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Math Active Learning Lab: Math 93 Notebook

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MALL

MATH ACTIVE LEARNING LAB

Math 93 Notebook

University of North Dakota

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Welcome to the MALL

Welcome to UND's Math Active Learning Lab (MALL)! As part of a nationwide movement, the UND Mathematics Department has redesigned our curriculum and pedagogy to reflect the current research on learning math. The MALL is based on the emporium model. The premise of this model is that the best way to learn math is by doing math, not by watching someone else do math. This means that most of your time in this course will be spent doing math, and your instructor will spend little time lecturing. Instructors and tutors are available in the MALL to support your learning during the required lab time. The philosophy of the MALL is well described by H. A. Simon's quote

> "Learning results from what the student does and thinks and ONLY from what the student does and thinks. The teacher can advance learning only by influencing what the student does to learn."

For many of you, this is your first college math course. Quite possibly, this course and our expectations may be quite different from your high school mathematics experiences. We cannot stress too strongly your role in ensuring your success in this class. More than anything else, your choices will determine your success in this course. Attending class regularly, diligently working in ALEKS, studying for exams, and seeking help when you need it will lead to success. Our approach includes cooperative learning. In class your instructor will facilitate group activities and discussion rather than repeating to you content of the text. We will be asking you to use the ALEKS resources and to work in your notebooks before coming to class. There will also be times when you will be expected to learn topics that will not be formally discussed in the classroom.

Instead of sitting in a lecture class for hours each week AND then being expected to do practice problems outside of class, part of your "class time" is spent doing homework in ALEKS. This provides instant feedback and links you to resources as needed. Using ALEKS allows us to individualize the student learning path. Students can move quickly through topics they are familiar with and take the time they need to learn more challenging topics. To help you get the most out of ALEKS, we have created this notebook. If ALEKS and the notebook are still leaving you confused about a topic, we expect you to ask an instructor or tutor for help.

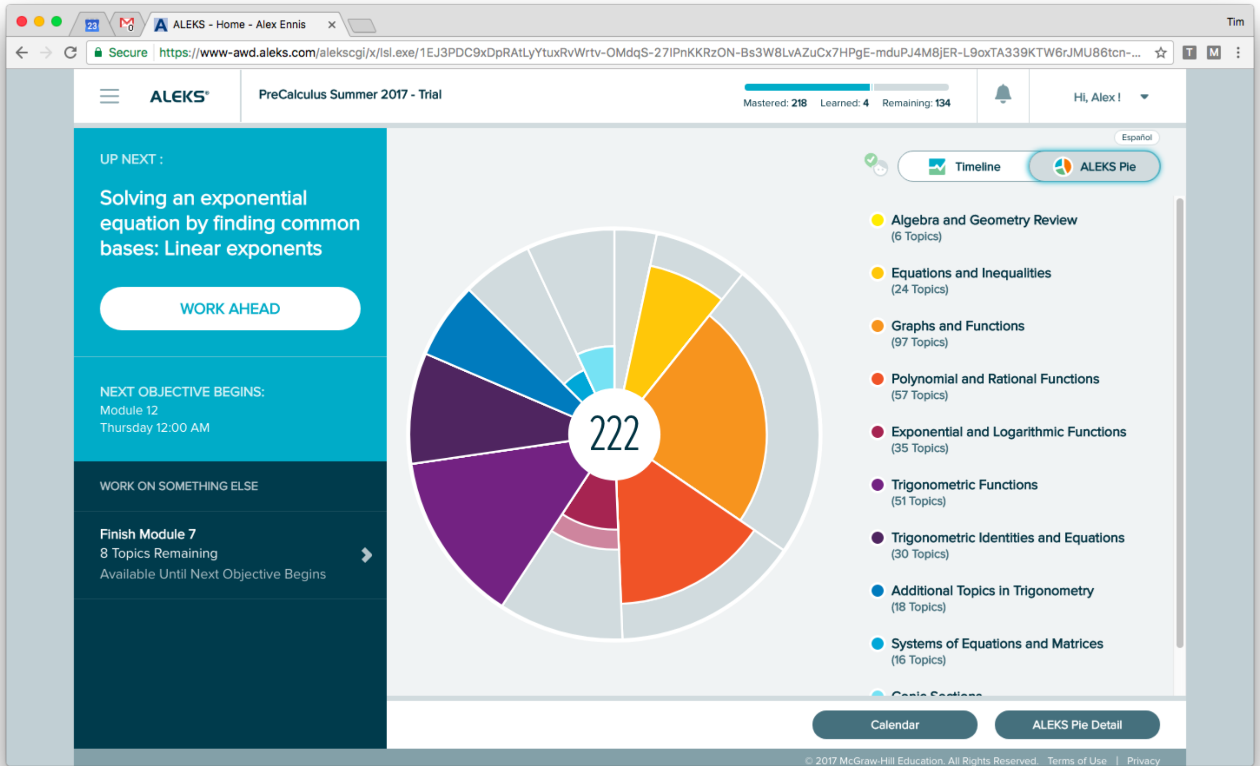
We are excited about this approach to teaching and learning mathematics, and we look forward to learning along with you this semester.

