shows the shear strain as a function of radius and time at $\theta = \pi/4, 5\pi/4$ by deviatoric loading, which is much larger than the induced radial and tangential strain and tends to cause shear failure. It decreases with time and changes from positive at short times to negative at long times. Figure 85 shows that the induced rotation component of strain about z-axis (Eqn. (105)) increases with time at $\theta = \pi/4, 5\pi/4$ by deviatoric loading, and has the same order of magnitude as the shear strain. For small strain theory, it is assumed that the rotation component of strain does not contribute to the strain field in the continuum body, so it is often neglected in the elastic analysis (it is zero for mode II):
\[ \omega_z = \frac{1}{2r} \left[ \frac{\partial (ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right] \] (105)

Figure 84. Maximum Induced Shear Strain as a Function of Radius at \( \theta = \frac{\pi}{4}, \frac{5\pi}{4} \) by Deviatoric Loading.

Figure 85. Induced Rotation Component of Strain as a Function of Radius at \( \theta = \frac{\pi}{4}, \frac{5\pi}{4} \) by Deviatoric Loading.

By comparing the magnitude of the induced rotation with that of the induced shear strain, it can be seen that the former is much smaller than the shear strain at large
time. This is not the case for small time. Therefore, the displacement field is not irrotational for small time but it does approach an irrotational field as time approaches infinity. For small strains, the rotation approximately represents a rigid body motion which does not cause any stresses. When not small, retaining it would mean that the stress tensor would no longer be symmetric.

*Induced Displacement by Deviatoric Loading*

The displacement of the wall of borehole after drilling is an important parameter for borehole completion. The symmetrical elastic stress loading (i.e., mode I) due to far-field stresses causes the rock to move into the hole, so that the wellbore size is reduced. The pore pressure loading (mode II) is also symmetrical and can expand or shrink the wellbore for drainage and pressurization, respectively. However, the deviatoric loading (mode III) causes the wellbore wall to move inward in the direction of the maximum horizontal stress, and expand in the direction of the minimum horizontal stress. The net result is that the shape of the wellbore changes and becomes elliptical.

Figure 86 shows that the wellbore wall moves *inward* (negative values) at $0=0$, $\pi$ and moves outward (positive values) to the formation at $0=\pi/2$, $3\pi/2$. The magnitude of the radial displacement in both directions changes from a constant value $7.6\times10^{-5} \text{ m}$ at short times to a steady-state value $1.39\times10^{-4} \text{ m}$ at long times. This can cause a stuck of pipe. Figure 87 shows the gentle impacts of chemical swelling coefficient on the induced radial displacements. The induced radial displacement on the wall increases with increasing $\omega_0$. Note that although the radial strain due to mode II is non-zero at the wall, the induced displacement at the wall is zero, $0=0$, $\pi$.  

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Figure 86. the Induced Displacement on the Wall by Deviatoric Loading.

Figure 87. Induced Radial Displacement on the Wall at $\theta = 0, \pi$ for Deviatoric Loading under Various Chemical Swelling Coefficients.

Summary

A coupled chemo-poro-thermoelasticity theory is applied to analyze the impacts of solute concentration, temperature on the responses of wellbore under symmetrical
loading and deviatoric loading. All of results in the graphs use compressive-positive
convention.

To explore the impacts of solute, temperature on the induced stress, pore pressure,
strain and displacement, only induced results are plotted and studied. The responses
of wellbore under symmetrical loading and deviatoric loading are investigated,
respectively.

(1) When drilling with a higher salinity mud, the negative pore pressure around
the wellbore is induced, and when drilling with a lower salinity, the positive pore
pressure around the wellbore is induced. The magnitude is positively proportional to
$\mathcal{R}$, but negatively proportional to $\omega_0$. When the chemically active system is subjected
to a nonisothermal condition, an addition pore pressure is also induced. Heating
increases the pore pressure and cooling reduces the pore pressure.

(2) When drilling with a higher salinity mud, the total stresses (including radial,
tangential and axial stress) are reduced, but the effective stresses are increased,
whereas when drilling with a lower salinity mud, the total stresses are increased, and
the effective stresses are reduced. The magnitude of induced stresses increases with
the increasing of $\mathcal{R}$, as well as $\omega_0$. When the chemically active system is subjected to
a nonisothermal condition, the total stresses are increases by heating, and reduced by
cooling. However, the effective stresses are reduced by heating and increased by
cooling. The results contradict the porothermoelasticity theory that predicts that
heating increases the effective stresses and cooling reduces the effective stresses.
Because chemical swelling is incorporated into the coupled chemo-poro-
thermoelasticity theory, and the temperature change affects the chemical swelling,
thereby affecting the effective stresses.
(3) Drilling with a higher salinity mud increases the strain (compression positive) including radial and tangential strain around the wellbore, whereas drilling with a lower salinity mud reduces the strain. For the chemically active system, heating reduces the strain and cooling increases the strain. The magnitude of induced strain increases with the increasing with $\mathcal{R}$, as well as $\omega_0$.

(4) When a deviatoric loading acts on the wall, the pore pressure in the direction of the maximum horizontal stress ($\theta = 0, \pi$) is decreased, but the pore pressure in the direction of the minimum horizontal stress ($\theta = \pi/2, 3\pi/2$) is increased. The total stresses including radial, tangential and axial stress in the direction of the maximum horizontal stress are reduced, whereas the total stresses in the direction of the minimum horizontal stress are increased. However, in the direction of the maximum horizontal stress the effective radial stress is increased at short term, but reduced at long term, and in the direction of the minimum horizontal stress the effective radial stress is reduced at short term, but increased at long term. The effective tangential and axial stress are reduced in the direction of the maximum horizontal stress and increased in the direction of the minimum horizontal stress. The induced radial strain in the direction of the maximum horizontal stress is increased at short term and reduced at long term, however, it is reduced in the direction of the minimum horizontal stress at short term and increased at long term. The tangential strain is reduced in the direction of the maximum horizontal stress and increased in the direction of the minimum horizontal stress. The shear strain is also induced and has its maximum at $\theta = \pi/4, 5\pi/4$. The wall of the wellbore moves inward (to the borehole) in the direction of the maximum horizontal stress and moves outward (faraway from the borehole) in the direction of the minimum horizontal stress. All of the deviatoric responses are slightly dependent on the chemical swelling coefficient $\omega_0$. 

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CHAPTER V
CHEMO-PORO-THERMOELASTIC WELLOBRE STABILITY ANALYSIS

Borehole instability is a widespread problem in the petroleum industry, especially when drilling in deep, low strength shale formations. The instability can be caused by the high compressive effective stress (shear failure) or tensile stress (tensile failure) due to the stress concentration and pore pressure increase while drilling. The temperature and salinity also directly and indirectly impact the distribution of stress and pore pressure around a wellbore while drilling. In this work, a chemo-poro-thermoelastic stress analysis is implemented into a wellbore stability code to quantitatively optimizing the drilling mud weight, salinity, and temperature. The results show that drilling with a cooler and more saline mud enhances wellbore stability and increases the range of safe mud weight window; whereas drilling with a warmer and lower salinity mud tends to reduce the range of mud weight window and increase the possibility of wellbore instability. The results indicate that manipulation of both temperature and chemistry makes it possible to maintain a wellbore stable by cooling a lower salinity mud or increasing the salinity of a warmer mud.

Rock Failure

Rock fails when it is subjected to an effective stress state that exceeds its strength. Rocks failure is classified as compressive shear failure and tensile failure. Both failure types have been observed in the laboratory (Fjaer et al., 1992) and field (e.g. Peska and Zoback, 1995) possibly in the rock formation around a wellbore after drilling due to insufficient support or over pressure.
Tensile Failure

Tensile failure will occur when the effective minimum horizontal principal stress $\sigma'_3$ exceeds the tensile strength of the rock, and the failure surface is perpendicular to the direction of $\sigma'_3$. Possible modes of tensile failure in a wellbore are illustrated in Figure 88 according to the direction of failure surface and the direction of $\sigma'_3$. Note that in Figure 88, failure in $a$, $b$, and $c$ is along the radial direction, the transverse direction, and the tangential direction, respectively. $a$ and $b$ represent the so-called hydraulic fracturing which can cause lost-circulation while drilling; and $c$ is radial spalling in which a fracture occurs inside the formation (this failure mode can not be predicted by an elastic stress analysis, but it is possible in poroelasticity, and chemoporoelasticity).

![Figure 88. Illustration of Tensile Failure Modes.](image)

The maximum principal stress criterion is used to evaluate for tensile failure, i.e., tensile failure will occur when the minimum effective principal stress exceeds the tensile strength of the rock. A failure potential (Eqn. (106)) is defined and tensile failure occurs when $TP$ is less than zero.

$$TP = \sigma'_3 + T_0 \quad \text{(106)}$$
$T_0$ is the tensile strength of the rock and is sometimes small and negligible. Both hydraulic fracturing and radial spalling are tensile failures, but they result from different stress states and cause different drilling problems; so the two are distinguished according to the direction of $\sigma'_{ij}$. For an inclined well, $\sigma'_n$, $\sigma'_\theta$, and $\sigma'_z$ are not the principal stresses inside the formation around the wellbore. If the failure direction is approximately along the direction of $\sigma'_n$ or $\sigma'_z$, the failure is called *hydraulic fracturing* which could cause lost-circulation. The failure is called radial spalling when the rupture plane is oriented approximately along the $\sigma'_{\theta\theta}$-direction.

**Shear Failure**

Shear failure occurs when the shear stress along some planes in the rock exceeds the shear strength of the material. In a downhole environment, shear failure may be characterized as *active* or *passive* depending on the movement of the failure plane (Mclennan et al., 1989). If the rock moves *inward*, active failure has occurred. On the other hand if the rock moves *outward*, passive shear failure has occurred. The Mohr-Coulomb criterion and Drucker-Prager criterion are the most common criteria used to judge whether shear failure occurs in a rock subjected to compressive stress.

