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D. THOMAS ON THE LIAR'S PARADOX

by Paul Mellema

Dr. Thomas' paper is to be applauded as an indication that some Wycliffe linguists have joined in the growing rapprochement between linguists and philosophers of language. Philosophers have devoted much attention to subtle analyses of the semantics of certain subparts of language, namely those which are "philosophically interesting." However limited this corpus may be, it behooves linguists to find out how much of the semantic work of philosophers can be salvaged for use in dealing with semantics in linguistics. Similarly, those logicians of wide vision who wish to expand the notation of symbolic logic to capture more of the detail of natural languages, can hope to profit from the work of linguists, now that the latter have begun to study semantics in a serious way. This mutually profitable arrangement depends, however, on linguists understanding the goals of philosophers (and vice versa, of course). It is my hope that what I say here will be of some help in promoting such understanding. The bibliography at the end of this paper also intended to steer linguists to philosophical literature having to do with matters of general linguistic interest, as well as to articles dealing specifically with the liar's paradox.

A Philosophical Bias against Grammar

Bertrand Russell, whose work was cited in the preceding piece, shared with many other philosophers of the early twentieth century a distrust of grammar and of everyday language. Russell claimed that a study of mere syntax would discover nothing wrong with the sentence, "Procrastination drinks quadruplicity." He was interested in hearing an account of why this sentence is odd, but apparently the grammar he learned in school told him that the sentence was perfectly well-formed. Thus the semantic anomaly of this sentence, which is of at least some philosophical interest, was outside the scope of grammar as Russell understood it. A philosopher could be misled if he listened to this sort of grammar.

Today, Russell's point is widely recognized, in various terms, by linguists. In tagmemics, a lexical heirarchy separate from the grammatical is required because of skewing between semantic and grammatical structures; transformational grammarians distinguish deep and surface structure; and stratificational grammar has lexemic and sememic strata to account for phenomena that are not grammatical in the usual narrow sense. Linguists generally have broadened their goals to include giving an account of semantic well-formedness, the paraphrase or synonymy relation among expressions, and so on.[1]

[1] W. V. Quine, in From a Logical Point of View, maintains that linguists need only give a correct account of the conditions under which a sentence has meaning (semantic well-formedness) and the conditions under which two expressions have the same meaning (synonymy). Quine feels that if a grammar can do these two things, it can do all that we have a right to ask of a grammar.

In addition to wanting an account of semantic ill-formedness, as exemplified in the sentence about procrastination, philosophers wanted to give (or get) an explanation of another class of sentences: those which, although well-formed in all respects, were necessarily false. These sentences, called contradictions, could be seen to be false by anyone who knew their meaning; no experience or observation (beyond that necessary to learn the meaning of the sentence) was needed to determine their falsity. The denial of a contradiction is on the whole necessarily true. Thus "Jim's father was a woman" is a contradiction, but it is necessarily true that "Jim's father was not a woman." One aim in formulating symbolic logic was to define these two related sets of sentences, contradictions and necessarily true sentences, using a notation that gave only the relevant structure of the corresponding natural-language sentences, leaving out irrelevant details of their meaning. Thus it was assumed that the native speaker (if not the grammarian) could often recognize a contradiction when he saw one, and that he similarly could recognize tautologies if they were not too complicated, or too encumbered with irrelevant details.

A further assumption was that the semantic sense of the native speaker was consistent, that no native speaker would recognize both a sentence and its negation as true, or necessarily true. Now the paradoxes were considered to be examples of inconsistency in the semantics of ordinary, intuitive language. There seems to be nothing very esoteric about the notion of a set; the term seems more the property of the man in the street than of the mathematician. It seems clear, moreover, that there are sets that have themselves as members: the set of all sets having more than two members clearly has more than two members itself, so this set (which contains every set having more than two members) is a member of itself. Also, there are some sets which do not contain themselves as members. The set of all countries in the United Nations is not itself a country belonging to the United Nations.