MALL staff

Contents

How to use ALEKS

After you complete the Initial Knowledge Check, each time you login to ALEKS, you will see your home screen, which looks like



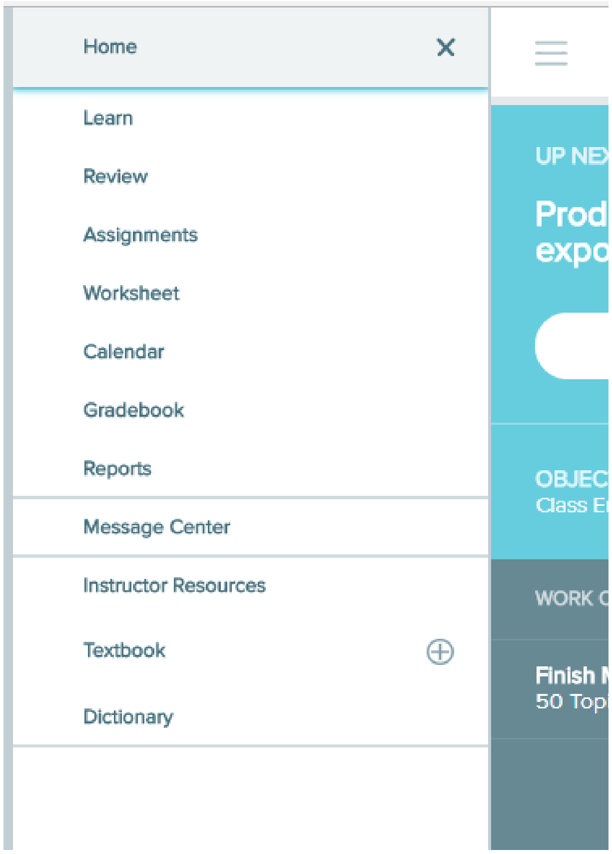
The big pie in the middle is your ALEKS pie. Your goal for the course is to fill your pie. Each slice of the pie is a "general topic objective", and is made of many sub-topics. Gray areas of the pie are topics that you've not yet learned, lightly colored areas are topics that you've learned but not mastered, and darkly colored areas are topics you've mastered.
Topics are mastered through **Knowledge Checks**. After learning 20 topics (or spending 5 hours in ALEKS), ALEKS will give you a Knowledge Check. This will focus on your learned topics, but will also ask about previously mastered topics and possibly future topics. Topics you demonstrate an understanding of become mastered and are added to your pie. Topics you don't understand are not added to your pie and may be removed from it if you miss questions on a topic you previously mastered.

Getting Help

ALEKS Technical Support is available at `https://www.aleks.com/support/contact_support` or by phone at (714) 619-7090. They won't help you learn a topic, but will help you if you have trouble accessing your account.

Navigating ALEKS

The blue menu on the left shows your current options for working in ALEKS. Links to additional information can be found under the *hamburger menu* at the top left of your screen; it looks like ☰
The entries in the menu are:



Home Takes you back to the home screen.

Learn Opens the next topic ALEKS has ready for you to learn. You can also filter the topics to focus on others.

Review Opens up topics you have learned or mastered for you to review. But since you've already learned or mastered these topics, they can't help fill your pie as well as learning new topics.

Assignments & Worksheet Shows links to the occasional item posted by your instructor.

Calendar Opens a calendar view of deadlines for weekly objectives, knowledge checks and tests.

Gradebook Shows your grades for ALEKS assignments and exams. The complete and official gradebook is in Blackboard.

Reports Opens a menu of reports that provide additional information about your progress in ALEKS. We encourage you to take a look at these pages.

Message Center You can send an email to your instructor or others in your class.

Textbook This link takes you to the E-Book.

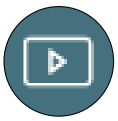
Dictionary This link takes you to a dictionary that is organized by pie slice categories.

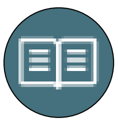
How to use this Notebook

This course Notebook, has been designed to help you get the most out of the ALEKS resources and your time.

- Topics in the Notebook are organized by weekly learning module.
- Space for notes from ALEKS learning pages, e-book and videos directs you to essential concepts.
- Examples and "You Try It" problems have been carefully chosen to help you focus on these essential concepts.
- Completed Notebook is an invaluable tool when studying for exams.

When you ask a tutor for assistance, the first thing she/he will ask is to see your Notebook. This is necessary for the tutor to determine how best to respond to your questions. The following icons will appear in the Notebook and on the ALEKS learning pages:


the play icon will show a video about the topic.


the book icon will go to the appropriate section of the e-book.

the dictionary icon will look up terms in the course dictionary.