The Mohr-Coulomb criterion is a generalization of the Tresca criterion (Maximum Shear-Stress Criterion) that accounts for the influence of hydrostatic stress (Boresi and Schmidt, 2002). The failure function is written in terms of the stress state and two material properties: the cohesion $c$ and the angle of internal friction. For principal stresses $\sigma_1 > \sigma_2 > \sigma_3$, the Mohr-Coulomb failure function is (Lubliner, 1990).

$$f = \sigma_1 - \sigma_3 + (\sigma_1 + \sigma_3)\sin\phi - 2c\cos\phi$$  \hspace{1cm} (107)

The Drucker-Prager failure criterion is a generalization of the Von Mises criterion that
includes the influence of hydraulic stress (Boreshi and Schmidt, 2002). The failure function can be written in the forms (Chen and Han, 1988).

\[ f = \sqrt{J_2} - m_d I_1 - b \quad (108) \]

Where \( I_1 = \sigma'_r + \sigma'_{\theta\theta} + \sigma'_{zz} \quad (109) \)

\[ J_2 = \frac{1}{6} \left[ (\sigma'_r - \sigma'_{\theta\theta})^2 + (\sigma'_r - \sigma'_{zz})^2 + (\sigma'_{\theta\theta} - \sigma'_{zz})^2 \right] + \sigma'_{r\theta}^2 + \sigma'_r^2 + \sigma'_{z\theta}^2 \quad (110) \]

\( m_d \) and \( b \) are coefficients that depend on the cohesion \( c \) and the angle of internal friction, \( \phi \).

In this thesis, Drucker-Prager failure criterion is used for shear failure analysis. In doing so, the shear failure potential is defined as

\[ SP = \sqrt{J_2} - m_d I_1 - b \quad (111) \]

Shear failure occurs when \( SP \) is greater than 0.

Influence of Poromechanical, Thermal and Chemical loads on Rock Failure around a Wellbore

Mud properties including weight, temperature and salinity impact the stress distribution around the wellbore, thereby affecting wellbore stability. Wellbore stability is determined according to the stability of rocks around the wellbore, therefore, for a given section of a wellbore, the maximum compressive shear failure potential around the wellbore (a function of polar angle) is defined as the shear failure potential of the wellbore, and the minimum tensile failure potential is defined as the tensile failure potential of the wellbore. The maximum compressive stress and minimum tensile stress occur only in the vicinity of wellbore or on the wall, so it is only necessary to search the compressive shear failure potential and Tensile failure.
potential in a circular zone with $1 \leq r/a \leq 2$ to find the maximum shear failure potential and the minimum tensile failure potential in the region.

Table 2. Properties of Shale Formation and Drilling Mud.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-situ stresses ($\sigma_v, \sigma_H, \sigma_h$)</td>
<td>25 MPa/km, 29MPa/km, 20MPa/km</td>
</tr>
<tr>
<td>Pore pressure</td>
<td>10 MPa/km</td>
</tr>
<tr>
<td>Shear modulus $G$</td>
<td>760 MPa</td>
</tr>
<tr>
<td>Biot’s coefficient $\alpha$</td>
<td>0.966</td>
</tr>
<tr>
<td>Drained Poisson’s ratio $\nu$</td>
<td>0.219</td>
</tr>
<tr>
<td>Undrained Poisson’s Ratio $\nu_u$</td>
<td>0.461</td>
</tr>
<tr>
<td>Permeability coefficient $\kappa$</td>
<td>0.333 x10$^{-17}$ m$^2$/Pa.s</td>
</tr>
<tr>
<td>Porosity $\phi$</td>
<td>0.2989</td>
</tr>
<tr>
<td>Fluid mass density $\bar{\rho}_f$</td>
<td>1111.11 kg/m$^3$</td>
</tr>
<tr>
<td>Fluid bulk modulus $K_f$</td>
<td>3291 MPa</td>
</tr>
<tr>
<td>Reflection coefficient $\Re$</td>
<td>0.2</td>
</tr>
<tr>
<td>Molar mass of solute (NaCl) $M'$</td>
<td>0.0585 kg/mole</td>
</tr>
<tr>
<td>Swelling coefficient $\omega_b$</td>
<td>1.5 MPa</td>
</tr>
<tr>
<td>Solute diffusivity $D^s$</td>
<td>2.0x10$^{-9}$ m$^2$/s</td>
</tr>
<tr>
<td>Thermal expansion coefficient of solid $\alpha_m$</td>
<td>1.8x10$^{-5}$ K$^{-1}$</td>
</tr>
<tr>
<td>Thermal expansion coefficient of fluid $\alpha_f$</td>
<td>3.0x10$^{-4}$ K$^{-1}$</td>
</tr>
<tr>
<td>Thermal diffusivity $\bar{c}_t$</td>
<td>1.6x10$^{-6}$ m$^2$/s</td>
</tr>
<tr>
<td>Coefficient of thermal diffusion $D^f$</td>
<td>6.0x10$^{-12}$ m$^2$/s/(s.K)</td>
</tr>
<tr>
<td>Specific entropy (NaCl, $C^0=0.15$) $S_0$</td>
<td>3686 J/(kg.K)</td>
</tr>
<tr>
<td>Skempton’s coefficient $B$</td>
<td>0.915</td>
</tr>
<tr>
<td>Fluid diffusivity $\bar{c}_f$</td>
<td>6.0 x10$^{-9}$ m$^2$/s</td>
</tr>
<tr>
<td>Fluid bulk modulus $K_f$</td>
<td>3290 MPa</td>
</tr>
<tr>
<td>Elastic modulus $E$</td>
<td>1853 MPa</td>
</tr>
<tr>
<td>Bulk modulus $K$</td>
<td>1099 MPa</td>
</tr>
<tr>
<td>Solid bulk modulus $K_s$</td>
<td>32600 MPa</td>
</tr>
<tr>
<td>Drucker-Prager material constant $m_d$</td>
<td>0.14</td>
</tr>
<tr>
<td>Drucker-Prager material constant $b$</td>
<td>12.0 MPa</td>
</tr>
</tbody>
</table>

To study the impact of various processes on wellbore stability, the failure potentials ($TP$, $SP$) for the section of a vertical wellbore at a depth of 1 km are calculated and analyzed under different hydraulic, thermal, and chemical conditions (the in-situ stress and the related rock and fluid parameters are listed in Table 2). Experimental data on the full range of chemo-poro-thermoelastic properties of shale
are not commonly available. The set used here has been extracted from Cui et al. (1997, 1998); van Oort et al. (1996); Tanner (1953) and Lide (2000).

**Pure Poroelastic Effects**

Drilling in an ideal elastic material causes an instantaneous stress concentration on the wellbore wall, however, drilling in a poroelastic material causes a transient hydro-mechanical process in the rock with coupled pore pressure and stress variations that can affect the wellbore stability.

![Diagram](image_url)

Figure 89. the Failure Potential for Elastic and Pure Poroelastic Models.

Figure 89 shows the failure potential for a range of mud pressures, the critical mud pressure (for failure) is indicated by the intersection of the curves with the line y=0. Note that initially the shear failure potential decreases with the mud pressure (at lower mud pressures), and then increases with mud pressure (at higher mud pressure). So, the failure potential function can intersect the line y=0 at two locations, at a critical low mud pressure and at a critical high mud pressure. Active shear failure is said to occur when the mud pressure is lower than the critical low mud pressure, and
passive shear failure occurs when the mud pressure is higher than the critical high mud pressure.

Also, it can be seen that the tensile failure potential increases with the mud pressure for lower mud pressures, and decreases with mud pressure at higher mud pressure. So it also may have two intersections with the line \( y = 0 \). Radial spalling occurs when the mud pressure is lower than the critical low mud pressure, and the so-called hydraulic fracturing occurs when the mud pressure is higher than the critical high mud pressure.

It can be observed in Figure 89 that according to the elastic model, shear failure occurs when the mud pressure is lower than 12.43 MPa (active shear failure) or higher than 30.24 (passive shear failure). It can also be seen that tensile failure occurs when the mud pressure is higher than 21 MPa (hydraulic fracturing) or lower than 10 MPa (a kick will enter the well at these well pressures). Coupled hydro-mechanical processes (poroelasticity) reduce both effective radial stress and tangential stress, therefore the critical low and high mud pressures for causing shear and tensile failure are different than the elastic predictions. The poroelastic model predicts that the shear failure occurs when mud pressure is lower than 11.3 MPa (active shear failure) or higher than 34.06 MPa (passive shear failure), thereby the safe operating zone (bounded by the critical low and high mud pressures) with respect to shear failure is expanded in comparison to the elastic result. However, the safe operating zone with respect to tensile failure is contracted, and the tensile failure occurs when the mud pressure is lower than 14.3 MPa (radial spalling) or higher than 22.41 MPa (hydraulic fracturing). Finally, with respect to both shear and tensile failure, the safe operating zone predicted by a poroelastic model is 14.3 ~ 22.4 MPa, which is narrower when compared with the elastic operating zone of 12.43 ~ 21 MPa.
Porothermoelastic Effects

A porothermoelastic model (McTigue, 1986; Kurashige, 1989; Li et al. 1998) was developed by incorporating the heat transport into Biot’s poroelastic theory and coupling the thermal expansion/contraction of shale matrix and pore fluid with the hydraulic-mechanic processes. Thermal expansion causes the increase of the tangential stress around the wellbore whereas thermal contraction cause the decrease of it, therefore the variation of mud temperature can affect the rock failure and the wellbore stability. Thermal expansion/contraction also changes the pore pressure which can affect the wellbore stability by changing the effective stresses.

Figure 90. the Failure Potential as a Function of Mud Pressure and Temperature Based on a Porothermoelastic Model.

Figure 90 shows the failure potential as a function of mud pressure with thermal loading. When considering shear failure, cooling reduces both the critical low mud pressure (causing active shear failure) and the critical high mud pressure (causing passive mud pressure) whereas heating increases them. However, in regard to tensile failure, cooling reduces the critical low mud pressure for radial spalling, and increases the critical high mud pressure for hydraulic fracturing, thereby expanding the safe
operating zone in regard to both shear and tensile failure. Heating increases the maximum effective principal stress (compression positive) in the vicinity of the wellbore wall, thereby it enhances the shear failure potential and increases the critical low mud pressure causing the active shear failure. Heating also reduces the minimum effective stress and increases the critical low mud pressure causing radial spalling, which almost approaches the critical high mud pressure causing hydraulic fracturing, so the safe operating zone is reduced to a narrow range (Figure 90).

**Chemoporoelastic Effects**

![Figure 91. the Failure Potential as a Function of Mud Pressure and Solute Mass Fraction Based on a chemoporoelastic Model.](image)

Chemical osmosis and chemical swelling can change the effective stresses around the wellbore (refer to Chapter IV), thereby affecting the failure potential and wellbore stability. Figure 91. shows the failure potential as a function of mud pressure when the wellbore is subjected to chemical loading. Drilling with a higher salinity mud tends to prevent both shear and tensile failure, and the safe operating zone with respect to both failure modes is expanded. On the contrary, drilling with a lower salinity mud tends to enhance the potential for both failures. In fact, Figure 90 shows
that there is no safe operating zone when drilling with a lower salinity mud \((C^m = 0.1, C^{sh} = 0.2)\), that is, the rock will fail in tension or compression over the entire range of possible mud pressures.

