2013

Research about low permeability measurement

Jun He
University of North Dakota

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RESEARCH ABOUT LOW PERMEABILITY MEASUREMENT

by

Jun He

Bachelor of Science, Southwest Petroleum University, 1995

A Thesis
Submitted to the Graduate Faculty
of the
University of North Dakota
In partial fulfillment of the requirements

for the degree of

Master of Science

Grand Forks, North Dakota
August
2013
This thesis, submitted by Jun He in partial fulfillment of the requirements for the Degree of Master of Science from the University of North Dakota, has been read by the Faculty Advisory Committee under whom the work has been done and is hereby approved.

Kegang Ling, Chairperson
07/15/2013

Richard LeFever
7/15/2013

Steve Benson
7/15/2013

This thesis is being submitted by the appointed advisory committee as having met all of the requirements of the Graduate School of the University of North Dakota, and is hereby approved.

Wayne S. Sandate
Dean of the Graduate School
July 17, 2013
Title: Research about Low Permeability Measurement

Department: Geology and Geological Engineering

Degree: Master of Science

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Jun He

July 16, 2013
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Petroleum exploration and production from shale formations have gained great attention throughout the world in the last decade. Producing the hydrocarbons from shale is challenging because of the low porosity and permeability. It is imperative to investigate permeability of the shale formations in order to better understand the performance of wells that are producing hydrocarbons from shale. Permeability is also one of key parameters in modeling fluids flow in matrix in reservoir simulation. Due to the low or very low permeability, the measurement of permeability is time consuming and expensive. These factors often limited the ability to perform permeability measurement on large numbers of samples. Thus, there is a great demand for a method that can significantly reduce the time of the measurement, which leads to lower cost in core analysis.

In this study a downstream pressure build-up method, which is more operational, as in this method the ratio of volume of the upstream reservoir, $V_1$, to volume of the downstream reservoir, $V_2$, approaches infinite.

In addition, we developed another new method to determine the permeability of low to very low permeability rock based on Darcy’s law and the radius-of-investigation concept, which has been used in the well test design and analysis. Our method evaluates the permeability under unsteady-state flow, which requires a shorter time to determine
flow capacity of low permeability rock. The new approach is different from the existing methods, such as GRI, oscillating pulse, and pulse decay methods. The significance of this investigation is that it overcomes the limitations in existing methods thus becomes an important supplement to the existing methods.
CHAPTER I

INTRODUCTION

Permeability is a property of a porous medium and is an indicator of its ability to allow fluids flow through its inter-connected pores. Permeability is an inherent characteristic of the porous media only. It depends on the effective porosity of the porous media (Triad, 2004).

The fundamental SI unit of permeability is m$^2$, but the Darcy (D), named after French engineer Henry Darcy, is a practical unit for permeability. One Darcy is defined as follows: a permeability of one Darcy will allow a flow of 1 cm$^3$/s of fluid of 1 centipoise (cp) viscosity through an area of 1 cm$^2$ under a pressure gradient of 1 atm/cm. One Darcy equals 0.986923 × 10$^{-12}$ m$^2$. In the oil and gas industry, a smaller unit of permeability, milli-Darcy (mD), is used more commonly because the permeability for most rocks is less than one Darcy, and for the low permeability rocks, the use of micro-Darcy (μD) or nano-Darcy(nD) is common.

The range of the permeability of the petroleum reservoir rocks may be from 0.1 to 1,000 mD. A rock is considered to be tight when its permeability is below 1 mD (Triad, 2004). However, this criterion has been lowered to values of 0.1mD (Law & Spencer, 1993) due to the application of the new stimulation techniques to increase oil and gas
production. Tight rocks have been extensively studied for a wide range of applications that include CO₂ geological storage, deep geological disposal of high-level, long-lived nuclear wastes, and production of oil and gas from unconventional reservoirs. In the recent years, the increasing demands for oil and gas have stimulated the explorations and productions of petroleum from low permeability formations, such as shale and limestone. Producing the hydrocarbons from those formations is challenging because of their low porosity and permeability. More realistic fluid flow simulation to model the process of producing the hydrocarbons in those formations requires more accurate measurements of permeability. Also it is urgent to investigate permeability of those low permeability formations in order to gain better understanding of the process of well producing hydrocarbons from shale.

Based on experimental work from Darcy, many permeability measurement methods have been presented in order to improve the measurement accuracy, and precision, and to reduce the measurement time. Some methods are used in laboratory, some are in field, and some can be used in both. In this study, we focused on the methods that can be used in the laboratory.

1.1. Previous Work

Conventional methods of measuring permeability in the laboratory utilize steady-state flow. Steady-state flow, or a constant pressure gradient flow, is established
through the core plug, and the permeability is calculated from the rate of the measured flow and the pressure gradient. But this method is not adequate when measuring permeability in the low permeability samples. Not only the low flow rates across the core plug are difficult to be measured and controlled, but the tests are also quite time consuming.

Because of the disadvantage of the steady-state flow, unsteady state flow, a condition under which the pressure gradient is a function of time, was studied. With the measurements of the volumetric flow rate, upstream, and downstream pressures, the permeability can be calculated. Since Brace et al. (1968) introduced a transient flow method to measure permeability of Westerly Granite, many methods that based on unsteady state flow theory have been proposed to measure the permeability of tight rocks.

Although many unsteady state methods have been developed to measure permeability, most of them can be categorized into three types, which are pulse decay method, oscillating pulse method, and Gas Research Institute (GRI) method (Tinni et al., 2012).

1.1.1. The Pulse-decay Method

The pulse decay method is a transient method. The experimental arrangement is shown schematically in Figure 1-1. The sample has both upstream reservoir and downstream reservoir with the initial condition of uniform pore, upstream and
downstream pressures. When a pressure pulse is applied at the upstream end of a core plug and propagates through core to the downstream reservoir, the pressure pulse will decay over time. The decay characteristics depend on the permeability, size of the sample, volumes of upstream and downstream reservoirs, and physical characteristics of the fluid. Permeability is estimated by analyzing the decay characteristics of the pressure pulse.

\[
\Delta p = \frac{kA}{\mu c L} \left( \frac{1}{V_1} + \frac{1}{V_2} \right) \cdot e^{-\alpha t}
\]

(1-1)

Where \( \alpha = \frac{kA}{\mu c L} \left( \frac{1}{V_1} + \frac{1}{V_2} \right) \)

(1-2)

\( A \) is cross-sectional area, \( L \) is length of sample, \( \mu \) is fluid viscosity, \( c \) is fluid compressibility, \( V_1 \) and \( V_2 \) are volumes of upstream and downstream reservoirs, \( p_1 \) and \( p_2 \)

Figure 1-1 Schematic diagram for pulse decay permeability system

Brace et al. (1968) suggested a transient flow, or pressure-pulse, technique to determine the permeability of granite. In their experiment the decay of pressure was measured and the permeability was estimated through the following equation.
are pressures of upstream and downstream reservoirs, \( p_f \) is final pressure, and \( \Delta p \) is the pressure difference between the upstream reservoir and the downstream reservoir at time \( t = 0 \).

