9-6-2023

Find the Distance to the Moon

Dean Smith
University of North Dakota, dean.smith@und.edu

Find the Distance to the Moon

You often look up numbers on the Internet, but do you know how that knowledge was gathered? This assignment will lead you through a series of observations and experiments to answer the question: "How do we know the distance to the Moon?" This has been measured multiple ways throughout history. A direct measurement can be done through measuring lunar parallax, but this can be a difficult measurement. The method here estimates the distance using our knowledge of how gravity works and observations of the Moon's orbit. Along the way, you will also find the size and mass of the Earth. The main tool you need is Kepler's 3rd law of planetary motion, published way back in 1611 or planets around the Sun but thanks to Isaac Newton, we can apply it to other orbits.

Kepler's 3rd Law is $T^2 = 4\pi^2/(GM)\ r^3$ where

- r, the distance to the Moon
- T, the period of the Moon's orbit
- M, the mass of the Earth
- G, the gravitational constant = $6.67\times10^{-11}\ m^3/(kg\ s^2)$

Solving for r gives: $r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$

To find the distance to the Moon, we need both the time it takes to orbit the Earth, T, and the mass of the Earth.

1. Finding the period of the Moon

The orbital period of the Moon has been known for a long time simply by observing the time it takes for the Moon to travel from a certain constellation back to that same constellation. I'll save you some time and let you make virtual observations using Stellarium or another planetarium program. Find a date and time when you can see the Moon in the south or southeast and move the time forward one day at a time until you see the Moon back in the same place relative to the background stars. It won't be exact but getting the time to the nearest day is good enough here. The Moon won't be the same phase after one orbit. For Kepler's Law, you will need to convert the time to seconds.

T = ________________ days

T = ________________ seconds

2. Finding the Mass of the Earth

Now for the hard part. You are going to estimate the mass of the Earth from measurements. To do this, you can use the acceleration due to gravity: $g = G\ M/R^2$. In this equation, big G is the same gravitational constant as in Kepler's 3rd Law, M is the mass of the Earth, and R is the radius of the Earth. Solving for M, we get:

$M = g\ R^2/G$

To find the mass of the Earth, you need to find little g, the acceleration due to gravity and R, the radius of the Earth.

3. Finding the acceleration due to gravity

By measuring the period of a pendulum, you can find the acceleration due to gravity.

The relationship for a simple pendulum is: $T = 2\pi\sqrt{\frac{L}{g}}$

Then $g = \frac{4\pi^2 L}{T^2}$ where L is the length of the pendulum and T is the period, or the time for one cycle.

Measure the time it takes for a pendulum to make 5 oscillations. This is 5 times the period, or 5T. L is the length of the pendulum.

5T = ________________ seconds

T = ________________ seconds

L = ________________ meters

Enter your data into the equation for g and find the acceleration due to gravity.

g = ________________ m/s^2

4. Finding the radius of the Earth

You need one more thing to find the mass of the Earth. You need to find the radius of the Earth. You might know that Eratosthenes did this in 200 BC by finding the difference in angular height of the Sun at two locations on Earth with similar longitudes. That same method can be used for any object, not just the Sun. For this project, you will instead use Polaris, which is visible any clear night in the northern hemisphere. Its angular height (known as its altitude) is the nearly the same as your latitude on Earth and does not change with the seasons. You will make this measurement in a simulated environment using Stellarium or other planetarium software.

Set the location in the software to Grand Forks, ND and change the time to any time at night so you can see the stars. Pause time in the software and click on Polaris. Look for the Az/Alt numbers in the upper left corner. The altitude is the second number. It is given as degrees, arcminutes (‘), arcseconds (“). You can ignore the arcseconds. Enter the degrees and arcminutes.

Polaris altitude (Grand Forks) = ________________

You want to convert this to just degrees. Since an arcminute is 1/60°, the formula is degrees + arcminutes/60. For example, 45° 15’ = 45 + 15/60 = 45.25°

Polaris altitude (Grand Forks) = ________________ degrees

Now change the location to Dallas, TX and find the altitude of Polaris from there.

Polaris altitude (Dallas) = _________________

Polaris altitude (Dallas) = _________________ degrees
What is the difference in degrees between the altitude of Polaris at these two locations?

Angular difference (θ) = _________________ degrees

You need to know the north-south distance between these two cities. I'm not going to make you drive that distance. Instead, you can use Google Earth. Go to earth.google.com and choose the ruler icon on the left side . Click Grand Forks, ND and draw a line down to Dallas, TX. In the box that shows the distance, click the arrow and choose meters from the dropdown box.

Distance (d) = _________________ meters

Now, we can set up a ratio to find the circumference of the Earth. There are 360° in a circle so

$C/d = 360°/\theta$

Enter your numbers and solve for C, the circumference of the Earth.

C = _________________ meters

At some point, you should have learned the relationship between the circumference of a circle and its radius: $C = 2\pi R$. Use this to find the radius of the Earth.

R = _________________ meters

Do you remember what all that was for? You now can find the mass of the Earth. Use the equation from way back in part 2 to find the mass in kilograms.

5. M = _________________ kg

Find the distance to the Moon!

Finally, enter your value for Earth's mass and the period of the Moon into Kepler's 3rd Law to find the distance to the Moon!

6. r = __________________________________ km

Instructor Notes

Depending on the mathematical level of your students, you may want to leave out the solved equations and have them do the algebra. You may also want to leave the equations out entirely and ask the student to provide the appropriate equation.

In part 3, the acceleration due to gravity can also be found through a more direct freefall experiment and use $h = \frac{1}{2} g t^2$ to find the 'g'. Dropping something from 2 meters takes less than a second to fall so a larger height is needed or better time measurements than a stopwatch. One method is to take a video and use computer software to find the time it takes to fall.

For part 4, you could find the radius of the Earth using a method like that of Eratosthenes. By measuring the length of the shadow of a meter stick at local noon, you can then find the altitude of the Sun using $\theta = \text{atan}(1/s)$ where 's' is the length of the shadow in meters. This may be impractical because you need to measure this in two locations. If you can make the measurement near an equinox, then you can use the equator as the second location where the Sun's altitude is 90°.