**Coupled Thermal and Chemical Effects**

Chemical swelling depends not only on the gradient of ion concentration (chemoporoelaticity) but also on the gradient of temperature (Ghassemi et al. 2005). Heat transport has a direct effect on the ionic flux through thermal filtration (Tanner, 1927). So, the above two models are unable to consider the rock failure and wellbore stability under combined thermal and chemical loadings. However, the coupled chemo-poro-thermoelastic model considers the couplings between the thermal and chemical processes, and is a robust tool to analyze the rock response and wellbore stability in non-isothermal chemical environments.

Figure 92 shows that increasing the mud salinity while maintaining a constant temperature difference between mud and shale tends to prevent both shear and tensile failure and expand the safe operating zone. Figure 93 shows that drilling with cooled, lower salinity mud tends to prevent both shear and tensile failure and expand the safe operating zone. Porothermoelastic model predicts that cooling tends to increase the critical high mud pressure causing hydraulic fracturing, but chemo-poro-thermoelastic gives counter results (Figure 93), because when drilling with cooler mud, it could reduce the swelling pressure and increase the effective stress, thereby preventing the hydraulic fracturing. There is no safe operating zone while drilling with a warmer mud \((T^m = 115^\circ C, T^{sh} = 65^\circ C)\).
Figure 92. Effects of Salinity on the Failure Potential.

Figure 93. The Failure Potential as a Function of Mud Pressure at Various Temperatures and Salinities for the Coupled chemoporoelastic Model.

Chemical osmosis and chemical swelling are two important processes that affect the stability of shale, and the magnitude of their effects depend on the reflection coefficient and swelling coefficient. When drilling with a cooled lower salinity mud, a higher reflection coefficient tends to enhance both shear failure and tensile failure.
and reduce the safe operating zone (Figure 94.a). This is because a lower salinity mud induces extra pore pressure inside the formation and adversely affects the shale stability around the wellbore due to chemical osmosis, and this effect is proportional to the reflection coefficient. When drilling with cooled higher salinity mud, however, a higher reflection coefficient tends to prevent both shear failure and tensile failure and enhance the safe operating zone (Figure 94.b). This is because more water is drawn into well from the shale formation by chemical osmosis.

Drilling with a lower salinity mud tends to increase swelling pressure and cause both shear and tensile failure (Figure 95.a), whereas drilling with a higher salinity tends to reduce swelling pressure and prevent both shear and tensile failure (Figure 95.b). The effect is proportional to the swelling coefficient (Figure 95). However, cooling a mud (of lower or higher salinity) reduces the induced swelling pressure and tends to prevent both failure types and enhances wellbore stability. This cooling effect is also proportional to the swelling coefficient (Figure 96).
Figure 94. Effects of Reflection Coefficient on the Failure Potential. (a) Cooler and Lower Salinity Mud; (b) Cooler and Higher Salinity Mud.
Figure 95. Effects of Swelling Coefficient on the Failure Potential. (a) Lower Salinity Mud; (b) Higher Salinity Mud.
Figure 96. Effects of Swelling Coefficient on the Failure Potential. (a) Cooler and Lower Salinity Mud; (b) Cooler and Higher Salinity Mud.

Transverse Failure

The diffusive processes of heat transport, ion transfer, and fluid flow are time-dependent, and any disturbance of stress, pore pressure, temperature, or ion concentration will result in time-dependent fluxes, and stress/pore pressure distribution. So, the rock failure determined by the local effective stresses is also time-dependent.

Figure 97 shows the transient critical mud pressure based on coupled chemo-poro-thermoelastic model. The critical low mud pressure corresponding to active shear decreases with time at early term, but increases with time at long term. The critical low mud pressure causing radial spalling also decreases with time and radial spalling disappears at large time for any mud pressure loading. The critical high mud pressure causing hydraulic fracturing decreases with time and approaches the elastic result at large time. This means that the hydraulic fracturing could be delayed and occur some time after drilling, unlike active shear failure which occurs instantly after
drilling. The critical high mud pressure causing passive shear failure slightly increases with time at the early stages, then decreases with time at large times.

Figure 97. Transient Critical Mud Pressure.

Temperature and Salinity Dependent Mud Weight Window

The impacts of chemical and thermal loading on the rock failure and safe operating zone were discussed in the above sections. The safe operating zone corresponding to both shear and tensile failure spans the interval between the critical low mud pressure and critical high mud pressure. Generally, the mud pressure is provided by the drilling mud column and controlled by mud weight (density of mud). The mud weight window is the range from the lower bound (critical low) mud weight to the upper bound (critical high) mud weight which can maintain the wellbore stable. Predicting the right mud weight window is critical to drilling successfully without wellbore instabilities, however, the impacts of thermal and chemical processes on rock failure make the design of correct mud weight difficult. Rock failure is not only dependent on the mud weight, but also on the mud temperature and salinity, so the
thermal and chemical effects ought not to be neglected to construct mud weight window (refer to Appendix E).

For a given azimuth, the mud weight window is constructed for wellbore inclination angles ranging from 0° to 90° in the $\sigma_v-\sigma_H$ plane. A zero inclination means a vertical wellbore, and 90° indicates a horizontal wellbore along the direction of the
maximum horizontal stress $\sigma_H$. All of relevant parameters are listed in Table 2. Figure 98 shows that the mud weight window contracts with the increasing of temperature when $C_m=0.2$, $C_{sh}=0.1$. Figure 99 shows that the mud weight window expands with the increasing of mud salinity when $T_m=85$, $T_{sh}=95$.

To predict and select a suitable drilling trajectory with respect to wellbore stability, the safe drilling mud weight for all of possible orientations and inclinations need to be investigated. Then, the safe operating mud weight as a function of the orientation and inclination of wellbore can be projected on a plane using the lower hemisphere projection (Peska and Zoback, 1995), (Figure 100-103). The radial distance from the center represents the wellbore inclination and the polar angle counter clockwise measured from the direction of maximum horizontal stress – 0° (Figure 100) represents the azimuth of the well (refer to Appendix F). So, the amount of inclination from the vertical increases along the radial lines, the center of the circle represents the 0° inclination, and the circular boundary corresponds to 90° inclination; the angle $\theta = 0^\circ, 180^\circ$ represents the direction of maximum horizontal stress.

Figure 100 shows the mud weight window based on the elastic model. According to the Figure, the critical low mud weight varies from $1.02 \rho_w$ to $1.27 \rho_w$ (Figure 100.a) and the critical high mud weight varies from $2.1 \rho_w$ to $4.0 \rho_w$(Figure 100.b). When drilling along the direction of the minimum horizontal stress ($\theta = 90^\circ, 270^\circ$), the critical low mud weight is greater than that when drilling along the direction of maximum horizontal stress with high inclination (Figure 100.a). This means that drilling along the direction of the minimum horizontal stress need a higher mud weight to prevent the possible active shear failure or radial spalling. The critical high mud weight attains its maximum values for highly inclined wells in the direction of the minimum horizontal stress (Figure 100.b). According to Figure 100.a and
100.b, a vertical well has a higher critical low mud weight and a lower critical high mud weight, so for the given in-situ stress field, the safe operating zone (1.27 $\rho_m \sim 2.1 \rho_w$) of a vertical well is smaller than that of deviated well.

Figure 100. Critical Mud Weight Based on the Elastic Model. (a) Critical Low Mud Weight; (b) Critical High Mud Weight.
Figure 101. Critical Mud Weight Based on the Pure Poroelastic Model. (a), Critical Low Mud Weight; (b), Critical High Mud Weight.
Figure 101 shows the mud weight window based on the purely poroelastic model. In the Figure, the critical low mud weight ranges from $0.75 \rho_w$ to $1.5 \rho_w$ and the critical high mud weight ranges from $2.3 \rho_w$ to $3.3 \rho_w$. Compared with the elastic result (Figure 100.a), the critical low mud weight has a wider range, whereas the critical high mud weight has a narrower range.

Drilling with lower salinity and cooler mud tends to prevent the rock failures and so contributes to reduce critical low mud weight or critical high mud weight, therefore increase the mud weight window. Figure 102 shows that the critical low mud weight varies from $0.5\rho_w$ to $1.2\rho_w$ and critical high mud weight varies from $2.3\rho_w$ to $3.31\rho_w$. Figure 103 shows that the critical low mud weight varies from $0.3\rho_w$ to $1.1\rho_w$, and the critical high mud weight varies from $2.3\rho_w$ to $3.35\rho_w$.
Figure 102. Critical Mud Weight Predicted From chemoporoelastic Model When Drilling With a Higher Salinity Mud, $C^m=0.2$, $C^{sh}=0.1$, Time=1 Hour. (a), Critical Low Mud Weight; (b), Critical High Mud Weight.
Summary

A coupled chemo-poro-thermoelastic theory that considers the thermal expansion/contraction, chemical osmosis, physico-chemical interactions between solutions and shale, and the couplings among them has been used to quantitatively analyze wellbore stability in shale while drilling at high temperature and high pressure environment. the analysis includes optimization of the mud temperature, salinity and weight.

Using the Drucker-Prager compressive failure criterion and the tensile failure criterion, the impacts of mud temperature, salinity, and pressure on the rock failure have been analyzed. the results suggest that cooling tends to prevent shear failure, radial spalling and hydraulic fracturing, whereas heating tends to enhance them. Also, drilling with a higher salinity mud reduces the swelling pressure, thereby enhancing...
the wellbore stability, and drilling with lower salinity reduces the wellbore stability. Furthermore, the interaction between thermal and chemical phenomena can be used to maintain a wellbore stable while drilling; lowering salinity when the mud is cooler than the formation and increasing salinity if the mud is warmer. The analytical nature of solutions used in the model facilitates real-time wellbore stability assessment.

Drilling with a lower mud pressure may cause active shear failure and radial spalling, while drilling with higher mud pressure may cause passive shear failure and hydraulic fracturing. So, predicting the right safe operating zone (mud weight window) that is the interval bounded by the critical low mud pressure and the critical high mud pressure is critical to drill successfully without wellbore instabilities. In addition to the in-situ stress, rock strength, and well trajectory, mud weight window is also affected by mud temperature and salinity. Cooling reduces the critical low mud pressure and increases the critical high mud pressure, thereby expanding mud weight window. On the contrary, heating increases the critical low mud pressure and reduces the critical high mud pressure, thereby contracting mud weight window. Drilling with higher salinity mud tends to reduce the critical low mud pressure and increase the critical high mud pressure, therefore expanding mud weight window; drilling with lower salinity mud contracts mud weight window by increasing the critical low mud pressure and reducing the critical high mud pressure.