Permeability is calculated from Equation (1-2) after obtaining \( \alpha \) from Equation (1-1), which is calculated as a function of pressure decay \( (P_1 - P_f) \) on a semi-logarithm scale against time. It should be noted that \( \Delta p \) must be small for this equation to be valid.

Dicker and Smits (1988) improved the pressure pulse-decay method by showing a general solution of the differential equation which describes the decay curve. Based on the solution, they pointed out that fast and accurate measurements are possible when the volumes of the upstream and downstream reservoirs in the equipment are equal to the pore volume of the sample. Jones (1997) developed a technique to reduce measurement time in pulse-decay experiment. In his approach, permeability is calculated from “late-time” measurements. Jones emphasized that the volumes of the upstream and downstream reservoirs should be equal and pointed out that the initial pressure equilibration step is the most time-consuming part of the pulse-decay technique. To avoid the equilibrium state, Johns’ method utilizes a smooth pressure gradient, which requires smaller upstream and downstream reservoirs. Metwally (2011) gave another pulse-decay method by keeping the upstream pressure constant, which leads to an infinitely large volume of the upstream reservoir so that the ratio of upstream volume over downstream volume is infinite. Thus, the solution of the pulse-decay
measurements can be simplified and the decay time can be reduced.

The followings are other important studies on the pulse-decay method: Hsieh et al. (1981) applied transient pulse test to measure the hydraulic properties of the rock samples with low permeability. Le Guen et al. (1993) employed pulse-decay method to measure permeability of rocksalt under thermo-mechanical stress. Luffel et al. (1993) reviewed three methods to measure shale permeability. Hildenbrand et al. (2002 and 2004) studied the gas effective permeability of fine-grained sedimentary rock using downstream pressure-time relationship under Darcy flow condition. Homand et al. (2004) applied the modified pulse test proposed by Hsieh (1981) to characterize permeability of low permeable rocks. Billiotte et al. (2008) used transient pulse technique to measure the permeability of mudstones. They observed that the gas permeability is decreasing with the increase of the confining stresses due to the closing of some micro fissures in the sample. This reduction is irreversible after a loading-unloading cycle. Cui et al. (2009) presented models to correct adsorption terms during pulse-decay measurements on crushed samples and in the field experiments. Cui et al. (2009) also used model to determine the permeability or diffusivity from on-site drill-core desorption test data. Metwally and Sondergeld (2011) built a new apparatus to simulate the permeability of tight rock samples over a range of the effective pressures based on the pulse decay technique.
In 1990, Kranz et al. presented an oscillating pulse method to determine the permeability and diffusivity of the rock samples. They provided one optimized measuring system and the oscillation frequency for each of the rock types. Fischer and Paterson (1992) measured permeability and storage capacity in three types of rocks (marble, limestone, and sandstone) during deformation at high pressure and temperature.

The oscillating pulse method estimates rock permeability by interpreting the amplitude attenuation and the phase retardation in the sinusoidal oscillation of the pore pressure as it propagates through a sample.

At the beginning of the experiment, the sample pore pressure, the upstream pressure, and the downstream pressure are stabilized. Then a sinusoidal pressure wave is generated in the upstream and propagates through a core plug. Using the information of the amplitude attenuation and phase shift between the upstream pressure wave and the derived downstream pressure wave at the downstream side of the sample, the permeability can be obtained (see Figure 1-2).

This method can measure the permeability of tight rock in a relative short time without destroying rock sample. The accuracy of permeability obtained from this method relies on the signal-to-noise ratio and data analysis techniques. The optimum frequency of the oscillation and the ratio of the downstream to upstream pore pressures depend upon the sample size and the magnitude of permeability (Kranz et al., 1990).
Therefore, the calculated permeability contains large uncertainty when measured under the condition of low signal-to-noise ratio. Moreover, different analysis techniques can result in different permeabilities in the same experiment.

![Figure 1-2 Changes of the pressure during a sinusoidal oscillation pulse method](image)

**1.1.3. Gas Research Institute (GRI) Method**

GRI method (Luffel, Hopkins, & Schettler Jr., 1993) differs from the previous methods by carrying out the measurement on crushed rock sample. The experimental arrangement is shown in Figure 1-3. The crushed rock particles are in Chamber 2. Initially the pressure in Chamber 1 is greater than the pressure in Chamber 2. Then open Gas Outlet Valve to allow gas flow from Chamber 1 to Chamber 2. The pressure
decay in the rock particles can be observed. Permeability is obtained through the analysis of this pressure decay over time.

Figure 1-3 Schematic diagram for GRI permeability system

GRI method requires shorter experimental time when compared with other methods. However the permeability measured from the crushed samples can differ by two to three orders of magnitude from the companion intact samples (Passey et al., 2010 and Tinni et al., 2012). Also microcracks in the crushed particle violate the assumption in the GRI method, which leads to an overestimate of permeability (Tinni et al., 2012).

1.2. Purpose/Thesis Statement

Due to the properties of the low or very low permeability, the measurement of the tight-rock permeability is time consuming and expensive. An inexpensive method that tremendously reduces the measurement time in core analysis is needed.
After reviewing the previous work, we chose pulse decay method as our base method to measure permeability in the tight rock. Because the pulse decay method does not destroy the core plug as GRI method and has higher confident level than oscillating pulse method.

We found that to improve the pulse decay method, alterations in the volumes of the sample pore space, the upstream reservoir, and the downstream reservoir are necessary. Theoretically, the perfect result can be obtained only when those volumes are equal, \( V_1 = V_2 = V_p \) (Dicker and Smits, 1988; and Jones, 1997), but it is not easy to obtain because the pore space of a core plug (2×1 inch) is very small. A long time for pre-balance is needed even if only the equal volumes between \( V_1 \) and \( V_2 \) are required.