Testing in ALEKS

To prepare for a test in ALEKS, in your Blackboard course, select "Syllabus & Textbooks" and download and install the "Respondus LockDown Browser".
To take a test, start the "LockDown Browser" application, connecting to the "UND Blackboard Learn" server. Log in to your Blackboard course, navigate to ALEKS, and a tutor will enter the password to start your exam.
An ALEKS test is another Knowledge Check, although it may have a few more questions. As with regular Knowledge Checks, these will ask about topics you've previously mastered (even from the beginning of the course) and possibly future topics. Topics where you show mastery will be added to your pie. Topics where you show that you have not learned the material will be subtracted from your pie.
ALEKS uses your responses to determine how many topics in your pie are mastered. Each test has a target number of topics. If you meet or exceed that number, your grade on the test is 100%. If you fall short, your grade is the percentage of topics that you've mastered out of the target. This means that it's possible ALEKS will say that you have lost a few topics from your pie, but that you're still ahead of the target and therefore earn 100%. On the other hand, it's also possible that you add several topics to your pie, but because you're still below the target, you don't earn as much for a grade.
The target number of topics is the number of topics in the modules on the exam (including the prerequisite topics). You can find the number of topics in each module by looking at ALEKS' syllabus for your course. This means that if you know all of the topics for the modules you've done so far, you'll earn 100% on the exam. It's also possible, however, to master topics from later modules that will take the place of topics from past modules.

Math ________ — ______________________
Fall 2018

Instructor: __________________
Email: __________________
Phone: __________________
Office: _____________
Focus Group: ______________________

Course prerequisites and content: [See your syllabus on Blackboard]

The Math Active Learning Lab (MALL): The MALL is based on the emporium model, which is based on the premise that the best way to learn math is by doing math, not watching someone else do math. This means that most of your time in this course will be spent doing math, and your instructor will spend little time lecturing. Instructors and tutors are available in the MALL to support your learning during the required MALL time.

All email correspondence will go to your official UND email address.

Outside of each scheduled class meeting (focus group) from _________ to _________ , you must spend at least _____ hours working in the MALL (O'Kelly 33).

- Credit for MALL time is based only on UND ID card swipes.
- Swipe your ID when entering **and** exiting the MALL.
- Swiping another student's ID is academic dishonesty.
- Minutes ____________________________ from one week to another.
- Class time ________________________ toward your MALL time.

MALL Expectations:

- The MALL is a math classroom. Please be considerate of others by keeping conversations focused on math and at a reasonable volume while in the MALL.
- Food, companions, and using your phone are NOT allowed in the MALL.
- Activities such as socializing, surfing the Internet, ____________________, doing work for another course, sleeping, etc. are not allowed in the MALL. If these activities are observed, you will be asked to leave the MALL.
- The use of a MALL computer is on a first-come first-serve basis; no reservation can be made.
- Please do not hesitate to ask questions in the MALL. Staff members in the MALL __________________________________.

ALEKS Access & Notebook: An ALEKS access code can be purchased from `https://www.aleks.com/` or the UND Bookstore. The course Notebook is only available at the Bookstore. You will be expected to bring the Notebook to your Focus Group meetings and the MALL. Graded Notebook checks will occur weekly.

Tests: There will be _____ along with the final exam. Notes, the book, calculators, and other electronic devices will not be allowed on any of the exams. Each test will have two parts.

- Paper-pencil portion will be given during the Focus Group meeting.
- Scheduled Knowledge Checks in ALEKS must be completed in the MALL testing area the __________________ the paper-pencil test.

Exam Dates: ________________________
____
____
____

Test Rules:

- Scheduled Knowledge Checks (tests and final exam) in ALEKS will be taken in the MALL.
- Do not wait until the last minute to take your ALEKS exams. You will not be allowed to start a test if the MALL is scheduled to close before the end of your full allotted time.
- Bring your ID and pencils with you. The MALL Testing Proctor will check your ID, give you scratch paper, and direct you to your seat. Once you have started the Lockdown Browser the proctor will input the test password. When you are finished, bring all your papers to the Testing Proctor
- Absolutely NO ____________________________________ (this includes cell phones) may be active in the testing area. Use of any electronic device during a test will be treated as academic dishonesty.
- Cellphones and other smart devices must be turned completely off and and placed on the testing table.
- You may not share any test information with anyone who hasn't taken the test. Violators will be charged with academic dishonesty.
- You may not leave your table during a test without permission. This includes getting water and using the restroom. Cell phones must be left with your belongings in the testing area.

Grading: Your course grade will be a weighted average of the following:

Tests	____%
Final Exam	____%
MALL Time & Focus Group Activities*	15%
Module Completion	15%

*Your lowest Focus Group score will be dropped. This will take into account any unexcused absences.