The mud weight window for all of wells at all of possible orientations and inclinations are investigated. The results suggest that a vertical well has a higher critical low mud weight and a lower critical high mud weight, so for the given in-situ stress field, the safe operating zone of a vertical well is smaller than that of deviated well. When drilling along the direction of the minimum horizontal stress, the critical low mud weight is greater than that when drilling along other orientations; when
drilling along the direction of the maximum horizontal stress with a high inclination, the critical high mud weight attains its maximum.
CHAPTER VI
SUMMARY AND CONCLUSIONS

A linear coupled chemo-poro-thermoelastic theory has been described and used to perform stress and pore pressure analysis around a well drilled in shale. The theory assumes the shale to be an isotropic, homogeneous, chemically active rock saturated by a binary solution. The rock is viewed as a geologic membrane, in which there exist three coupled fluxes: water flow, solute flow and heat flow. Due to the very low permeability (nanodarcy) of shale, all of conductive fluxes are ignored. This theory allows the analysis of coupled impacts of hydraulic, mechanical, thermal and chemical loading on the solution-filled shale system.

The field equations of the chemo-poro-thermoelasticity theory were solved for distribution of stress, pore pressure, strain and displacement around a wellbore for non-isothermal drilling in water sensitive shale. The analytical solutions were derived for the case of a suddenly drilled well with constant pressure, temperature and salinity applied on its walls. The solution was obtained analytically in the Laplacian space, and numerically transformed into the time domain. The stress analysis was then implemented in a wellbore stability model to optimize the mud properties for maintaining a wellbore stable. The resulting stress and mud weight analysis algorithms were applied using available rock and mud data. The results show that:

(1) Thermal filtration has a direct impact on solute transfer. Solutes tend to move to the cooler side of a membrane, thus heating the mud increases the solute mass fraction in the formation and cooling decreases it. The effect of thermal gradient on the solute flux is dependent on the thermal filtration coefficient $D_T$;
(2) Both thermal and chemical loading directly and indirectly impact the pore pressure distribution. Mud salinity induces a negative pore pressure by chemical osmosis, i.e., a higher salinity mud lowers the pore pressure in the. The chemical osmosis is also dependent on the reflection coefficient, $\mathcal{R}$. The temperature has a positive effect on pore pressure, i.e., a higher temperature difference between the mud and formation induces a higher pore pressure.

(3) Drilling with a higher salinity mud induces tensile total stresses (radial and tangential), but compressive effective stresses, and vice versa. The magnitudes are proportional to the reflection coefficient, $\mathcal{R}$, and chemical swelling coefficient, $\omega_0$. While drilling with a higher salinity mud in chemically-active shale, heating induces compressive total stresses, but tensile effective stresses; and cooling induces tensile total stresses and compressive effective stresses. The thermal impact is contrary to the result by porothermoelasticity for chemically inert rocks. This is because chemo-poro-thermoelasticity considers chemical swelling, and heating increases chemical swelling pressure with is tensile which causes tensile effective stresses.

(4) Drilling with higher salinity mud induces positive strains including radial and tangential strains, and vice versa. The magnitude of induced strain increases with the increasing the reflection coefficient, $\mathcal{R}$, and chemical swelling coefficient $\omega_0$. While drilling with a higher salinity mud, heating induces tensile strain, and cooling induces compressive strains. This is also in contrast to the result predicted by porothermoelasticity for chemically inert rocks by the same reason as the above.

(5) The deviatoric responses of the rock also weakly depend on the chemical swelling coefficient $\omega_0$. When drilling through the shale with a higher chemical swelling coefficient, $\omega_0$, the deviatoric loading induces a slightly higher pore pressure.
and effective stress, and a lower total stress and strain inside the rock. But, at the wall the response is independent on $\omega_h$.

The coupled chemo-poro-thermoelasticity theory was used to analyze the chemical and thermal effects on wellbore stability and safe mud weight. The results suggest that cooling the mud tends to prevent shear failure, radial spalling and hydraulic fracturing, whereas heating tends to enhance them. Also, drilling with a higher salinity mud reduces the swelling pressure thereby enhancing wellbore stability, and drilling with lower salinity reduces wellbore stability. Furthermore, the interaction between thermal and chemical phenomena can be used to maintain a wellbore stable while drilling namely, lowering salinity when the mud is cooler than the formation and increasing salinity if the mud is warmer.

The coupled chemo-poro-thermoelasticity solutions were also implemented to analyze the chemical and thermal impacts on the mud weight window. The results suggest that cooling reduces the critical low mud pressure and increases the critical high mud pressure, thereby expanding mud weight window. On the other hand, heating increases the critical low mud pressure and reduces the critical high mud pressure, thereby contracting mud weight window. Drilling with a higher salinity mud tends to reduce the critical low mud pressure and increase the critical high mud pressure, therefore expanding the mud weight window. Drilling with a lower salinity mud contracts the mud weight window by increasing the critical low mud pressure and reducing the critical high mud pressure.
Appendix A

Derivation of $H' = H''$

According to the assumption of reversibility of hydraulic and mechanical processes in the theory of poroelasticity, the work done in an increment of deformation is

$$dW = \sigma_g d\varepsilon_g + pd\zeta = \varepsilon_g d\sigma_g + \zeta dp$$  \hspace{1cm} (A-1)

For the case, when the system deforms from one state to another, through a series of intermediate states, and then return back to the original state, the work increment should be zero.

$$0 = \oint dW = \oint \varepsilon_{ij} d\sigma_{ij} + \zeta dp$$ \hspace{1cm} (A-2)

According to Green’s theorem

$$\oint \varepsilon_{ij} d\sigma_{ij} + \zeta dp = \iint_s \left( \frac{\partial \varepsilon_{ij}}{\partial p} - \frac{\partial \zeta}{\partial \sigma_{ij}} \right) d\sigma_{ij} dp$$ \hspace{1cm} (A-3)

Put Eqn. (A-3) into Eqn. (A-2)

$$\iint_s \left( \frac{\partial \varepsilon_{ij}}{\partial p} - \frac{\partial \zeta}{\partial \sigma_{ij}} \right) d\sigma_{ij} dp = 0$$ \hspace{1cm} (A-4)

Since the integral is valid for any volume, the relation

$$\frac{\partial \varepsilon_{ij}}{\partial p} = \frac{\partial \zeta}{\partial \sigma_{ij}}$$ \hspace{1cm} (A-5)

can be directly derived from Eqn. (A-4)
Eqn. (A-6) and (A-7) can be obtained by taking the partial derivation of Eqns. (2) and (3) in Chapter II.

\[
\frac{\partial \varepsilon_{ij}}{\partial \sigma} = \frac{1}{3H'} 
\]  
(A-6)

\[
\frac{\partial \zeta}{\partial \sigma_{ij}} = \frac{1}{3H'} 
\]  
(A-7)

Put Eqn. (A-6) and Eqn. (A-7) into Eqn. (A-5) and obtain

\[ H' = H'' \]  
(A-8)
Constitutive equations

\[
\sigma_{ij} = 2G \dot{e}_{ij} + (K - \frac{2G}{3}) \varepsilon_{kk} \delta_{ij} - \alpha \dot{\psi} \delta_{ij} + \chi \dot{C}^S \delta_{ij} - \gamma_1 \ddot{T} \delta_{ij} \quad \text{(B-1)}
\]

\[
\zeta = \alpha \dot{e} + \beta \dot{\psi} + \chi \dot{C}^S - \gamma_2 \ddot{T} \quad \text{(B-2)}
\]

Where

\[
\dot{\alpha} = \alpha - \frac{M^S \omega_0}{C^D \rho_f R T_0}
\]

\[
\beta = \frac{\eta_0}{K_f} + \frac{\omega_0 (\alpha - 1) M^S}{K C^D \rho_f R T_0}
\]

\[
\chi = \frac{\omega_0}{C^S} \left( 1 - \frac{C^S}{C^D} \right)
\]

\[
\dot{\chi} = \frac{\alpha - 1}{K} \chi
\]

\[
\gamma_1 = K \alpha_m + \frac{s_f a^D \omega_0 M^S}{R T_0}
\]

\[
\gamma_2 = \alpha \alpha_m + (\alpha_f - \alpha_m) \phi_0 + \frac{s_f a^D \omega_0 (\alpha - 1) M^S}{K R T_0}
\]
\[ a^D = \frac{1}{C^D} \]

Conservation equation of fluid mass

\[ \dot{\zeta} = k \nabla^2 p - k \mathcal{R}A \nabla^2 C^S \]  \hspace{1cm} (B-3)

\[ A = \frac{\bar{\rho}_f RT_0}{C^S C^D M^S} \]

Conservation equation of solute mass

\[ \phi \dot{C}^S = D^S \nabla^2 C^S + \overline{C^S} D^T \nabla^2 T \]  \hspace{1cm} (B-4)

Thermal diffusion equation

\[ \dot{T} = c_T \nabla^2 T \]  \hspace{1cm} (B-5)

Equilibrium equation (Body forces are neglected)

\[ \sigma_{i,j,i} = 0 \]  \hspace{1cm} (B-6)

Put (B-1) into (B-6)

\[ G \nabla^2 u_i + \frac{G}{1-2\nu} e_{i} - \alpha' p_{i,j} + \chi C^s_{i,j} - \gamma_1 T_{i,j} = 0 \]  \hspace{1cm} (B-7)

The following equation can be derived from equation (B-7)

\[ u_{k,k} = e = \frac{\eta}{G} \frac{\alpha'}{\alpha} p + \frac{\eta}{G} \frac{\gamma_1}{\alpha} T - \frac{\eta}{G} \frac{\chi}{\alpha} C^S + g(t) \]  \hspace{1cm} (B-8)

With \( g(t) = 0 \) (at infinite, all of the change of \( p, T, C^S \) and \( e \) are 0, so \( g(t) \) must be 0)

(B-9)

In the cylindrical coordinates (axisymmetry),

\[ u_{k,k} = \frac{1}{r} \frac{\partial(ru_r)}{\partial r} \]  \hspace{1cm} (B-10)
Combine (B-8), (B-9) and (B-10)

\[ u_r = A \frac{1}{r} + \frac{1}{r} \frac{\eta}{G} \alpha \int_{a}^{r} p \, dr + \frac{1}{r} \frac{\eta}{G} \alpha \int_{a}^{r} rT \, dr - \frac{1}{r} \frac{\eta}{G} \alpha \int_{a}^{r} C^s \, dr \] (B-11)

Put (B-8) into (B-2)

\[ \dot{\xi} = \left( \frac{\eta \alpha}{G} + \beta \right) \dot{p} + (\dot{\xi} - \frac{\eta}{G} \chi) \dot{c}^s + \left( \frac{\eta \chi}{G} - \gamma_2 \right) \dot{T} \] (B-12)