In our study, we first investigated the downstream pressure build-up method, which belongs to pulse decay method, but it is more operational since it does not require the equal volumes between \( V_1 \) and \( V_2 \). We also developed another method, which is called radius-of-investigation method, to determine the permeability of low to very low permeability rock, by utilizing Darcy law and the radius-of-investigation concept. This radius-of-investigation method is very useful in decreasing the time required for a test.
CHAPTER II

METHOD

2.1. Sampler and Equipment

Middle Bakken core samples, supplied by the North Dakota Geological Survey's Wilson M. Laird Core and Sample Library, were chosen as the specimens to represent the tight rocks.

Due to the fragile nature of the core, before preparing the core plugs, we pre-cooled the core at -85 °C for 20 days and drilled with liquid N₂ coolant (Figure 2-1). The core plugs are cylindrical with dimension of one inch in diameter and two inches long.

Figure 2-1 Core plug sampling system used in this study
The equipment that is used to perform our experiments is AutoLab-1500, which is made by New England Research Inc. Figure 2-2 presents a conceptual diagram of the gas flow in AutoLab-1500. The cylindrical core plug was covered with copper sheet (Figure 2-3) in order to make a gas-tight seal on the cylindrical wall of the sample, and for applying radial confining pressure. Then the core plug was mounted in a sample holder with flexible rubber sleeves at both ends of the plug (Figure 2-3). At last, the sample holder was put into a vessel flooded with mineral oil, in which the sample can be hydrostatically compressed by applying force to plug by hydraulic means.

Figure 2-2 Experimental setup for permeability measurement under stresses
Figure 2-3 Images of core and core holder for low permeability test system

Figure 2-4 Images of the end caps (left: downstream cap, right: upstream cap)

Figure 2-4 shows that the two end caps contain two axial ports for transporting gas to and from the sample and each of them has radial and circular grooves for distributing gas to its entire surface. The upstream end-cap connects to a servo-controlled hydraulic intensifier, which is used to control and monitor the upstream pressure ($p_1$). The downstream pressure at the other end of the sample is monitored by a miniature pressure transducer, which is located in the downstream end-cap. To minimize the volume of the downstream reservoir, a small pocket is implemented inside the downstream end-cap.
(Figure 2-5). The volume of downstream reservoir in the AutoLab-1500 is 0.63 cc.

Figure 2-5 Schematic diagram of AutoLab-1500 low permeability system

2.2. Derivation of Diffusivity Equation

Because the permeability of tight rock is low, gas (nitrogen) is used as the test fluid in our experiment. The gas flows from the left-side of the core, through the core, and out of the right-side of the core as shown in Figure 2-6.
To derive the diffusivity equation of the gas flow in the core, following assumptions are made: 1) the core is homogeneous, 2) the properties of the rock are constant, 3) the flow in the cylindrical core is laminar, and 4) the flow in the core is isothermal.

Considering a control volume (from \( x \) to \( x + \Delta x \)), which is the volume that the gas flows in from \( x \) and out at \( x + \Delta x \) during a certain time period \( \Delta t \), the law of the mass conservation provides:

\[
\text{Mass}_{\text{in}} - \text{Mass}_{\text{out}} = \text{Accumulated Mass} \tag{2-1}
\]

The mass of gas that flows into the section is:

\[
\text{Mass}_{\text{in}} = \rho_x v_x A \Delta t \tag{2-2}
\]

where \( \Delta t \) is the time period, \( \rho_x \) is the gas density at location of \( x \) during \( \Delta t \), \( v_x \) is the gas velocity at \( x \) in the \( x \) direction during \( \Delta t \), and \( A \) is the cross-sectional area of the core plug.

The mass of gas that flows out of the section is:

\[
\text{Mass}_{\text{out}} = \rho_{x+\Delta x} v_{x+\Delta x} A \Delta t \tag{2-3}
\]
where \( \Delta t \) is the time period, \( \rho_{x+\Delta x} \) is the gas density at location of \( x+\Delta x \) during \( \Delta t \), \( v_{x+\Delta x} \) is the gas velocity at \( x+\Delta x \) position during \( \Delta t \), and \( A \) is the area of the cross section of the core plug.

The mass of gas that accumulates inside the section is:

\[
\text{Accumulated Mass} = \rho_{t+\Delta t} \phi_{t+\Delta t} A \Delta x - \rho_t \phi_t A \Delta x
\]

Where \( \rho_{t+\Delta t} \) is the gas density in the control volume at \( t+\Delta t \), \( \phi_{t+\Delta t} \) is the rock porosity of the control volume at \( t+\Delta t \), \( \rho_t \) is the gas density in the control volume at \( t \), \( \phi_t \) is the rock porosity of the control volume at \( t \), \( \Delta x \) is the incremental distance in \( x \) direction, and \( A \) is the area of cross section of the core plug.

Substituting Equations (2-2), (2-3), and (2-4) into Equation (2-1), we have

\[
\rho_x v_x A \Delta t - \rho_{x+\Delta x} v_{x+\Delta x} A \Delta t = \rho_{t+\Delta t} \phi_{t+\Delta t} A \Delta x - \rho_t \phi_t A \Delta x
\]

Dividing both sides with \( A \Delta x \Delta t \) and taking the limits as \( \Delta x \) and \( \Delta t \) go towards zero, the resulting equation becomes a linear diffusivity equation, which is

\[
\frac{\lim_{\Delta x \to 0}}{\Delta x} \frac{\rho_{x+\Delta x} v_{x+\Delta x} - \rho_x v_x}{\Delta x} = \frac{\lim_{\Delta t \to 0}}{\Delta t} \frac{\rho_{t+\Delta t} \phi_{t+\Delta t} - \rho_t \phi_t}{\Delta t}
\]

\[
\frac{\partial}{\partial x} (\rho v) = -\frac{\partial}{\partial t} (\rho \phi)
\]

Using Darcy’s law, we can express gas velocity as

\[
v = -\frac{k}{\mu} \frac{\partial p}{\partial x}
\]

where \( k \) is permeability, \( \mu \) is gas viscosity, \( \frac{\partial p}{\partial x} \) is the pressure drop along \( x \) direction.