There is, then, at least one set which is not a member of itself. Consider now the set of all non-self-membered sets.

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It is possible, using the method of reductio ad absurdum or indirect proof, to show that this set is a member of itself. We assume to begin with that

(1) The set of all sets that are not members of themselves is not a member of itself.

This set (call it N, for "non-self-membered") has the property, according to (1), of not being a member of itself. But the set N is defined to include every set having this property, so on the assumption that (1) is true, we have to conclude that N is a member of N — a fact which (1) denies. This is the "absurdity" we need to show that we were wrong in assuming (1). So (1) is false, and its negation is true: N is a member of itself.[1]

[1] This last sentence is the step where appeal is made to the principle of indirect proof, and intuitionist logicians do not accept this principle. In particular, they refuse to grant that if not-P is false, then P is true. It has been shown that this leads to an infinite number of truth-values instead of the usual two (true and false).

Unfortunately, we can in the same way prove (1) by assuming its negation,

(1') The set of all sets which are not members of themselves is a member of itself.

According to (1'), the set N is a member of itself. But in this case, the set N lacks the property required of all members of N, namely the property of not being self-membered. Since according to (1') the set N does not have the property common to all and only the members of N, it follows that the set N cannot be in the set N — that is, (1') is false because it leads to a contradictory conclusion. Hence the negation of (1'), namely (1), must be true. The upshot, then, of these two indirect proofs is that the most natural, intuitive definition of 'set' and related terms leads us to a contradiction. Beneath the innocuous-looking surface of the notions of set and membership, we have uncovered inconsistency; apparently everyday language is not, after all, a consistent semantic system.

This, at any rate, is the moral which many philosophers draw from set-theoretical and other paradoxes.[2] And if the

[2] Russell himself seems to criticize the paradoxes, however, as being based on a perversion of the ordinary use of 'set.' He claims that it never makes any sense, in any language, to assert (or deny) that a set is a member of itself. Thus he seems to be finding fault not with English or any other language, but rather with mathematicians' formalization and exegesis of the notions involved in the paradoxes.

discovery of paradoxes in the grammar of such garden-variety words as 'set' and 'member' was not enough to arouse the interest of logicians, the mathematical importance of the theory of sets was good reason for them to try to resolve the paradoxes. Georg Cantor, a German mathematician of the nineteenth century, had discovered the versatility of set theory (without bothering to formalize the theory into any set of axioms), and Russell himself participated in a demonstration that all[1] the theorems of the theory of natural, integral, rational, real, and complex numbers, as well as the theorems of the calculus, were proveable using only the

[1] This statement is subject to certain strictures imposed by Goedel's theorem about the incompleteness and incompletability of the theory of natural numbers.

assumptions of set theory. What, then, if the foundation of most mathematics up to the eighteenth century turned out to be inconsistent? Could Mathematics be saved? If so, it would have to be through a resolution of the paradoxes of set theory.

The Discreteness Assumption of General Logic

In pointing out the effect that vagueness can have on arguments, Dr. Thomas is joining the good company of Max Black, Ludwig Wittgenstein, and John Locke (as well, I believe, as Russell himself). Locke pointed out that though we know the difference between a horse and a lump of lead, we might be quite at a loss if a horse were to be changed by barely perceptible degrees into a lump of lead: at what point does the thing cease to be a horse, and when does it become a lump of lead? (How much hair must a man lose to become bald?) For similar reasons, Wittgenstein held that for most words, it is impossible to give necessary and sufficient criteria for their application to items in our experience. He held, for example, that there are not any characteristics shared by all games, and only games, by virtue of which we can define the word 'game' in a rigorous way.[2] Rather, Wittgenstein

[2] Similarly, some phonologists hold that phonemes may not be rigorously definable in terms of features: there may be no set of features shared by all allophones of one phoneme, and by no allophone of any other phoneme.

says, games bear a family resemblance to one another. Max Black has made some proposals about measuring the vagueness of words, and about incorporating vagueness coefficients into the apparatus of symbolic logic.[3] I understand that some

[3] See his book, Language and Philosophy. The first essay in the book is intended to illustrate and explain what is meant by the "linguistic method" in philosophy.