Combine (B-3) and (B-12)

\[ c \cdot \nabla^2 p - \frac{\partial p}{\partial t} = -c \cdot \frac{\partial C^s}{\partial t} - c \cdot \frac{\partial T}{\partial t} \] (B-13)

\[ c \cdot \left[ \frac{k}{\left( \frac{\eta}{G} \alpha + \beta \right)} - \frac{k}{\eta} \frac{M^s \alpha_0}{M^s} \left( \frac{1}{\frac{1}{G}(1-v^2)} \right) \right] = \frac{c_f}{1 - c_0} \] (B-14)

\[ c_0 = c_f \left[ \frac{1}{k} \left( \frac{\alpha(1-2v)}{G(1-v^2)} \right) \right] \frac{\alpha_0 M^s}{C^D R T_0 \rho_f} = c_f \left[ \frac{1}{k} \left( \frac{2\eta (1-2v) G}{(1+v)} \right) \right] \frac{\alpha_0 M^s}{C^D R T_0 \rho_f} \] (B-15)

\[ c_f = \frac{2kG(1-v)(v_u - v)}{\alpha^2 (1-v_u)(1-2v)^2} \] (B-16)

\[ c^1 = \frac{k \Re \phi \rho_f \rho_f RT_0}{s M^s C^D \frac{1}{k} \left( \frac{\alpha-1}{G} - \frac{\eta}{G} \right) \chi} \]
\[
\frac{\overline{\rho}_f RT}{M^S} \left[ - \frac{C_f \Re \phi}{D^S C^S C_D^f} + \frac{C_D^f c_0}{C^S} \left( 1 - \frac{C^S}{C^D} \right) \right] = \frac{1}{1 - c_0}
\]
(B-17)

\[
c_{\beta}' = -\frac{\eta}{G} (\gamma_1 - \gamma_2) \left( \frac{\eta}{G} \alpha' + \beta \right) + \frac{k \Re C^S D^T}{C^T D^S C_D^f} \overline{s_f M^S} \left( \frac{\eta}{G} \alpha' + \beta \right)
\]
(B-18)

\[
c_{\beta}' = \frac{c_f}{k} \left( \frac{4\eta}{3} - \phi_0 \right) \alpha_m + \alpha_f \phi_0 - c_0 s_f \overline{\rho}_f + \frac{c_f \Re C^S D^T}{C^T D^S C_D^f} \overline{s_f M^S} \left( \frac{\eta}{G} \alpha' + \beta \right)
\]
(B-19)

\[
\alpha_m^c = \frac{s_f M^S}{C_D^f RT_0}
\]
(B-20)

Solve equation (B-5)

\[
\tilde{T} = \frac{T_m - T_{sh}}{s K_0(\xi_T)} K_0(\xi_T)
\]
(B-21)

\[
\xi_T = r \sqrt{\frac{s}{c_r}} \beta_T = r_0 \sqrt{\frac{s}{c_r}}
\]

Combining (B-4) and (B-5)

\[
\phi \tilde{C}^S = D^S \nabla^2 C^S + \frac{C^S D^T}{C_T} \tilde{T}
\]
(B-22)

Take Laplace transform on Eqn. (B-22)

\[
s \phi \tilde{C}^S = D^S \nabla^2 \tilde{C}^S + \frac{s C^S D^T}{C_T} \tilde{T}
\]
(B-23)
Put (B-21) into (B-23)

\[ \nabla^2 \tilde{C}^S - \frac{s \phi}{D^S} \tilde{C}^S = -\frac{\tilde{C}^S D^T (T_M - T_{sh})}{C_T D^S K_0(\beta_T)} K_0(\xi_T) \]  \hspace{1cm} (B-24)

Eq (B-24) is rewritten in cylindrical coordinate.

\[ \frac{d^2 \tilde{C}^S}{d^2 r} + \frac{1}{r} \frac{d \tilde{C}^S}{dr} - \frac{s \phi}{D^S} \tilde{C}^S = -\frac{\tilde{C}^S D^T (T_M - T_{sh})}{C_T D^S K_0(\beta_T)} K_0(\xi_T) \]  \hspace{1cm} (B-25)

Set \( \xi_{ch} = r \frac{s \phi}{D^S} \) \( \beta_{ch} = r_0 \frac{s \phi}{D^S} \)

Eq (B-25) can be written as

\[ \xi_{ch}^2 \frac{d^2 \tilde{C}^S}{d^2 r} + \xi_{ch} \frac{d \tilde{C}^S}{d \xi_{ch}} - \xi_{ch}^2 \frac{s \phi}{D^S} \tilde{C}^S = -\frac{\tilde{C}^S D^T (T_M - T_{sh})}{C_T D^S K_0(\beta_T)} \xi_{ch}^2 K_0(\xi_T) \]  \hspace{1cm} (B-26)

This can be solved using a procedure similar to Detournay and Cheng (1988).

With BC:

\[ r \to \infty \quad \tilde{C}^S = 0 \]

On the wall of well bore: \( \tilde{C}^S = \frac{C^M_S - C^S_{sh}}{s} \);

So that:

\[ \tilde{C}^S = \frac{K_0(\xi_{ch})}{s K_0(\beta_{ch})} [(C^S_M - C^S_{sh}) + \frac{\tilde{C}^S D^T (T_M - T_{sh})}{C_T \phi \left( \frac{D^S}{C_T \phi} - 1 \right)}] - \frac{\tilde{C}^S D^T (T_M - T_{sh})}{s C_T \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} K_0(\beta_T) \]  \hspace{1cm} (B-27)

\[ \xi_{ch} = r \frac{s \phi}{D^S} \] \( \beta_{ch} = r_0 \frac{s \phi}{D^S} \)

Put (B-21) and (B-27) into (B-13)

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\[ c' \nabla^2 \tilde{p} - s \tilde{p} = -Y_{ch} \frac{K_0(\xi_{ch})}{K_0(\beta_{ch})} + Y_T \frac{K_0(\xi_T)}{K_0(\beta_T)} \]  
(B-28)

\[ Y_{ch} = c'^\circ \left( C_{sh}^c - C_{ch}^c \right) + \frac{C^c}{C_T} \left( D^s (T_M - T_{sh}) \right) \left( C_T \phi \left( \frac{D^s}{C_T} - 1 \right) - c' \phi \right) \]  
(B-29)

\[ Y_T = (T_M - T_{sh}) \left[ \frac{c'^\circ C^c}{C_T} D^s \phi \left( \frac{D^s}{C_T} - 1 \right) \right] - c' \phi \]  
(B-30)

To write (B-28) in cylindrical coordinate (\( r \) is independent of \( \theta \))

\[ r^2 \frac{d^2 \tilde{p}}{dr^2} + r \frac{d \tilde{p}}{dr} - \frac{r^2}{c'} \tilde{p} = -r^2 Y_{ch} \frac{K_0(\xi_{ch})}{K_0(\beta_{ch})} + r^2 Y_T \frac{K_0(\xi_T)}{K_0(\beta_T)} \]  
(B-31)

Eq (B-31) can also be solved using the procedure similar to before. It has the following general solution

\[ \tilde{p} = H_1 I_0(\xi) + H_2 K_0(\xi) - \frac{Y_{ch}}{s} \frac{K_0(\xi_{ch})}{K_0(\beta_{ch})} - \frac{Y_T}{s} \frac{K_0(\xi_T)}{K_0(\beta_T)} \]  
(B-32)

\[ \xi = r \sqrt{\frac{s}{c'}} \quad \beta = a \sqrt{\frac{s}{c'}} \]

\[ r \to \infty \quad K_0(\xi_T) = 0 \quad K_0(\xi_{ch}) = 0 \quad K_0(\xi) = 0 \quad \tilde{p} = 0 \quad I_0(\xi) \neq 0 \]  
(B-33)

Put (B-33) into (B-32), \( H_1 \) is solved

\[ H_1 = 0 \]  
(B-34)

At the wall of wellbore

\[ r = a \]
\[
\frac{p_m - p_0}{s} = H_2 K_0(\xi) - \frac{Y_{ch}}{s \left( \frac{c \phi}{D^s} - 1 \right)} + \frac{Y_T}{s \left( \frac{c}{C_T} - 1 \right)} 
\]

(B-35)

So \( H_2 = \frac{1}{sK_0(\beta)} \left[ \frac{p_m - p_0}{s} + \frac{Y_{ch}}{\left( \frac{c \phi}{D^s} - 1 \right)} - \frac{Y_T}{\left( \frac{c}{C_T} - 1 \right)} \right] \)

(B-36)

So Eq (B-31) has the following solution

\[
\bar{p}_r = \left[ \frac{p_m - p_0}{s} + \frac{Y_{ch}}{s \left( \frac{c \phi}{D^s} - 1 \right)} - \frac{Y_T}{s \left( \frac{c}{C_T} - 1 \right)} \right] \frac{K_0(\xi)}{K_0(\beta)} - \frac{Y_{ch}}{s \left( \frac{c \phi}{D^s} - 1 \right)} \left( \frac{K_0(\xi_{ch})}{K_0(\beta_{ch})} - \frac{Y_T}{\left( \frac{c}{C_T} - 1 \right)} \right)
\]

(B-37)

The radial displacement and the bulk strain have known (Eqn. (B-8) and Eqn. (B-11)).

\[
u_r = A \frac{1}{r} + \frac{1}{r} \frac{\eta \alpha}{G} \int_a^r p dr + \frac{1}{r} \frac{\eta \gamma_1}{G} \int_a^r T dr - \frac{\eta \chi}{G} \int_a^r C^s dr
\]

\[
u_{kr} = e = \frac{\eta \alpha}{G} p + \frac{\eta \gamma_1}{G} T - \frac{\eta \chi}{G} C^s + g(t)
\]

Radial strain

\[
e_{rr} = \frac{\partial u_r}{\partial r} = -\frac{A}{r^2} - \eta \frac{\alpha'}{G} \frac{1}{r^2} \int_a^r pr dr - \frac{\eta \gamma_1}{r^2 G} \int_a^r T r dr + \frac{\eta \chi}{r^2 G} \int_a^r C^s r dr + \frac{\eta \alpha'}{G} p + \frac{\eta \gamma_1}{G} T - \frac{\eta \chi}{G} C^s
\]

(A-0) (At infinite, all of the change of \(p, T, C^s\) and \(e_{rr}\) are 0, so \(A\) must be 0)

The tangential strain
\[
e_{\theta} = \frac{u_r}{r} = \frac{A}{r^2} + \frac{\eta \alpha^2}{r^2 G \alpha} \int_a^r p r dr + \frac{\eta \gamma_i}{r^2 G \alpha} \int_a^r T r dr - \frac{\eta \chi}{r^2 G \alpha} \int_a^r C^s r dr \quad (B-39)
\]