According to the real gas law, we can calculate gas density using
\[ \rho = \frac{M}{RT} \cdot \frac{p}{z} \]  

(2-7)

where \( M \) is molar mass, \( R \) is the ideal gas constant, \( T \) is the temperature of the gas, \( p \) is the pressure of the gas, and \( z \) is gas \( z \)-factor.

Substituting Equations (2-6) and (2-7) into Equation (2-5), we get

\[ \frac{\partial}{\partial x} \left[ \frac{M}{RT} \cdot \frac{p}{z} \cdot \left( -\frac{k}{\mu} \frac{\partial p}{\partial x} \right) \right] = -\frac{\partial}{\partial t} \left( \frac{M}{RT} \cdot \frac{p}{z} \cdot \phi \right) \]

Based on our assumptions above, temperature \( (T) \) and permeability \( (k) \) are constants, we divide both sides by \( \frac{M}{RT} \) to get

\[ \frac{\partial}{\partial x} \left[ \frac{k}{\mu z} \cdot \frac{\partial p}{\partial x} \right] = \frac{\partial}{\partial t} \left( \frac{p}{z} \cdot \phi \right) \]

which can be simplified as

\[ k \cdot \frac{\partial}{\partial x} \left[ \frac{p}{\mu z} \cdot \frac{\partial p}{\partial x} \right] = \frac{\partial}{\partial t} \left( \frac{p}{z} \cdot \phi \right) \]

(2-8)

Using gas pseudo-pressure concept (Al-Hussainy, 1966), the gas pseudo-pressure is defined as

\[ m(p) = \int_{p_b}^{p} \frac{2p}{\mu z} dp \]

(2-9)

where \( p \) is the pressure and \( p_b \) is the low base pressure.

Assuming isothermal and small pressure drop, we take the derivatives with respect to \( x \), and \( t \) at both sides of the Equation (2-9). The equation becomes

\[ \frac{\partial}{\partial x} \left[ m(p) \right] = \frac{\partial}{\partial x} \left( \int_{p_b}^{p} \frac{2p}{\mu z} dp \right) \]
which can be simplified as

$$\frac{\partial}{\partial x} [m(p)] = \frac{2p}{\mu z} \cdot \frac{\partial p}{\partial x}$$

.................................................................(2-10)

and

$$\frac{\partial}{\partial t} [m(p)] = \frac{\partial}{\partial t} \left( \int_{\mu z}^{\phi} \frac{2p}{d p} \right)$$

which can be simplified as

$$\frac{\partial}{\partial t} [m(p)] = \frac{2p}{\mu z} \cdot \frac{\partial p}{\partial t}$$

.................................................................(2-11)

If we use $m(p)$ to rearrange Equation (2-8), it becomes

$$k \cdot \frac{\partial}{\partial x} \left[ \frac{2p}{\mu z} \cdot \frac{\partial p}{\partial x} \right] = \mu \cdot \frac{\partial}{\partial t} \left( \frac{2p}{\mu z} \cdot \phi \right)$$

which can be written as

$$k \cdot \frac{\partial^2}{\partial x^2} [m(p)] = \mu \phi \cdot \frac{\partial}{\partial t} \left( \frac{2p}{\mu z} \right) + \mu \cdot \frac{2p}{\mu z} \cdot \frac{\partial \phi}{\partial t}$$

.................................................................(2-12)

Assuming $\mu$ is constant, Equation (2-12) can be rearranged to

$$k \cdot \frac{\partial^2}{\partial x^2} [m(p)] = \frac{2\phi}{z} \cdot \frac{\partial}{\partial t} \left( \frac{p}{z} \right) + \frac{2p}{z} \cdot \frac{\partial \phi}{\partial t}$$

.................................................................(2-13)

If the right-hand side (RHS) of the Equation (2-13) can be transformed into a new form so that the only variable required to be differentiated is $m(p)$, then solving the equation will be much easier.

Based on this hypothesis, the first item of the RHS of the Equation (2-13),

$$\frac{2\phi}{z} \cdot \frac{\partial}{\partial t} \left( \frac{p}{z} \right)$$

needs to be modified. Recalling real gas law, we have
\[ P V = znRT \]

\[ \frac{p}{z} = \frac{1}{V} \cdot nRT \]

Inserting \( \frac{p}{z} = \frac{1}{V} \cdot nRT \) into \( \frac{\partial}{\partial t} \left( \frac{p}{z} \right) \), we get

\[ \frac{\partial}{\partial t} \left( \frac{p}{z} \right) = \frac{\partial}{\partial t} \left( \frac{nRT}{V} \right) = \frac{\partial}{\partial p} \left( \frac{1}{V} \right) \frac{\partial p}{\partial t} = nRT \cdot \left( -\frac{1}{V^2} \cdot \frac{\partial V}{\partial p} \right) \frac{\partial p}{\partial t} \]

The coefficient of isothermal compressibility of gas is \( c_g = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right) \). Substituting \( c_g \)

into Equation (2-14), we get

\[ \frac{\partial}{\partial t} \left( \frac{p}{z} \right) = \frac{nRT}{V} \cdot c_g \cdot \frac{\partial p}{\partial t} = \frac{p}{z} \cdot c_g \cdot \frac{\partial p}{\partial t} \]

The second term of the RHS of the Equation (2-13) is \( \frac{2p}{z} \cdot \frac{\partial \phi}{\partial t} \). According to the

rock pore compressibility, \( c_s = \frac{1}{\phi} \left( \frac{\partial \phi}{\partial p} \right) \), we have

\[ \frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial p} \cdot \frac{\partial p}{\partial t} = \phi c_s \cdot \frac{\partial p}{\partial t} \]

Finally, substituting Equation (2-15), and (2-16) into (2-13) gives

\[ k \cdot \frac{\partial^2}{\partial x^2} \left[ m(p) \right] = \frac{2\phi}{z} \left( \frac{p}{z} \cdot c_g \cdot \frac{\partial p}{\partial t} \right) + \frac{2p}{z} \left( \phi c_s \cdot \frac{\partial p}{\partial t} \right) \]

which can be simplified as

\[ k \cdot \frac{\partial^2}{\partial x^2} \left[ m(p) \right] = \phi \mu (c_g + c_s) \left( \frac{2p}{\mu E} \frac{\partial p}{\partial t} \right) \]

Substituting Equation (2-11) into the above equation gives

\[ k \cdot \frac{\partial^2}{\partial x^2} \left[ m(p) \right] = \phi \mu c_s \frac{\partial}{\partial t} \left[ m(p) \right] \]

which can be written as
\[
\frac{\partial^2}{\partial x^2} [m(p)] = \frac{\varphi \mu c_l}{k} \frac{\partial}{\partial t} [m(p)] \]

...(2-17)

where \( c_t \) is the total compressibility \( (c_t = c_g + c_s) \).