British philosophers of the "ordinary language" school have argued that vagueness is a property of considerable value to language communities and users, that it is not at all a liability.

With regard to some versions of the liar's paradox (e.g., "All men are liars", or "I'm a liar but that's the truth"), the charge of vagueness which Dr. Thomas brings is certainly telling, and these versions are worthy of no more space than Dr. Thomas devotes to them, because they are so exceedingly weak and unconvincing.

Self-reference and the Liar's Paradox

It is not strictly speaking accurate to say that general logic pays no attention to discourse structure. One of the principal goals of logic, in fact, is precisely to define what constitutes a valid deductive proof, which is a discourse of a certain sort. However, Dr. Thomas' mistake at the beginning of section 2 of his paper is by no means fatal to his argument in that section. Just what is that argument?

According to Thomas, there are two syntactic requirements for use of words such as 'lie' (whether verb or noun) and 'know': first, there must be some clause or assertion which is said to be a lie, or which is said to be known, etc.; and second, the clause or assertion which is referred to as a lie must be <u>different</u> from the clause in which the word 'lie' itself appears. In a word, utterances about lying or falsity cannot, in English, be self-referring. Hence the "sentences" which exemplify the liar's paradox are not well-formed sentences of English at all, and proving contradictions by making use of these non-sentences does not in the least show that English is inconsistent (in the logician's sense), or that if English is consistent then it consists of an infinite number of different languages.

I should like to make three comments on this position. First, it is not clear to me why Thomas refers to the alleged ill-formedness of the paradoxical sentences as syntactic or grammatical, rather than semantic. If it is correct and necessary to state the second requirement in terms of reference, that is what the word 'lie' <u>refers</u> to, then we are dealing with the semantic relation <u>par excellence</u>. Truth and falsity are semantic properties of assertions, because it is necessary to know what an assertion means in order to check its truth value, and ordinarily it is necessary to look at the real-world situation which the assertion purports to describe (refer to) if we want to know whether or not a particular assertion is a lie. For example, we can hardly rely on syntax alone to determine the well-formedness (on Dr. Thomas' criteria) of the sentence,

(2) What I say at 4:35 is false.

This sentence, Thomas would presumably agree, is perfectly good English if (a) I say <u>something</u> at that moment and (b) what I say is something different from (2) itself. But to determine whether either of these requirements is met, I must determine what time it is, say by looking at the hands of a clock, and I must find out whether at the right moment my lips are moving, producing some assertion other than (2). Thus to determine whether (2) is well-formed, on Dr. Thomas' cr teria, it necessary to have information that is clearly not syntactic information about this or any other sentence. For this reason, it seems to me that the kind of ill-formedness (if such it is) that Dr. Thomas is talking about is semantic rather than syntactic.

Second, I suspect that though Thomas only claims that these are facts in English, he would be interested in defending any human language against paradox-hunters like Russel. It is not clear from the paper, but probably Thomas feels that if the rules of some language did permit words meaning 'lie' or 'false' to be used in a self-referring way, then that language would contain paradoxes. Possibly Thomas would be willing to say, then, that the requirement that words like 'lie' not refer to the clauses they appear in is a requirement of the semantics of all languages, and not just English.[1]

[1]On the whole, philosophers have been interested in semantic facts about all languages rather than in grammatical facts peculiar to only a few languages. In a way, the development of notation in symbolic logic can be seen as an attempt to transcribe or represent, in a language-independent way, the semantic structure of linguisitc structures, just as phonetic notation permits us to represent the sounds of utterances in a language-independent way.