Put (B-8), (B-38) and (B-39) into (B-1)

\[
\dot{\sigma}_r = -\frac{2\eta \alpha}{r^2 \alpha} \int_a^r p r dr - \frac{2\eta \gamma_i}{r^2 \alpha} \int_a^r T r dr + \frac{2\eta \chi}{r^2 \alpha} \int_a^r C^s r dr \quad (B-40)
\]

\[
\dot{\sigma}_\theta = 2G \left[ \frac{A}{r^2} + \frac{\eta \alpha}{r^2 G \alpha} \int_a^r p r dr + \frac{\eta \gamma_i}{r^2 G \alpha} \int_a^r T r dr - \frac{\eta \chi}{r^2 G \alpha} \int_a^r C^s r dr \right] + \left( K - \frac{2G}{3} \right) \left( \frac{\eta}{G \alpha} \dot{p} + \frac{\eta \gamma_i}{G \alpha} \dot{T} - \frac{\eta \chi}{G \alpha} C^s \right) - \dot{\alpha} \dot{\theta} + \dot{\chi} \dot{C}^s - \gamma_i \dot{T}
\]

\[
\dot{\sigma}_\theta = \frac{2GA}{r^2} + \frac{2\eta \alpha}{r^2 \alpha} \int_a^r p r dr + \frac{2\eta \gamma_i}{r^2 \alpha} \int_a^r T r dr - \frac{2\eta \chi}{r^2 \alpha} \int_a^r C^s r dr + \left[ \left( K - \frac{2G}{3} \right) \frac{\eta}{G \alpha} - 1 \right] \left( \dot{\alpha} \dot{p} - \dot{\chi} \dot{C}^s + \gamma_i \dot{T} \right)
\]

\[
\left( K - \frac{2G}{3} \right) \frac{\eta}{G \alpha} - 1 = \frac{2G(1+v)}{3(1-2v)} - \frac{2G}{3} \frac{\eta}{G \alpha} - 1 = \frac{2G}{3} \frac{1+v-1+2v}{1-2v} - \frac{\eta}{G \alpha} - 1 = \frac{v}{1-v}
\]

\[
\dot{\sigma}_\theta = \frac{2\eta \alpha}{r^2 \alpha} \int_a^r p r dr + \frac{2\eta \gamma_i}{r^2 \alpha} \int_a^r T r dr - \frac{2\eta \chi}{r^2 \alpha} \int_a^r C^s r dr = \frac{2\eta}{\alpha} (\dot{\alpha} \dot{p} - \dot{\chi} \dot{C}^s + \gamma_i \dot{T}) \quad (B-41)
\]

Take Laplace transform on EQ (B-40) and (B-41)

\[
\ddot{\sigma}_r = -\frac{2\eta \alpha}{r^2 \alpha} \int_a^r \ddot{p} r dr - \frac{2\eta \gamma_i}{r^2 \alpha} \int_a^r \ddot{T} r dr + \frac{2\eta \chi}{r^2 \alpha} \int_a^r \ddot{C}^s r dr \quad (B-42)
\]

\[
\ddot{\sigma}_\theta = \frac{2\eta \alpha}{r^2 \alpha} \int_a^r \ddot{p} r dr + \frac{2\eta \gamma_i}{r^2 \alpha} \int_a^r \ddot{T} r dr - \frac{2\eta \chi}{r^2 \alpha} \int_a^r \ddot{C}^s r dr = \frac{2\eta}{\alpha} (\dot{\alpha} \ddot{p} - \dot{\chi} \ddot{C}^s + \gamma_i \ddot{T}) \quad (B-43)
\]

The solutions of T, Cs and P in Laplace space are

\[
\ddot{T} = \frac{T_M}{s K_0(\beta_T)} K_0(\frac{e_T}{s_T}) \quad (B-44)
\]
\[
\tilde{C}^{S''} = \frac{C_1 K_0(\xi_{ch})}{s K_0(\beta_{ch})} - \frac{C_1 K_0(\xi_T)}{s K_0(\beta_T)} \tag{B-45}
\]

\[
\tilde{p}'' = \frac{C_3 K_0(\xi)}{s K_0(\beta)} + \frac{C_4 K_0(\xi_{ch})}{s K_0(\beta_{ch})} + \frac{C_5 K_0(\xi_T)}{s K_0(\beta_T)} \tag{B-46}
\]

\[
C_1 = (C_M^S - C_{sh}^S) + \frac{\tilde{C}^S \bar{D}^T (T_M - T_{sh})}{C_T \phi \left( \frac{D^S}{C_T \phi} - 1 \right)}
\]

\[
C_2 = \frac{\tilde{C}^S \bar{D}^T (T_M - T_{sh})}{s C_T \phi \left( \frac{D^S}{C_T \phi} - 1 \right)}
\]

\[
C_3 = p_M - p_0 + \frac{Y_{ch}}{\left( \frac{c'}{\phi} - 1 \right)} - \frac{Y_T}{\left( \frac{c'}{c_T} - 1 \right)}
\]

\[
C_4 = -\frac{Y_{ch}}{\frac{c'}{\phi} - 1}
\]

\[
C_5 = \frac{Y_T}{\frac{c'}{C_T} - 1}
\]

\[
Y_{ch} = \frac{c'}{c} \left[ (C_M^S - C_{sh}^S) + \frac{\tilde{C}^S \bar{D}^T (T_M - T_{sh})}{C_T \phi \left( \frac{D^S}{C_T \phi} - 1 \right)} \right]
\]

\[
Y_T = (T_M - T_{sh}) \left[ \frac{c'}{C_T \phi \left( \frac{D^S}{C_T \phi} - 1 \right) - \frac{c'}{c_T'}} \right]
\]

Put (B-44), (B-45) and (B-46) into (B-11), (B-42) and (B-43)
\[ \tilde{U}_{rr} \equiv \frac{a \eta \alpha C_4}{G \alpha s} \left[ \frac{a}{r} K_i(\beta) \frac{1}{\beta K_0(\beta)} \right] + \left( \frac{a \eta \alpha C_4}{G \alpha s} - \frac{a \eta \chi C_1}{G \alpha s} \right) \left[ \frac{a}{r} K_i(\beta_{ch}) \frac{1}{r \beta_{ch} K_0(\beta_{ch})} \right] \]  
\[ + \left( \frac{a \eta \alpha C_5}{G \alpha s} + a \eta \gamma (T_M - T_{sh}) + \frac{a \eta \chi C_2}{G \alpha s} \right) \left[ \frac{a}{r} K_i(\beta_T) \frac{1}{r \beta_T K_0(\beta_T)} \right] \]  
\[ \Box \]  
(B-47)

\[ \tilde{\sigma}_{rr} \equiv -\frac{2 \eta \alpha C_3}{\alpha s} \left[ \frac{a^2}{r^2} K_i(\beta) \frac{a}{r^2 \beta K_0(\beta)} \right] + \left( \frac{2 \eta \alpha C_4}{\alpha s} - \frac{2 \eta \chi C_1}{\alpha s} \right) \left[ \frac{a^2}{r^2} K_i(\beta_{ch}) \frac{a}{r^2 \beta_{ch} K_0(\beta_{ch})} \right] \]  
\[ - \left( \frac{2 \eta \alpha C_5}{\alpha s} + 2 \eta \gamma (T_M - T_{sh}) + \frac{2 \eta \chi C_2}{\alpha s} \right) \left[ \frac{a^2}{r^2} K_i(\beta_T) \frac{a}{r^2 \beta_T K_0(\beta_T)} \right] \]  
\[ \Box \]  
(B-48)

\[ \tilde{\sigma}_{\theta \theta} \equiv \frac{2 \eta \alpha C_3}{\alpha s} \left[ \frac{a^2}{r^2} K_i(\beta) \frac{a}{r^2 \beta K_0(\beta)} \right] + \left( \frac{2 \eta \alpha C_4}{\alpha s} - \frac{2 \eta \chi C_1}{\alpha s} \right) \left[ \frac{a^2}{r^2} K_i(\beta_{ch}) \frac{a}{r^2 \beta_{ch} K_0(\beta_{ch})} \right] \]  
\[ + \left( \frac{2 \eta \alpha C_5}{\alpha s} + 2 \eta \gamma (T_M - T_{sh}) + \frac{2 \eta \chi C_2}{\alpha s} \right) \left[ \frac{a^2}{r^2} K_i(\beta_T) \frac{a}{r^2 \beta_T K_0(\beta_T)} \right] \]  
\[ \Box \]  
(B-49)

The fluid, ion and temperature fluxes are then given by:

\[ J^f = -\frac{\bar{\rho}_f}{\eta} \nabla p - \mathcal{R} \nabla C^S \]  
(B-50)

\[ J^s = -\bar{\rho}_f D^S \nabla C^S - \bar{\rho}_f D^T \nabla T \]  
(B-51)

\[ J^q = -k^T \nabla T \]  
(B-52)

Put (B-21), (B-29) and (B-49) into (B-62), (B-63) and (B-64)
\[
\tilde{J}^{\text{eff}}_f = \tilde{\rho}_f k \left[ C_3 \beta K_1(\xi) \frac{K_0(\beta_{ch})}{sa} + \beta_{ch} \left( C_4 - C_1 \frac{\Re \tilde{\rho}_f \tilde{R} \tilde{T}_0}{M^S C^S D^D} \right) \frac{K_0(\tilde{\xi}_{ch})}{K_0(\beta_{ch})} + \beta_T (C_2 + C_3) \frac{K_0(\tilde{\xi}_T)}{K_0(\beta_T)} \right]
\]

(B-53)

\[
\tilde{J}^{\text{eff}}_s = \tilde{\rho}_f \left[ C_1 D^S \beta_{ch} \frac{K_1(\tilde{\xi}_{ch})}{K_0(\beta_{ch})} + \beta_T \left[ D^T (T_M - T_{ch}) - D^S C_2 \right] \frac{K_0(\tilde{\xi}_T)}{K_0(\beta_T)} \right]
\]

(B-54)

\[
\tilde{J}^{\text{eff}}_q = -\frac{k^T \beta_T (T_M - T_{ch})}{sa} \frac{K_1(\tilde{\xi}_T)}{K_0(\beta_T)}
\]