Try Score: Your Try Score reflects your effort in this course. The Try Score is composed of:

- focus group participation,
- notebook completion,
- attempting every exam and retaking when your first attempt is less than 80%,
- spending at least _____ hours per week working in the MALL, and
- completing the module or spending sufficient time working in ALEKS.

This is **not** included in your course grade, but will be shared with your academic advisor.

Working in ALEKS at home: You can work in ALEKS anywhere you have internet access. This does NOT count toward your ______________________. Work well ahead of deadlines to be safe. Deadlines will NOT be extended because of home computer/internet issues.

Attendance & Participation:

- Students who do not attend the first class meeting, or contact the instructor the first week, will be dropped from the course.
- Students who do not complete their Initial Knowledge Check within two full days of their first class meeting will be dropped from the course.
- Assignments given during the Focus Group meetings will be completed in small groups and will require your full attention.
  - Regular and on-time attendance ____________________. Repeated absences or late arrivals will significantly impact your Focus Group grade.
  - Unless required for the Focus Group activity, cell-phone or computer use will result in a zero for the day.
  - Absences will usually be excused if due to serious emergency. An emergency serious enough to cause an absence from a Focus Group activity or test is also serious enough to ____________ documentation.
  - Students with valid excuse approved prior to or within ________________ of a test will be able to make up **one** test on reading and review day.
  - Students anticipating absences due to athletic commitments (or any other type of university sanctioned commitment) must document their need to be absent from class the ______________________________ prior to the absence.

Absences will be dealt with on a case-by-case basis; however, two situations occur commonly enough to merit mention here. Travel plans __________________________________ cause for an excused absence. *In particular, having bought a plane ticket is not sufficient reason to reschedule a student's final exam.* Also, an activity related to social functions (including those that involve a students' residence hall, apartment complex, sorority or fraternity) is never sufficient for an excused absence.

Disability Accommodations: Contact me to request disability accommodations, discuss medical information, or plan for an emergency evacuation. To get confidential guidance and support for disability accommodation requests, students are expected to register with DSS at `http://und.edu/disability-services/`, 190 McCannel Hall, or 701.777.3425.

Academic Honesty: All students in attendance at the University of North Dakota are expected to be honorable and to observe standards of conduct appropriate to a community of scholars. Academic misconduct includes all acts of dishonesty in any academically related matter and any knowing or intentional help or attempt to help, or conspiracy to help, another student. The UND Academic Dishonesty Policy will be followed in the event of academic dishonesty.

Module 1

Product rule with positive exponents: Univariate

Watch *Video 1: Multiplying Monomials* to complete the following.

Multiply the monomials.

1.

2.

3.

Multiplying binomials with leading coefficients greater than 1

Watch *Exercise: Multiplying Binomials* to complete the following.

Multiply the polynomials by using the distributive property.

YOU TRY IT:
Multiply the polynomials.

1. $(2x-3)(-3x+5)$

Factoring a quadratic with leading coefficient 1

Watch *Video 9: Factoring Trinomials with a Leading Coefficient of 1* to complete the following.

Factor completely.

YOU TRY IT: Factor completely.

2. $x^2 - 12x + 27$

Factoring out a constant before factoring a quadratic

EXAMPLE:
Factor $8x^2 + 4x - 60$.

We begin by factoring out a 4.

$$\begin{aligned} 8x^2 + 4x - 60 &= 4(2x^2 + x - 15) \\ &= 4(x+3)(2x-5) \end{aligned}$$

YOU TRY IT:

3. Factor $-10y^2 + 35y - 15$.

Factoring a quadratic with leading coefficient greater than 1: Problem type 1

Watch *Video 5: Factoring a Trinomial with Leading Coefficient Not Equal to 1 (Trial and Error Method)* to complete the following.

Factor.

YOU TRY IT: Factor completely.

4. $-2 + 9x + 5x^2$

Factoring a quadratic with leading coefficient greater than 1: Problem type 2

Watch *Video 4: Factoring a Trinomial by the Trial-and Error Method (Leading Coefficient Not Equal to 1)* to complete the following.

Factor completely by using the trial-and-error method.

YOU TRY IT: Factor completely.

5. $2x^2 - 7x - 15$

Factoring a difference of squares in one variable: Advanced

Factored Form of a Difference of Squares

$$a^2 - b^2 = \underline{\hspace{6cm}}$$

EXAMPLE: Factor completely, if possible.

$$81 - 49x^2$$

$$\begin{aligned} 81 - 49x^2 &= (9)^2 - (7x)^2 \\ &= (9 - 7x)(9 + 7x) \end{aligned}$$

We can check our factored form by multiplying the resulting binomials.

$$\begin{aligned} (9 - 7x)(9 + 7x) &= (9)^2 + 9(7x) - 7x(9) - (7x)^2 \\ &= (9)^2 + 0x - (7x)^2 \\ &= 81 - 49x^2 \end{aligned}$$

YOU TRY IT: Factor completely, if possible.