Certain philosophers, in discussing Gödel's proof of the incompleteness of arithmetic, have argued that the device of self-referring expressions, which is crucial to the proof, makes no sense (not even contradictory sense).[2] Thus there

[2]On this question, see Popper (1954) and Ross (1969).

have been philosophers that have taken Thomas' position with respect to the "distribution" of 'false,' 'lie,' etc. in semantic structures in any language.

My third point is that Russell is one of these philosophers, and that there is no fundamental disagreement between Russell and Thomas, except on the use of the locution "infinite set of different languages" to describe a language built on the assumption that self-referring expressions are ill-formed in some way (perhaps semantically). What Russell means by this phrase is merely that a language must distinguish an infinite number of semantic levels or types of expressions: there are in Russell's set theory expressions that designate objects that are not sets, and these can be members of first-level sets. First-level sets, however, cannot be members of first-level sets, Russell says or we run the risk of generating paradoxes; for this reason, there must be second-level sets, and a separate set of expressions in set theory to refer to these. Still, though, Russell would surely not have been justified in saying that set theory requires an infinite hierarchy of languages for its formulation; nor is there any reason to say that natural languages are any worse off than the formal language of set theory in this respect.

This is just to say that more than Russell's intuition was sound when he said (if he did) that English was an infinite set of different languages: the only thing that wasn't sound was his choice of words, since his intuition had lead him to analyze words like 'lie' in exactly the way that Thomas proposes. It may be, of course, that I have been guilty of the fault I thought to find in Thomas: it may be that I have simply taken too narrowly Thomas' use of the term intuition.

One more thing remains to be said. I did not mean, in the preceding paragraph, to be endorsing Russell's particular approach to the paradoxes of set theory, nor Thomas' entirely analogous approach to semantic paradoxes like the liar's paradox. I confess that I have nothing very enlightening to say about alternative set theories such as Zermelo's and von Neumann's, but I know that these do not involve the radical position taken by Russell, that self-membership is a meaningless concept, and I do know that these alternatives are able to avoid the paradoxes of set theory just as well as Russell's theory of types. Perhaps, then, these more moderate versions of set theory could be adapted to the needs of semantics. In this way, linguists would be freed from the onus of having to argue for a rather radical philosophical position, viz. that self-referring expressions are always and in all languages semantically ill-formed, ie. meaningless.

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BIBLIOGRAPHY

Austin, J. L. How to Do Things with Words. Cambridge: Harvard University Press, 1962

Black, Max. Language and Philosophy. Ithaca: Cornell University Press, 1949.

Davidson, Donald, "The Logical Form of Action Sentences," in The Logic of Action and Preference, ed. N. Rescher. Pittsburg: Pittsburg University Press, 1967.

______, "Causal Relations," in The Journal of Philosophy, v. 64 (1967), pp. 691-703.

Findlay, J., "Goedelian Sentences: A Non-numerical Approach," in Mind, v. 51 (1942), pp. 259-65.

Fitch, F. B., "Self-Reference in Philosophy," in Mind, v. 55 (1946), pp. 64-73.

Kirk, Robert, "Translation and Indeterminacy," in Mind, v. 78 (1969), pp. 321-41.

______, "Quine's Indeterminacy Thesis," in Mind, v. 78 (1969), pp. 607-8.

Popper, Karl R., "Self-Reference and Meaning in Ordinary Language," in Mind, v. 63 (1954), pp. 162-9.

Quine, W. V. From a Logical Point of View. Cambridge: Harvard University Press, 1953.

______, Word and Object. Cambridge: MIT Press, 1960.

Ross, Alf, "On Self-Reference and a Puzzle in Constitutional Law," in Mind, v. 78 (1969), pp. 1-24.

Russell, Bertrand, "Mathematical Logic as Based on the Theory of Types," in Logic and Knowledge, ed. R. C. Marsh. London: George Allen and Unwin, 1956.

Searle, John R. Speech Acts. London: Cambridge University Press, 1969.

Ushenko, A., "Undecidable Statements and Metalanguage," in Mind, v. 53 (1944), pp. 258-62.