(B-55)
APPENDIX C

SOLUTIONS FOR MODE III

Constitutive equations

\[ \sigma_j = 2G\dot{e}_j + \left( K - \frac{2G}{3} \right) e_{kk}\delta_{ij} - \alpha \dot{p}\delta_{ij} + \chi \dot{C}^S - \gamma_1 \dot{T} \delta_{ij} \]  
\( \text{(C-1)} \)

\[ \dot{\zeta} = \alpha \dot{e} + \beta \dot{p} + \chi \dot{C}^S - \gamma_2 \dot{T} \]  
\( \text{(C-2)} \)

\[ \alpha' = \left( \alpha - \frac{M^S \omega_0}{C^D RT_0 \rho_f} \right) \]

\[ \beta = Q + \frac{\phi_0}{K_f} + \frac{\omega_0 (\alpha - 1) M^S}{K C^D RT_0 \rho_f} \]

\[ \chi = \frac{\omega_0}{C^S} \left( 1 - \frac{C^S}{C^D} \right) \]

\[ \chi' = \frac{\alpha - 1}{K} \chi \]

\[ \gamma_1 = K \alpha_m + \frac{\tilde{s}_f}{RT_0} a^D \omega_0 M^S \]

\[ \gamma_2 = \alpha \alpha_m + (\alpha_f - \alpha_m) \phi_0 + \frac{s_f}{RT_0} a^D \omega_0 (\alpha - 1) M^S \]

\[ a^D = \frac{1}{C^D} \]
Conservation equation of fluid mass

\[ \dot{\zeta} = k \nabla^2 p - k \mathcal{R} A \nabla^2 C^S \]  

\[ \dot{A} = \frac{\rho_f RT_0}{C^S C^D M^S} \]  

For mode 3, the temperature and solute mass fraction are constant, so the constitutive equation and conservation equations can be reduced to

\[ \sigma_{ij} = 2G e_{ij} + (K - \frac{2G}{3}) e_{kk} \delta_{ij} - \alpha' p \delta_{ij} \]  

\[ \zeta = \alpha e + \beta' p \]  

\[ \dot{\zeta} = k \nabla^2 p \]

Put (C-5) into (C-4)

\[ \sigma_{ij} = 2G e_{ij} + (K - \frac{2G}{3} + \frac{\alpha \alpha'}{\beta'} e_{kk} \delta_{ij} - \frac{\alpha'}{\beta'} \zeta \delta_{ij} \]  

Equilibrium equation (body forces are neglected)

\[ \sigma_{ij,j} = 0 \]  

Put (C-7) into (C-8)

\[ \sigma_{11,1} = 2G e_{11,1} + (K - \frac{2G}{3} + \frac{\alpha \alpha'}{\beta'} e_{,1} - \frac{\alpha'}{\beta'} \zeta_{,1} \]  

\[ \sigma_{12,2} = 2G e_{12,2} \]

\[ \sigma_{13,3} = 2G e_{13,3} \]

\[ \ldots \ldots \]
\[ \sigma_{ij,j} = 2G e_{ij,j} + (K - \frac{2G}{3} + \frac{\alpha \alpha'}{\beta'}) e_{ik,j} \delta_{ij} - \frac{\alpha'}{\beta'} \zeta_{j,j} \delta_{ij} = 0 \quad (C-11) \]

\[ e_{ij} = \frac{U_{i,j} + U_{j,i}}{2} \quad (C-12) \]

Put (C-12) into (C-11)

\[ G \nabla^2 u_i + \left( \frac{G}{1-2v} + \frac{\alpha \alpha'}{\beta'} \right) e_{j,j} - \frac{\alpha'}{\beta'} \zeta_{j,j} = 0 \quad (C-13) \]

Set \[ u_i = \phi_{,i} \]

\[ \nabla^2 u_i = \phi_{,kk} \]

\[ e_{,i} = \phi_{,kk} \quad (C-14) \]

Put (C-14) into (C-13)

\[ \left( \frac{2G(1-v)}{1-2v} + \frac{\alpha \alpha'}{\beta'} \right) \phi_{,kk} - \frac{\alpha'}{\beta'} \zeta_{,j} = 0 \quad (C-15) \]

Integrate (C-15)

\[ e = \phi_{,kk} = \frac{1}{2G(1-v) + \frac{\alpha \alpha'}{\beta'}} \frac{\alpha'}{\beta'} \zeta + g(t) \quad (C-16) \]

\[ g(t) = 0 \text{ (at infinite, both } e \text{ and } \zeta \text{ are 0, so } g(t) \text{ must be 0)} \]

Combine (C-5),(C-6) and (C-16)

\[ \frac{\partial \zeta}{\partial t} = c' \nabla^2 \zeta \quad (C-17) \]

\[ c' = \frac{k}{\left( \frac{\eta}{G} \alpha' + \beta' \right)} \]

Equations (C-13) and (C-17) are field equations. They can be written as:
\[ \nabla^2 u_i + Q_i e_{,i} - Q_2 \xi_{,i} = 0 \quad (C-18) \]
\[ \frac{\partial \xi}{\partial t} = c' \nabla^2 \xi \quad (C-19) \]
\[ Q_1 = \frac{1}{G} \left( \frac{G}{1-2\nu} + \alpha \right) \]
\[ Q_2 = \frac{1}{G} \frac{\alpha}{\beta} \]

In the cylinder coordinates
\[ \nabla^2 U_r = \cos \theta \left( \frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r} \frac{\partial U_r}{\partial r} - \frac{U_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 U_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial^2 U_\theta}{\partial \theta} \right) \]
\[ + \sin \theta \left( - \frac{\partial^2 U_\theta}{\partial r^2} - \frac{1}{r} \frac{\partial U_\theta}{\partial r} - \frac{2}{r^2} \frac{\partial U_r}{\partial \theta} + \frac{U_\theta}{r^2} - \frac{1}{r^2} \frac{\partial^2 U_\theta}{\partial \theta^2} \right) \quad (C-20) \]
\[ U_{,r,r} = \cos \theta \left( \frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r} \frac{\partial U_r}{\partial r} - \frac{U_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 U_\theta}{\partial \theta} - \frac{1}{r^2} \frac{\partial U_\theta}{\partial \theta} \right) \]
\[ + \sin \theta \left( - \frac{1}{r} \frac{\partial U_\theta}{\partial \theta} - \frac{1}{r^2} \frac{\partial^2 U_r}{\partial \theta} - \frac{1}{r^2} \frac{\partial^2 U_\theta}{\partial \theta^2} \right) \quad (C-21) \]

So equations (C-18) and (C-19) can be rewritten in cylinder coordinates as
\[ (1 + Q_1) \frac{\partial e}{\partial r} - \frac{2}{r} \frac{\partial \omega}{\partial \theta} - Q_2 \frac{\partial \xi}{\partial r} = 0 \quad (C-22) \]
\[ \frac{1+Q_1}{r} \frac{\partial e}{\partial \theta} + \frac{2}{r} \frac{\partial \omega}{\partial r} - Q_2 \frac{1}{r} \frac{\partial \xi}{\partial \theta} = 0 \quad (C-23) \]
\[ \frac{\partial^2 \xi}{\partial r^2} + \frac{1}{r} \frac{\partial \xi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \xi}{\partial \theta^2} - \frac{1}{c} \frac{\partial \xi}{\partial t} = 0 \quad (C-24) \]

The displacements, stress and pore pressure have the following dependence on $\theta$
\[ (\tilde{\zeta}^{(3)}, \tilde{e}^{(3)}, \tilde{u}_r^{(3)}, \tilde{\sigma}_{rr}^{(3)}, \tilde{\sigma}_{\theta\theta}^{(3)}, \tilde{p}^{(3)}) = (\tilde{\xi}, \tilde{\eta}, \tilde{u}_r, \tilde{\sigma}_{rr}, \tilde{\sigma}_{\theta\theta}, \tilde{p}) \cos 2\theta \quad (C-25) \]
\[ (\tilde{\omega}^{(3)}, \tilde{\omega}_\theta^{(3)}, \tilde{\sigma}_{r\theta}^{(3)}) = (\tilde{\omega}_r, \tilde{\omega}_{\theta}, \tilde{\sigma}_{r\theta}) \sin 2\theta \quad (C-26) \]
Put (C-25) and (C-26) into (C-22)–(C-24), and Laplace transform

\[ (1 + Q_1) \frac{d E}{dr} - \frac{4}{r} W - Q_2 \frac{d Z}{dr} = 0 \]  
(C-27)

\[ \frac{1+Q_1}{r} E - \frac{d W}{dr} - Q_2 \frac{1}{r} Z = 0 \]  
(C-28)

\[ r^2 \frac{d^2 Z}{dr^2} + r \frac{d Z}{dr} - \left( \frac{s}{c} r^2 + 4 \right) Z = 0 \]  
(C-29)

This set of Equation is solved (e.g., Detournay and Cheng, 1988):

\[ \frac{d U_r}{dr} = \frac{A_2}{r^2} - \frac{3 A_4}{r^4} + \frac{Q_2 C_1}{s(1 + Q_1)} \left( 1 + \frac{6}{\xi^2} \right) K_1(\xi) + \frac{Q_2 C_1}{s(1 + Q_1)} K_1(\xi) \]  
(C-30)

\[ \frac{U_r}{r} + \frac{1}{r} \frac{d U_0}{d \theta} = -1 \frac{Q_1 A_2}{1 + Q_1} - \frac{3 A_4}{r^4} - \frac{Q_2 C_1}{s(1 + Q_1)} \left( \frac{6}{\xi^2} K_2(\xi) + \frac{K_1(\xi)}{\xi} \right) \]  
(C-31)

\[ \frac{d U_r}{r} + \frac{d U_0}{dr} - \frac{U_0}{r} = - \frac{2 Q_1 A_2}{r^2(1 + Q_1)} + \frac{6 A_4}{s(1 + Q_1)} \frac{K_2(\xi)}{\xi^2} + \frac{4 Q_2 C_1}{s(1 + Q_1)} \frac{K_1(\xi)}{\xi} \]  
(C-32)

Put (C-30), (C-31) and (C-32) into (C-7)

\[ \frac{S}{r} = \frac{2K + 2G/3 + 2GQ_1 + 2\alpha \alpha'/\beta'}{(1 + Q_1)} \frac{A_2}{r^2} - \frac{6 G A_4}{s(1 + Q_1)} \left[ \frac{K + 4G/3 + 12G}{\xi^2} - \frac{\alpha'(1 + Q_1 - \alpha Q_2)}{\beta' Q_2} \right] K_2(\xi) + \frac{2G}{\xi} K_1(\xi) \]  
(C-33)
\[
\tilde{S}_{00} = \frac{(2K + 2G/3 - 2GQ_1 + 2\alpha\alpha'/\beta') A_2}{r^2} + \frac{6GA_4}{r^4} + \frac{Q_2 C_1}{s(1+Q_1)} [K \frac{2G}{3} - 12G - \xi^2 - \xi] (C-34)
\]

\[
\tilde{S}_{\rho\theta} = -\frac{Q_1}{r^2} A_2 - \frac{6GA_4}{r^4} + \frac{6G}{s(1+Q_1)} C_2 [\frac{6G}{\xi^2} K_2(\xi) + \frac{2G}{\xi} K_1(\xi)] (C-35)
\]