6. $16x^2 - 49$

7. $64x^2 + 25$

Solving an equation written in factored form

Watch the video *Solving a Quadratic Equation Using the Zero Product Rule* to complete the following.

PROPERTY **Zero Product Rule**

If $ab = 0$, then ________________ or ________________.

Solve the equation ______________________.

EXAMPLE:
Solve the equation $(x-4)(8-x) = 0$.

$$(x-4)(8-x) = 0$$

$$\begin{aligned} x - 4 &= 0 \quad \text{or} \quad 8 - x = 0 \\ x &= 4 \quad \text{or} \quad 8 = x \end{aligned}$$

The solution is $x = 4, 8$.

YOU TRY IT:

8. Solve the equation $(x+5)(2x-3) = 0$.

Finding the roots of a quadratic equation

We can find the roots of a quadratic equation by the following three methods.

1. Factoring
2. Completing the square
3. Using the quadratic formula.

Finding the roots of a quadratic equation of the form $ax^2 + bx = 0$

Watch *Video 5: Solving a Quadratic Equation using the Zero Product Rule* to complete the following.

Solve the equation.

EXAMPLE:
Solve the equation $2x^2 + 8x = 0$.

$$2x^2 + 8x = 0$$
$$2x(x+4) = 0$$

$$2x = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = 0 \quad \text{or} \quad x = -4$$

The solution is $x = 0, -4$.

YOU TRY IT:

9. Solve the equation $4x^2 - 20x = 0$.

Finding the roots of a quadratic equation with leading coefficient 1

Watch *Video 4: Solving a Quadratic Equation Using the Zero Product Rule* to complete the following.

Solve the equation.

EXAMPLE:
Solve the equation $x^2 + 8x = -15$.

$$x^2 + 8x = -15$$
$$x^2 + 8x + 15 = 0$$
$$(x+3)(x+5) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = -3 \quad \text{or} \quad x = -5$$

The solution is $x = -3, -5$.

YOU TRY IT:

10. Solve the equation $x^2 + 4x - 21 = 0$.

Finding the roots of a quadratic equation with leading coefficient greater than 1

Solving a Quadratic Equation by Factoring

Step 1 Write the equation in the form ____________________.

Step 2 ____________________ completely.

Step 3 Apply the ______________________________. That is, set each factor equal to __________ and solve the resulting equations.

EXAMPLE:
Solve the equation $15x^2 + 7x - 4 = 0$.

$$15x^2 + 7x - 4 = 0$$
$$(5x+4)(3x-1) = 0$$

$$5x + 4 = 0 \quad \text{or} \quad 3x - 1 = 0$$
$$x = -\frac{4}{5} \quad \text{or} \quad x = \frac{1}{3}$$

The solution is $x = -\frac{4}{5}, \frac{1}{3}$.

YOU TRY IT:

11. Solve the equation $4x^2 - x - 3 = 0$.

Solving a quadratic equation needing simplification

Watch *Video 6: Solving a Quadratic Equation Using the Zero Product Rule* to complete the following.

Solve the equation.

EXAMPLE: Solve: $2x^2 - x - 3 = (x+1)^2$.

$$\begin{aligned} 2x^2 - x - 3 &= (x+1)^2 \\ 2x^2 - x - 3 &= x^2 + 2x + 1 \\ x^2 - 3x - 4 &= 0 \\ (x-4)(x+1) &= 0 \\ x &= 4, -1 \end{aligned}$$

YOU TRY IT:

12. Solve: $2x^2 + x = (x-2)^2 - 10$

Solving a word problem using a quadratic equation with rational roots

Watch *Video 10: Solving a Geometry Application Using a Quadratic Equation(Area of a Rectangle)* to complete the following.

The area of a rectangular field is __________. The length is ______________________________.

Find the ______________________________.

YOU TRY IT:

13. The length of a rectangular photograph is 7 in more than the width. If the area is 78 in^2, what are the dimensions of the photograph?

Restriction on a variable in a denominator: Linear

Division by ________ is not defined.

A rational expression is undefined when its _______________ is 0.

Restriction on a variable in a denominator: Quadratic

EXAMPLE:

Find all excluded values for $\dfrac{y+2}{y^2-9}$.

We must exclude values when $y^2 - 9 = 0$.

$$\begin{aligned} y^2 - 9 &= 0 \\ y^2 &= 9 \\ y &= 3, -3 \end{aligned}$$

$\dfrac{y+2}{y^2-9}$ is undefined when $y = 3$ or $y = -3$.