Now the diffusivity Equation (2-5) for linear gas flow becomes Equation (2-17).

When pressure difference between the two sides of the core is small, the coefficient in the RHS of the Equation (2-17) can be considered as constant. Based on this assumption, Equation (2-17) can be treated as a linear partial differential equation.

2.3. Method 1: Downstream Pressure Build-up Measurement Method

In our downstream pressure build-up method, a constant pressure is applied at the upstream end of the core plug throughout the entire test and the pressure build-up is observed in the downstream reservoir when the gas flows into it. Figure 2-7 shows an example of the pressure change in the downstream reservoir as a function of time.
Figure 2-7 Changes of the pressures from a downstream pressure build-up method. The upstream pressure \( p_1 \) is constant, and the downstream pressure \( p_2 \) builds up through the time.

### 2.3.1. Formula Derivation

To calculate the permeability from the build-up curve of the measured downstream pressure, the solution of the diffusivity Equation (2-17) needs to be known. According to the solution of this problem from Hsieh et al. (1981) and Dicker and Smits (1988), the exact solution for the pressure in the downstream reservoir is

\[
\frac{m[p_2(t)] - m[p_2(0)]}{m[p_1(0)] - m[p_2(0)]} = \frac{b}{a + b + ab} + 2 \sum_{m=1}^{\infty} \left( \frac{e^{-t \theta_m^2} (ab^2 - b \theta_m^2)}{[\theta_m^4 + \theta_m^2 (a + a^2 + b + b^2) + ab(a + b + ab)] \cos \theta_m} \right)
\]

where \( \theta_m \) can be calculated from the following equation
\[ \tan \theta = \frac{(a + b) \theta}{\theta^2 - ab} \]

where \( a \) is the ratio of the sample pore volume \((V_p)\) over the upstream reservoir volume \((V_1)\), and \( b \) is the ratio of the sample pore volume over the downstream reservoir volume \((V_2)\), \((a = \frac{V_p}{V_1}, b = \frac{V_p}{V_2})\).

In the Equation (2-18) the dimensionless time, \( t_D \), is defined as:

\[ t_D = \frac{kt}{\phi \mu c L^2} \]

By careful observation we find that \( \mu \) can be treated as constant at low pressure range, which is the pressure conditions used in this study. According to Equation (2-9),

\[ m(p) = \int_{\mu \sigma}^{\rho} \frac{2p}{d p} \]

which is, the left-hand side (LHS) of Equation (2-18) can be written as

\[ m[p_1(t)] - m[p_1(0)] = \int_{p,0}^{p,0} \frac{2p}{\mu \sigma} d \sigma = \int_{p,0}^{p,0} \frac{2p}{\mu \sigma} d \sigma = \frac{p_2^2(t) - p_2^2(0)}{p_1^2(t) - p_1^2(0)} \]

Next, we simplify the RHS of Equation (2-18). The upstream pressure \( p_1 \) is invariant throughout the test, which implies that the upstream volume \( V_1 \) leads to infinity, so the ratio of the sample pore volume to the upstream volume \((a = \frac{V_p}{V_1})\) can be considered as zero. Substituting \( a \) as zero and the Equation (2-21) into Equation (2-18), we obtain:

\[ \frac{p_2^2(t) - p_2^2(0)}{p_1^2(0) - p_2^2(0)} = 1 + 2 \sum_{m=1}^{\infty} \frac{e^{-i\phi_n^2} (-b \phi_n^2)}{[\phi_n^4 + \phi_n^2 (b + b^2)] \cos \phi_n} \]
which can be written as

\[
\frac{p_i^2(0) - p_i^2(t)}{p_i^2(0) - p_i^2(0)} = 2 \sum_{m=1}^{\infty} \left( e^{-bt_m^2} \cdot \frac{b\theta_m^2}{[\theta_m^4 + \theta_m^2 (b+b^2)] \cos \theta_m} \right)
\]

For \( a = 0 \), Equation (2-19) becomes

\[
tan \theta = \frac{b}{\theta}
\]

This equation contains an infinite numbers of solution \( \theta_m \) and the values of the solutions increase monotonically. Thus

\[
\cos \theta_m = (-1)^{m-1} \frac{\theta_m}{\sqrt{\theta_m^2 + b^2}}.
\]

Dicker and Smits (1988) mentioned that Equation (2-24) is not fully single exponentially decreasing because the volume of upstream reservoir is much larger than the volume of downstream reservoir. But they indicated that the experiment under this condition is rapid. In addition, a single exponential equation fit very well with the downstream pressure build-up curve which was built later if a right interval was selected. Thus, we simplified Equation (2-24) to:

\[
\frac{p_i^2(0) - p_i^2(t)}{p_i^2(0) - p_i^2(0)} = e^{-t_i\theta_i^2} \left( 2 \frac{b\sqrt{\theta_i^4 + \theta_i^2 b^2}}{\theta_i^4 + \theta_i^2 (b+b^2)} \right)
\]
Let \( \Delta p(t) = p_i^2(0) - p_i^2(t) \), we get

\[
\frac{\Delta p(t)}{\Delta p(0)} = \left( 2 \frac{b\sqrt{\theta_i^4 + \theta_i^2 b^2}}{\theta_i^4 + \theta_i^2 (b + b^2)} \right) e^{-t_0 \theta_i^2}
\]

which can be written as

\[
\Delta p(t) = \left( 2 \Delta p(0) \cdot \frac{b\sqrt{\theta_i^4 + \theta_i^2 b^2}}{\theta_i^4 + \theta_i^2 (b + b^2)} \right) e^{-t_0 \theta_i^2}
\]

Taking the natural log of Equation (2-25) yields

\[
\ln[\Delta p(t)] = \ln \left( 2 \Delta p(0) \cdot \frac{b\sqrt{\theta_i^4 + \theta_i^2 b^2}}{\theta_i^4 + \theta_i^2 (b + b^2)} \right) + (-t_0 \theta_i^2) \]