The boundary condition

\[
r = a
\]

\[
p = 0 \quad \tilde{S}_{\nu\rho} = \frac{-S_0}{s} \quad \tilde{S}_{\rho\theta} = \frac{S_0}{s} \quad (C-36)
\]

using boundary condition (C-36), the constant coefficients are solved,

\[
C_1 = -\frac{2S_0}{G} \frac{K_2(\beta)}{\alpha(1-2\nu)} + \frac{Q_2}{(1+Q_1)} [K_2(\beta) - 2K_1(\beta)/\beta] (C-37)
\]

\[
A_2 = -\frac{(1+Q_1 - \alpha Q_2) a^2 S_0 K_2(\beta)}{s\alpha G \{ \frac{K_2(\beta)}{\alpha(1-2\nu)} + \frac{Q_2}{(1+Q_1)} [K_2(\beta) - 2K_1(\beta)/\beta] \}} (C-38)
\]

\[
A_4 = -\frac{a^4 S_0}{6sG} \frac{1}{(1+Q_1)((1-\nu)K_2(\beta) - 2(v_u - \nu) K_1(\beta)/\beta) \times \{ 4Q_2(1-\nu)(1-\nu) K_2(\beta) + 8Q_2\alpha(1-2\nu)(1-\nu) [K_1(\beta)/\beta + 3K_2(\beta)/\beta^2] + (1+Q_1)((1-\nu)K_2(\beta) - 2(v_u - \nu) K_1(\beta)/\beta] \}} (C-39)
\]

Redefining the coefficients and using \( C_1, C_2 \) and \( C_3 \) instead of \( C_1, A_2 \) and \( A_4 \) respectively, and the solution can be expressed as the following.
\[
\begin{align*}
\frac{s p}{S_0 \cos 2\theta} &= \frac{2(1-v)\gamma_{1,2}(\xi_1,\xi_2) / K_2(\beta) + 2(1-2v)\gamma_{1,2}/r^2}{(1-2v)((1-2v)\alpha^2/(v-u) - 3(1-v)/(1+v))} \\
\frac{s S_{\rho\rho}}{S_0 \cos 2\theta} &= \frac{2(v-u)(\alpha-A)\gamma_{1,2}}{(1-u)((1-2v)\alpha^2 + [2\alpha(1-2v)+v]A/(1+v))} \\
&\times \left[ \frac{K_1(\xi) / \xi + (1+6/\xi^2)K_2(\xi)}{K_2(\beta)} \right] \\
\frac{s S_{\rho\theta}}{S_0 \sin 2\theta} &= \frac{2(v-u)(\alpha-A)\gamma_{1,2}}{(1-u)((1-2v)\alpha^2 + [2\alpha(1-2v)+v]A/(1+v))} \\
&\times \left[ \frac{K_1(\xi) / \xi + (1+6/\xi^2)K_2(\xi)}{K_2(\beta)} \right] \\
\frac{s G_{Ex}}{S_0 \cos 2\theta} &= \frac{2(v-u)(\alpha-A)\gamma_{1,2}}{(1-u)((1-2v)\alpha^2 + [2\alpha(1-2v)+v]A/(1+v))} \\
&\times \left[ \frac{K_1(\xi) / \xi + (1+6/\xi^2)K_2(\xi)}{K_2(\beta)} \right] \\
\frac{s G_{E\theta}}{S_0 \cos 2\theta} &= \frac{2(v-u)(\alpha-A)\gamma_{1,2}}{(1-u)((1-2v)\alpha^2 + [2\alpha(1-2v)+v]A/(1+v))} \\
&\times \left[ \frac{K_1(\xi) / \xi + (1+6/\xi^2)K_2(\xi)}{K_2(\beta)} \right]
\end{align*}
\]
\[ s \overline{G E}_{\theta}^{\text{III}} = \frac{2(v_u - v)(\alpha - A)C_1}{S_0 \sin 2\theta (1 - v_u)(1 - 2v)\alpha^2 + [2\alpha(1 - 2v) + (v - 3)](v_u - v)A/(1 + v)} \left[ K_1(\xi) / \xi + (1 + 6 / \xi^2)K_2(\xi) \right] K_2(\beta) \]

\[
\frac{s \overline{G W}_{r\theta}}{S_0 \sin 2\theta} = C_2 a^2 \frac{r^2}{r^4}
\]

\[ s \overline{G U}_{r}^{\text{III}} = \frac{(v_u - v)(\alpha - A)C_1}{aS_0 \cos 2\theta (1 - v_u)(1 - 2v)\alpha^2 + [2\alpha(1 - 2v) + (v - 3)](v_u - v)A/(1 + v)} \left[ K_1(\xi) / \xi + 2K_2(\xi) / \xi^2 \right] K_2(\beta) \]

\[
\frac{s \overline{G U}_{\theta}}{aS_0 \cos 2\theta} = -\frac{(v_u - v)(\alpha - A)C_1}{2(1 - v)/(1 - 2v) + \alpha Q_2} \left[ \frac{1}{2} \left( \frac{C_2 a^2}{r} + \frac{C_3 a^4}{r^3} \right) \right]
\]

Where \( S_0 = (\sigma_H - \sigma_n)/2 \)

\[ A = \frac{M S \omega_o}{C^D RT_0 \rho_f} \]

\[ Q_2 = \frac{(\alpha - A)/G}{1/M + (\alpha - 1)A/K} \]

\[ C_1 = -\frac{2(1 - v)(1 - 2v) + \alpha Q_2}{\alpha(1 - 2v)^2 + (1 - 2v)Q_2 K_1(\beta)} - \frac{\alpha Q_2 K_1(\beta)}{\beta K_2(\beta)} \]

\[ K_2(\beta) = \frac{2K_2(\xi) / \xi^2}{K_2(\beta)} \]

\[ \theta = \theta \]
\[ C_2 = \frac{(1 - \nu)[2(1 - \nu)/(1 - 2\nu) + \alpha Q_2]}{(1 - \nu)/(1 - 2\nu) + \alpha(1 - \nu)Q_2} - \frac{\alpha(1 - 2\nu)Q_2 K_1(\beta)}{\beta K_2(\beta)} \]

\[ C_3 = -\frac{1}{6} \frac{(1 - \nu)(5 - 4\nu)}{(1 - 2\nu)} + 3\alpha(1 - \nu)Q_2 + \frac{3\alpha(1 - 2\nu)Q_2 K_1(\beta)}{\beta K_2(\beta)} + \frac{12\alpha(1 - 2\nu)Q_2}{\beta^2} \]

(C-50)
APPENDIX D

SOLUTION FOR $\sigma_{zz}$

The 3-D constitutive equations in a cylindrical coordinate are

$$
\sigma_{rr} = 2G\dot{e}_{rr} + (K - \frac{2G}{3})\dot{e}_{kk} - \alpha' \dot{p} + \chi \dot{C} + \gamma_i \dot{T}
$$ (D-1)

$$
\sigma_{\theta\theta} = 2G\dot{e}_{\theta\theta} + (K - \frac{2G}{3})\dot{e}_{kk} - \alpha' \dot{p} + \chi \dot{C} + \gamma_i \dot{T}
$$ (D-2)

$$
\sigma_{zz} = 2G\dot{e}_{zz} + (K - \frac{2G}{3})\dot{e}_{kk} - \alpha' \dot{p} + \chi \dot{C} + \gamma_i \dot{T}
$$ (D-3)

The total compressive stress and volumetric strain are

$$
\sigma_{kk} = \sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz}
$$ (D-4)

$$
e_{kk} = e_{rr} + e_{\theta\theta} + e_{zz}
$$ (D-5)

Put (D-1), (D-2) and (D-3) into (D-4)

$$
\sigma_{kk} = 3K\dot{e}_{kk} - 3\alpha' \dot{p} + 3\chi \dot{C} + 3\gamma_i \dot{T}
$$ (D-6)

Combine (D-3) and (D-6), and substitute $\sigma_{kk}$ for $e_{kk}$

$$
\sigma_{zz} = 2G\dot{e}_{zz} + \frac{v}{1 + v} \sigma_{kk} - \frac{1 - 2v}{1 + v} (\alpha' \dot{p} - \chi \dot{C} + \gamma_i \dot{T})
$$ (D-7)

Use the plane strain assumption ($\dot{e}_{zz} = 0$) and (D-4)

$$
\sigma_{zz} = v(\sigma_{rr} + \sigma_{\theta\theta}) - (1 - 2v)(\alpha' \dot{p} - \chi \dot{C} + \gamma_i \dot{T})
$$ (D-8)

For inclined wellbore, the solution is only valid at the vicinity of the wellbore.
APPENDIX E

ALGORITHM USED TO FIND THE SAFE MUD WINDOW
Search for mud weight window

Input rock and fluid parameters and initial mud weight (0 MPa)

Input borehole geometry and calculate borehole stresses

Calculate principal stresses

Evaluate initial shear failure potential $f(i)/$tensile failure potential $TP(i)$

Maximize $f(i)/$Minimize $TP(i)$ with respect to $\theta$ and $r$

Incrementally increase mud weight $P(i+1)=P(i)+\Delta P$

Calculate principal stresses

Evaluate new shear failure potential $f(i+1)/$tensile failure potential $TP(i)$

Minimize/maximize $f(i+1)/T(i+1)$ with respect to $\theta$ and $r$

Is $f(i)>0$ and $f(i+1)\leq 0$ ?
Is $f(i)<0$ and $f(i+1)\geq 0$ ?

The critical low mud weight for shear failure/tensile failure exists between $(P(i), P(i+1))$

Input $P(i)$ and $P(i+1)$ into root searching routine to determine critical low mud weight within a specific tolerance

Is $P(i+1)\geq$the mud weight limit?

End analysis

Is $P(i+1)\leq$the mud weight limit?
APPENDIX F

LOWER HEMISPHERE PROJECTION

Lower hemisphere projection is used to express a 3-D borehole in a 2-D plane according to the orientation and inclination of the borehole. The radial distance $r$ from the center represents the wellbore inclination and the polar angle $\theta$ counter clockwise measured from the direction of $\theta = 0^\circ$ represents the azimuth of the well (Figure 104). So, the amount inclination from the vertical increases along the radial lines, the center of the circle represents the $0^\circ$ inclination, and the circular boundary corresponds to $90^\circ$ inclination; the angle $\theta = 0^\circ$, $360^\circ$ represents the orientation.

Figure 104. Lower Hemisphere Projection.
REFERENCES