YOU TRY IT:

14. Find all excluded values of $\dfrac{u+7}{u^2-4u+4}$.

Evaluating a rational function: Problem type 1

Watch *Video 2: Evaluating a Rational Function for Selected Values in the Domain* to complete the following.

Evaluate the function for the given value of x. $f(x) =$ ______________

1.

2.

3.

4.

The function is ___________ at ________.

The value 3 is ___________ in the ________________ of the function.

Evaluating a rational function: Problem type 2

Definition of a Rational Function

A function is a ____________________ if it can be written in the form _______________. Where p and q are ____________________ and __________.

EXAMPLE:
Given $f(x) = \dfrac{x+3}{x^2-3x}$, find the following.

a. $f(4)$

$$f(4) = \frac{4+3}{4^2-3(4)} = \frac{7}{16-12} = \frac{7}{4}$$

b. $f(-5)$

$$\begin{aligned} f(-5) &= \frac{-5+3}{(-5)^2-3(-5)} \\ &= \frac{-2}{25-(-15)} \\ &= \frac{-2}{40} = -\frac{1}{20} \end{aligned}$$

c. $f(0)$

$f(0) = \dfrac{0+3}{0^2-3(0)} = \dfrac{3}{0} \implies f(0)$ is undefined because the function is not defined at $x=0$.

YOU TRY IT:
Given $g(x) = \dfrac{x-7}{x^2-4}$, find the following.

15. $g(1)$

16. $g(0)$

17. $g(2)$

18. $g(-2)$

Additional Notes:

Module 2

Simplifying a ratio of factored polynomials: Linear factors

Watch *Video 5: Simplifying a Rational Expression* and complete the following.

PROPERTY **Fundamental Principle of Rational Expressions**

Let p, q, and r represent polynomials such that $q \neq 0$ and $r \neq 0$. Then

$$\frac{pr}{qr} = \underline{\hspace{3cm}} = \underline{\hspace{3cm}} = \underline{\hspace{2cm}}$$

Simplify the expression.

The expressions $\dfrac{2x-6}{x^2-8x+15}$ and $\dfrac{2}{x-5}$ are equivalent for all real numbers except __________ and __________ because they make the denominator equal to __________.

EXAMPLE: Simplify.

$$\frac{5(2x+1)(x-4)}{35(x-4)(x-3)}$$

.

Divide the numerator and denominator by 5.

$$\frac{\cancel{5}^1(2x+1)(x-4)}{\cancel{35}_7(x-4)(x-3)} = \frac{(2x+1)(x-4)}{7(x-4)(x-3)}$$

Divide the numerator and denominator by $x-4$.

$$\frac{(2x+1)\cancel{(x-4)}^1}{7\cancel{(x-4)}_1(x-3)} = \frac{2x+1}{7(x-3)}$$

YOU TRY IT:

19. Simplify $\dfrac{8(2x+3)(x-7)}{18(2x+3)(x+7)}$.

Simplifying a ratio of polynomials using GCF factoring

EXAMPLE:

Simplify $\dfrac{18x^2 - 24x}{18x^2 - 36x}$.

We factor out the greatest common factor (GCF) from the numerator and the denominator.

$$\frac{18x^2 - 24x}{18x^2 - 36x} = \frac{6x(3x-4)}{18x(x-2)}$$

Divide numerator and denominator by $6x$.

$$\frac{\cancel{6x}^1(3x-4)}{\cancel{18x}_3(x-2)} = \frac{3x-4}{3(x-2)}$$

YOU TRY IT:

20. Simplify $\dfrac{24x^2 + 2x}{12x^2 + x}$.

Simplifying a ratio of linear polynomials: $1, -1$, and no simplification

Watch *Video 7: Recognizing a ratio of* -1 to complete the following.

Recognizing a ratio of 1

$\dfrac{5}{5} = \underline{\qquad}$ $\qquad$ $\underline{\qquad\qquad} = 1$

Recognizing a ratio of -1

$\dfrac{5}{-5} = \underline{\qquad}$ $\qquad$ $\underline{\qquad\qquad} = -1$

Write all of the ratios equal to -1 other than the ones above shown in the video.

EXAMPLE: Simplify.

a. $\dfrac{x-4}{4-x}$

$$\begin{aligned}\frac{x-4}{4-x} &= \frac{x-4}{-1(-4+x)} \\ &= \frac{\cancel{x-4}}{-1\cancel{(x-4)}} \\ &= -1\end{aligned}$$

b. $\dfrac{3x-6y}{2y+x}$

$$\frac{3x-6y}{2y+x} = \frac{3(x-2y)}{2y+x}$$

Cannot be simplified.

YOU TRY IT: Simplify.

21. $\dfrac{x+2}{2-x}$

22. $\dfrac{4y-6x}{3x-2y}$

Simplifying a ratio of polynomials by factoring a quadratic with leading coefficient 1

EXAMPLE:
Simplify $\dfrac{x^2+x-6}{4x-8}$.