Substituting \( t_0 \) from Equation (2-20) into Equation (2-26), we get

\[
\ln[\Delta p(t)] = \ln \left( 2 \Delta p(0) \cdot \frac{b\sqrt{\theta_i^4 + \theta_i^2 b^2}}{\theta_i^4 + \theta_i^2 (b + b^2)} \right) - \frac{\theta_i^2 k}{\varphi \mu c \ell} t \]

Figure 2-8 \( \ln(\Delta p) \) vs. time plot for core sample 1
Assigning $s = \frac{-\theta_i^2 k}{\varphi \mu c L^2}$, which is the slope of the pressure difference in a logarithm as a function of time based on Equation (2-27) (Figure 2-7); permeability can be easily obtained from Equation (2-28) when $s$ is fitted in Figure 2-8.

$$k = \frac{-\varphi \mu c L^2}{\theta_i^2} \cdot s \quad \text{ .......................................................... (2-28)}$$

Using the Taylor series of $\tan \theta$, $\tan \theta \approx \theta + \frac{\theta^3}{3}$, we can calculated $\theta_1$ from Equation (2-23): $\theta_1 + \frac{\theta_1^3}{3} = \frac{b}{\theta_i}$, and

$$\theta_1^2 = \frac{3}{2} (-1 + \sqrt{1 + \frac{4}{3} b}) \quad \text{ .......................................................... (2-29)}$$

Substituting Equation (2-29) into Equation (2-28), we obtained:

$$k = \frac{2 \varphi \mu c L^2 s}{3 - 3 \sqrt{1 + \frac{4}{3} b}} \quad \text{ .......................................................... (2-30)}$$

Considering that $b = \frac{V_p}{V_2} = \frac{\varphi A L}{V_2}$, the equation becomes

$$k = \frac{2 \varphi \mu c L^2 s}{3 - 3 \sqrt{1 + \frac{4 \varphi A L}{3 V_2}}} \quad \text{ .......................................................... (2-31)}$$

2.3.2. Measurement Procedure

The determination of the permeability is a three-step process, namely installing the core plug into AutoLab-1500, running the test, and analyzing the resultant data.
1) Installing the core plug into AutoLab-1500

First, the core holder is placed into the vessel, then the vessel is filled with mineral oil and the confining pressure is increased to the desired level \((p_c)\). The valve between the core plug and the upstream reservoir is closed. Dry nitrogen is used to fill the upstream reservoir and the upstream pressure is increased to the desired level \((p_I)\). The downstream pressure is atmospheric pressure. Notice that the confining pressure must be greater than the upstream pressure.

2) Running the test

The starting time is recorded when the valve between the core plug and the upstream reservoir is opened. During the whole test, the upstream and the confining pressures are constant. The pressures are monitored and recorded at both the upstream and downstream ends of the sample. The test is end when the downstream pressure is equal to the upstream pressure, which is at the point ‘A’ in the Figure 2-7.

3) Analyzing the resultant data

First the pressure difference is calculated in a logarithm scale from equation:

\[
\ln[\Delta p(t)] = \ln[p_1^2(0) - p_2^2(t)].
\]

Then form the plot by function fitting (Figure 2-8), we get the slope, \(s\). Finally, using Equation (2-31), we can obtain the permeability of the rock.

2.4. Method 2: Radius-of-Investigation Measurement Method

Based on the radius-of-investigation concept (Lee, 1982), we proposed a new
laboratory core permeability measurement method.

When doing the permeability test using the downstream pressure build-up method, we observed that the downstream pressure did not increase immediately when the upstream reservoir connected with the core plug. The lower the permeability is, the longer delay time is observed.

Based on this phenomenon, a correlation can be found between the permeability and the delaying time. Through this correlation, the permeability can be measured in a much shorter time (Figure 2-8) when compared with the previous method.

In our research, we discovered that the radius-of-investigation concept (Lee, 1982) could be useful for uncovering the relationship between the permeability of rock and the waiting time before the downstream pressure increases. Radius of investigation is the distance that a pressure disturbance moves into a formation when it is caused from the well. Lee pointed out that it is possible to calculate the maximum distance that a pressure disturbance can reach at any time, if we know the properties of rock and fluid, such as the rock permeability and porosity, fluid viscosity, and the compression of both rock and fluid. This means that the maximum distance of pressure disturbance is a function of permeability and time, when other parameters are constants.

Thus, the time that a pressure disturbance spends in a rock is a function of the permeability of the rock, if we know the length of the rock. Our hypothesis is that we can calculate the low permeability in laboratory by measuring the delaying time, which is
the time that the pressure disturbance propagates from the upstream end of the core plug to the downstream end (in case pressure disturbance is generated in upstream), or the pressure disturbance propagates from the downstream end of the core plug to the upstream end (in case pressure disturbance is generated in downstream).

2.4.1. Formula Derivation

The pressure disturbance concept is applied here to estimate the propagation of pressure in the core plug. First, we introduced a pressure disturbance by either increasing the upstream pressure or decreasing the downstream pressure instantaneously, and then we attempted to find the time, \( t_m \), at which the disturbance at location \( x \) will reach its maximum.

According to the solution to the diffusivity Equation (2-17), for an instantaneous pressure disturbance in an infinite linear system (Carslaw, 1959), we have

\[
m( p) = \frac{Q}{\sqrt{t}} \exp \left( \frac{-x^2}{4k} \right) \exp \left( \frac{-Q\mu c_t}{t} \right)
\]

where \( Q \) is a constant, which is related to the strength of the instantaneous pressure disturbance.