We factor out the GCF from the numerator and factor the denominator.

$$\frac{x^2+x-6}{4x-8} = \frac{(x+3)\cancel{(x-2)}}{4\cancel{(x-2)}} = \frac{x+3}{4}$$

YOU TRY IT:

23. Simplify $\dfrac{x^2+3x+2}{3x+6}$.

Simplifying a ratio of polynomials: Problem type 2

Watch *Video 8: Rational Expressions in which a Factor of* -1 *is present* to complete the following.

Simplify the expression.

Multiplying Rational Expressions

Multiplication Property of Rational Expressions

Let p, q, r, and s represent polynomials, such that $q \neq 0$ and $s \neq 0$. Then

$$\frac{p}{q} \cdot \frac{r}{s} = \underline{\hspace{3cm}}$$

Multiplying Rational Expressions

Step 1 Factor the ____________ and ____________ of each expression.

Step 2 ____________ the numerators and multiply the ____________.

Step 3 Reduce the ratios of ________________ to 1 or -1 and ____________.

Multiplying rational expressions involving multivariate monomials

Watch *Video 1: Multiplying Rational Expressions* to complete the following

$\boxed{\textit{PROCEDURE}}$ To **multiply rational expressions**, multiply the ____________ and multiply the ____________. Then simplify if possible.

$$\frac{p}{q} \cdot \frac{r}{s} = \underline{\hspace{5cm}} \text{ provided that } q \neq 0 \text{ and } s \neq 0$$

Multiply the rational expressions.

1.

2.

EXAMPLE:
Multiply $\frac{4m^4n^2}{3mn^5} \cdot \frac{15n}{2m^2}$.

$$\frac{4m^4n^2}{3mn^5} \cdot \frac{15n}{2m^2} = \frac{2 \cdot 2 \cdot m^4 \cdot n^2 \cdot 3 \cdot 5 \cdot n}{3 \cdot m \cdot n^5 \cdot 2 \cdot m^2}$$

Divide numerator and denominator by 2 and 3.

$$= \frac{\cancel{2} \cdot 2 \cdot m^4 \cdot n^2 \cdot \cancel{3} \cdot 5 \cdot n}{\cancel{3} \cdot m \cdot n^5 \cdot \cancel{2} \cdot m^2}$$

$$= \frac{2 \cdot m^4 \cdot n^2 \cdot 5 \cdot n}{m \cdot n^5 \cdot m^2}$$

Divide numerator and denominator by n^3.

$$= \frac{2 \cdot m^4 \cdot 5}{m \cdot n^2 \cdot m^2}$$

Divide numerator and denominator by m^3.

$$= \frac{2 \cdot m \cdot 5}{\cdot n^2}$$

$$= \frac{10m}{n^2}$$

YOU TRY IT:

24. Multiply $\frac{2a}{3b^2} \cdot \frac{9b}{14a^2}$.

Multiplying rational expressions made up of linear expressions

EXAMPLE:

Multiply $\dfrac{3x+6}{5x-10} \cdot \dfrac{x-2}{4x+8}$.

Factor out the GCF in each term then simplify.

$$\begin{aligned}\frac{3x+6}{5x-10} \cdot \frac{x-2}{4x+8} &= \frac{3(x+2)}{5(x-2)} \cdot \frac{x-2}{4(x+2)} \\ &= \frac{3\cancel{(x+2)}}{5\cancel{(x-2)}} \cdot \frac{\cancel{x-2}}{4\cancel{(x+2)}} \\ &= \frac{3}{20}\end{aligned}$$

YOU TRY IT:

25. Multiply $\dfrac{-5x+15}{2x+18} \cdot \dfrac{3x+27}{x-3}$.

Multiplying rational expressions involving quadratics - leading coefficients greater than 1

EXAMPLE:

Multiply $\dfrac{25x^2-9}{5x+10} \cdot \dfrac{2x^2+11x+14}{5x+3}$.

$$\begin{aligned}\frac{25x^2-9}{5x+10} \cdot \frac{2x^2+11x+14}{5x+3} &= \frac{(5x+3)(5x-3)}{5(x+2)} \cdot \frac{(2x+7)(x+2)}{(5x+3)} && \text{Factor completely} \\ &= \frac{\cancel{(5x+3)}(5x-3)}{5\cancel{(x+2)}} \cdot \frac{(2x+7)\cancel{(x+2)}}{\cancel{(5x+3)}} && \text{Divide out the common factors} \\ &= \frac{(5x-3)(2x+7)}{5}\end{aligned}$$

YOU TRY IT:

26. Multiply $\dfrac{6x^2-42x+60}{x^2-x-6} \cdot \dfrac{x-3}{18x-36}$.

Dividing rational expressions involving linear expressions

Division Property of Rational Expressions

Let p, q, r, and s represent polynomials, such that $q \neq 0$ and $s \neq 0$. Then

$$\frac{p}{q} \div \frac{r}{s} = \underline{\hspace{6em}} = \underline{\hspace{6em}}$$