It is a physics problem of extreme value to find the time at which the pressure disturbance reaches its maximum. The maximum solution can be solved when the time derivative of the Equation (2-32) equals to zero:
\[
\frac{d[m(p)]}{dt} = \left[ \frac{Q}{\sqrt{t}} \exp \left( -\frac{x^2}{4 \frac{k}{\phi \mu_i} t} \right) \right] = 0
\]

which is

\[
\frac{d[m(p)]}{dt} = \exp \left( -\frac{x^2}{4 \frac{k}{\phi \mu_i} t} \right) \frac{d}{dt} \left( \frac{Q}{\sqrt{t}} \right) + \frac{Q}{\sqrt{t}} \frac{d}{dt} \left[ \exp \left( -\frac{x^2}{4 \frac{k}{\phi \mu_i} t} \right) \right] = 0
\]

Simplifying the above equations lead to

\[
\frac{d[m(p)]}{dt} = \left[ -\frac{Q}{2t^{\frac{3}{2}}} \exp \left( -\frac{x^2}{4 \frac{k}{\phi \mu_i} t} \right) + \frac{x^2 Q}{4 \frac{k}{\phi \mu_i} t^{\frac{5}{2}}} \right] \cdot \exp \left( -\frac{x^2}{4 \frac{k}{\phi \mu_i} t} \right) = 0
\]

Finally we got Equation (2-33) as

\[
\frac{d[m(p)]}{dt} = \frac{Q}{2t^{\frac{3}{2}}} \cdot \exp \left( -\frac{x^2}{4 \frac{k}{\phi \mu_i} t} \right) \left[ -1 + \frac{x^2}{2 \frac{k}{\phi \mu_i} t} \right] = 0 \quad \text{..................................................(2-33)}
\]

Considering the initial condition at \( t=0 \) and \( p(x,t=0) = p_i, \ t=0 \) is a trivial solution to

Equation (2-33). Dividing both sides of the Equation (2-32) by \( \frac{Q}{2t^{\frac{3}{2}}} \cdot \exp \left( -\frac{x^2}{4 \frac{k}{\phi \mu_i} t} \right) \)

yields

\[-1 + \frac{x^2}{2 \frac{k}{\phi \mu_i} t} = 0.\]
Rearranging the equation, we get the time

\[ t_m = t = \frac{\phi \mu c x^2}{2k} \] .................................................................(2-34)

Expressing permeability in terms of porosity, viscosity, total compressibility, location, and time, Equation (2-34) can be written as

\[ k = \frac{\phi \mu c x^2}{2t_m} \] .................................................................(2-35)

Converting Equation (2-35) into the U.S. field units we have

\[ k = \frac{1896\phi \mu c x^2}{t_m} \] .................................................................(2-36)

where permeability \( k \) is in mD, porosity \( \phi \) is dimensionless (in fraction), viscosity \( \mu \) is in cp, total compressibility \( c_t \) is in psi\(^{-1}\), time \( t_m \) is in hour, and location (or distance) \( x \) is in ft.

Equations (2-35) and (2-36) are the governing equations to measure the rock permeability. They are used to calculate the permeability of any rock that meets the aforementioned assumptions and can be used for high-permeability rocks as well. The proposed method evaluates the permeability under unsteady-state flow and requires short time period to determine the flow capacity of the low-permeability rock.

2.4.2. Measurement Procedure

The procedure for this method is as the same as that of the downstream pressure build-up method, which are installing the core plug into AutoLab-1500, running the test,
and analyzing the resultant data.

The difference between these two methods is that the new method is much faster than the downstream pressure build-up method. Theoretically when the disturbance reached the end of the core, the test is ended.

Figure 2-9 End times of downstream pressure build-up method and radius of investigation method
Point ‘A’ marks the time at which the downstream pressure build-up method stops
Point ‘B’ marks the time at which the radius of investigation method stops

Figure 2-9 shows that the total experiment time for the downstream pressure build-up method is about 8000 seconds finishing at point ‘A’ and the new method only requires no more than 800 second, \( t_m \), finishing at point ‘B’. It is ten times faster than the build-up method. The time at point ‘B’ is \( t_m \), when the pressure disturbance reaches downstream
end of the core plug. This means that the pressure disturbance sensed by pressure gauge is not caused by arbitrary disturbance but real pressure disturbance from upstream.
CHAPTER III

RESULTS & DISCUSSION

Autolab-1500 system provides an oscillating pulse method to measure the low permeability. The installing processes for the oscillating pulse method are the same as the processes for the previous two methods. In our study, we measured the low permeability using these three methods, and compared the results that were calculated by our methods with the results that are provided by Autolab-1500. Figure 3-1 shows the changes of pore pressures during the three tests. The oscillating method started at point ‘A’, when the initial pressure equilibration is reached, and ended at the point ‘C’.

![Figure 3-1 Changes of the pressure during one experiment](image)

Point ‘A’ marks the time that the downstream pressure build-up method stops at Point ‘B’ marks the time that the radius of the investigation method stops at Point ‘C’ marks the time that the oscillating pulse method stops at...
We measured the permeabilities of the six core plugs using the oscillating pulse method, downstream pressure build-up method, and the radius-of-investigation method. Figures (2-8, 3-2, 3-3, 3-4, 3-5, and 3-6) were used to obtain the slopes for the downstream pressure build-up method.

The parameters that were used in the tests and experiment results are shown in Tables (3-1) through (3-3). The permeabilities from the downstream pressure build up method and radius of investigation method are close to those from the oscillating pulse method which have been validated by New England Research Inc (see Figure 3-7). Therefore the downstream pressure build up method and the radius of investigation

![Figure 3-2 ln(Δp) vs. time plot for core sample 2](image)
Figure 3-3 $\ln(\Delta p)$ vs. time plot for core sample 3

Figure 3-4 $\ln(\Delta p)$ vs. time plot for core sample 4
Figure 3-5 \( \ln(\Delta p) \) vs. time plot for core sample 5

Figure 3-6 \( \ln(\Delta p) \) vs. time plot for core sample 6
method provide reliable ways to estimate permeability of tight rocks.

Traditionally, the oscillating pulse method, which is provided by the Autolab-1500 system, is the fastest way. Figure 3-1 shows that although the measured time of the oscillating pulse method is only from point ‘A’ to ‘C’, this method still requires the system to reach the equilibrium state, which can be quite time consuming.

Table 3-1 Permeability measured by oscillating pulse method

<table>
<thead>
<tr>
<th>Parameter</th>
<th>unit</th>
<th>Core 1</th>
<th>Core 2</th>
<th>Core 3</th>
<th>Core 4</th>
<th>Core 5</th>
<th>Core 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>µd</td>
<td>0.108</td>
<td>0.046</td>
<td>0.0724</td>
<td>0.0438</td>
<td>0.11</td>
<td>2.25</td>
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Table 3-2 Permeability measured by downstream pressure build-up method

<table>
<thead>
<tr>
<th>unit</th>
<th>Core 1</th>
<th>Core 2</th>
<th>Core 3</th>
<th>Core 4</th>
<th>Core 5</th>
<th>Core 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
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<td>2.7224</td>
<td>2.7008</td>
<td>2.3882</td>
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<tr>
<td>D</td>
<td>in</td>
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<td>1.0394</td>
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</tr>
<tr>
<td>φ</td>
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<td>0.000009</td>
<td>0.000009</td>
<td>0.000009</td>
<td>0.000009</td>
</tr>
<tr>
<td>c_g</td>
<td>1/psi</td>
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<td>0.000125</td>
<td>0.000125</td>
<td>0.000125</td>
<td>0.000125</td>
</tr>
<tr>
<td>c_t</td>
<td>1/psi</td>
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<tr>
<td>µ</td>
<td>cp</td>
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<td>0.0293</td>
<td>0.0293</td>
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<td>0.0293</td>
</tr>
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<td>V_2</td>
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<td>2.22E-05</td>
<td>2.22E-05</td>
</tr>
<tr>
<td>s</td>
<td>Ln(µd²)/h</td>
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<td>-1.3644</td>
<td>-1.818</td>
<td>-1.2528</td>
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<tr>
<td>k</td>
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<td>0.09</td>
<td>0.1164</td>
<td>0.0791</td>
<td>0.158</td>
</tr>
</tbody>
</table>
Moreover, the oscillating pulse method is inconvenient. Not only does the range of the frequency but also the shape of the sine wave need to be chosen carefully, in order to match the range of the permeability. Choosing the wrong frequency will lead to the failure in the experiment. Thus, the oscillating pulse method from the Autolab-1500 may not be the optimum option for measuring the low permeability.

The pressure build-up method, which is based on the pulse decay method, is the transformation of a mature technique to measure the low permeability. Our study

<table>
<thead>
<tr>
<th>Table 3-3 Permeability measured by radius-of-investigation method</th>
</tr>
</thead>
<tbody>
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<td>unit</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>$L$ in</td>
</tr>
<tr>
<td>$\varphi$ fraction</td>
</tr>
<tr>
<td>$c_s$ 1/psi</td>
</tr>
<tr>
<td>$c_g$ 1/psi</td>
</tr>
<tr>
<td>$c_t$ 1/psi</td>
</tr>
<tr>
<td>$\mu$ cp</td>
</tr>
<tr>
<td>$t$ h</td>
</tr>
<tr>
<td>$k$ $\mu$d</td>
</tr>
</tbody>
</table>
managed to reduce the downstream volume as much as possible in order to reduce the operation time. It requires less time than the oscillating pulse method and is a faster alternative of the stable and commonly accepted method. It is worth mentioning that during this procedure, to reduce the uncertainty, the data that are chosen to calculate the slope, s, should be part of the data between point ‘B’ and ‘A’ in Figure 2-9 and Figure 3-1. Doing so ensures the consistency of the calculation of slope, s, and reduces the uncertainty in the obtained permeability.

The radius-of-investigation method requires the least amount of the time to perform and results in a reliable answer. It utilizes the propagation speed of the pressure wave in a certain media to calculate the permeability. This method made a fast measurement of the low permeability become a reality in the laboratory. The measure time for this
method is 10 times less than the pressure build-up method in our study. Not only the radius-of-investigation method can be used to measure the low permeability, it can also be used to measure the high permeability by replacing the gas fluid with the liquid fluid. Other than the human introduced random error, the major uncertainty source in this method is mainly from the selection of point ‘B’. In this method, the beginning of the responding time is manually selected. However, selection of point ‘B’ can be done by comparing the slopes between the nearby measurements and selecting the largest changing rate of these slopes. In order to automate the analyzing procedures, this job needs to be done as part of the future works.

In radius-of-investigation method, we used gas fluid instead of the liquid fluid to measure low permeability rock. It should be noted that liquid will be used for high permeability rocks. Replacing gas with liquid, we can still derive the same governing equations as Equations (2-35) and (2-36) with liquid properties replacing gas properties. Therefore, Equations (2-35) and (2-36) are capable of estimating permeability of any rock that meets the aforementioned assumptions. They evaluate the permeability under unsteady-state flow and require shorter time comparing with other methods.
CHAPTER IV
CONCLUSIONS

In this study, we developed two methods to measure the low permeability in the tight rock, namely the pressure build-up method and the radius-of-investigation method. The derivation processes were presented and the results from the two measurements were shown and compared. Our results show that both methods have the capability of measuring the low permeability and one of them can obtain the measurement in a very short amount of the time. The key conclusions of our study are listed below:

1). The pressure build-up method was developed based on the pulse decay method, which is the most commonly used method to measure the low permeability.

2). The radius-of-investigation method was developed using the delayed responding time from the beginning time that the pressure disturbance entered the sample to the time that the pressure disturbance propagates to the end of the sample.

3). Both methods provide reliable measurements of the permeability in our study.

4). The radius-of-investigation method can make the measurements within a very short period of the time, which is 10 times less than that of the commonly used pulse
decay method in our experiment.
REFERENCES


NOMENCLATURE

\[ A : \] area of the cross section of the core plug

\[ c_S : \] formation compressibility

\[ c_g : \] gas isothermal compressibility

\[ c_t : \] total compressibility

\[ D : \] diameter of core

\[ k : \] permeability

\[ L : \] length of core

\[ M : \] molecular weight

\[ m(p) : \] gas pseudopressure

\[ Q : \] the strength of the instantaneous pressure disturbance

\[ p : \] pressure

\[ p_b : \] base pressure

\[ P_2 : \] downstream pressure

\[ p_1 : \] upstream pressure

\[ \Delta p : \] pressure difference

\[ q_g : \] gas rate

\[ R : \] universal gas constant
\( s \) : the slope of the pressure difference in a logarithm as a function of time

\( T \) : temperature

\( t \) : time

\( t_m \) : time at which the pressure disturbance is a maximum at \( x \)

\( \Delta t \) : time period

\( V_1 \) : volume of the upstream reservoir

\( V_2 \) : volume of the downstream reservoir

\( V_p \) : pore volume of the core

\( v_x \) : gas velocity in \( x \) direction

\( x \) : distance from original point in \( x \) direction

\( \Delta x \) : incremental distance in \( x \) direction

\( z \) : gas \( z \)-factor

\( \varphi \) : porosity

\( \rho_g \) : gas density

\( \mu \) : viscosity

\( \mu_g \) : gas viscosity