EXAMPLE:

Divide $\dfrac{8z-16}{-20} \div \dfrac{3z-6}{40}$.

$$\begin{aligned}
\frac{8z-16}{-20} \div \frac{3z-6}{40} &= \frac{8z-16}{-20} \cdot \frac{40}{3z-6} \\
&= \frac{8(z-2)}{-1\cdot 20} \cdot \frac{2\cdot 20}{2(z-2)} \\
&= \frac{8\cancel{(z-2)}}{-1\cdot \cancel{20}} \cdot \frac{\cancel{2}\cdot \cancel{20}}{\cancel{2}\cancel{(z-2)}} \\
&= -8
\end{aligned}$$

YOU TRY IT:

27. Divide $\dfrac{4x}{8x+4} \div \dfrac{6}{14x+7}$.

Dividing rational expressions involving quadratics with leading coefficients of 1

Watch *Video 4: Dividing Rational Expressions* to complete the following.

Divide.

EXAMPLE:
Divide $\dfrac{t^2-49}{t^2+4t-21} \div \dfrac{t^2-2t-35}{t^2+8t+15}$.

$$\begin{aligned}
\frac{t^2-49}{t^2+4t-21} \div \frac{t^2-2t-35}{t^2+8t+15} &= \frac{t^2-49}{t^2+4t-21} \cdot \frac{t^2+8t+15}{t^2-2t-35} \\
&= \frac{(t-7)(t+7)}{(t+7)(t-3)} \cdot \frac{(t+3)(t+5)}{(t-7)(t+5)} \\
&= \frac{\cancel{(t-7)}\cancel{(t+7)}}{\cancel{(t+7)}(t-3)} \cdot \frac{(t+3)\cancel{(t+5)}}{\cancel{(t-7)}\cancel{(t+5)}} \\
&= \frac{t+3}{t-3}
\end{aligned}$$

YOU TRY IT:

28. Divide $\dfrac{x^2-25}{x^2+x-20} \div \dfrac{x^2-2x-15}{x^2+7x+12}$.

Multiplication and division of 3 rational expressions

Watch *Video 5: Dividing Rational Expressions and Using the Order of Operations* to complete the following.

Divide.

YOU TRY IT:

29. Divide $\dfrac{t^2-49}{t^2+4t-21} \div \dfrac{t^2-2t-35}{t^2+8t+15} \cdot (4t-12)$.

Introduction to the LCM of two monomials

Watch *Video 3: Determining the Least Common Denominator Between Two Rational Expressions* to complete the following.

PROCEDURE Determining the LCD of Two or More Expressions

- ____________________
- The LCD is the ________________ from the ________, where each factor is raised to its ____________.

Determine the LCD for each group of fractions.

1.

2.

Least common multiple two monomials

EXAMPLE:
Find the LCM of $6a^3b^2$ and $4ab^5$.

List the multiples of 6 and 4
6: $6, 12, 18, \ldots$ and $4: 4, 8, 12, 16, \ldots$

We see that 12 is the LEAST COMMON multiple. Thus, 12 the LCM of 6 and 4.

The LCM of a^3 and a is a^3 since $a^3 = \mathbf{a} \cdot a^2$

The LCM of b^2 and b^5 is b^5 since $b^5 = \mathbf{b^2} \cdot b^3$

The LCM is $12a^3b^5$.

YOU TRY IT:

30. Find the LCM of $15xy$ and $20x^2y^3z$.

Additional Notes:

Module 3

Finding the LCD of rational expressions with quadratic denominators

Watch the video *Exercise: Finding the Least Common Denominator* to complete the following.

Find the least common denominator (LCD).

The LCD is the ______________________________ from the ____________, where each factor is raised to its _______________ to which it appears in any denominator.

EXAMPLE:
Find the LCD of $\dfrac{8}{x^2-16}$ and $\dfrac{1}{x^2-x-20}$.

We begin by factoring the denominators.

$$\frac{8}{x^2-16} = \frac{8}{(x-4)(x+4)}$$
$$\frac{1}{x^2-x-20} = \frac{1}{(x+4)(x-5)}$$

The LCD is $(x-4)(x+4)(x-5)$.

YOU TRY IT:

31. Find the LCD of $\dfrac{3}{x^2-x-6}$ and $\dfrac{4}{x^2+9x+14}$.

Finding the LCD of rational expression with linear denominators: Relatively prime

EXAMPLE:
Find the LCD of $\dfrac{8}{3x+6}$ and $\dfrac{x}{4x-8}$.

We begin by factoring the denominators.

$$\frac{8}{3x+6} = \frac{8}{3(x+2)} \qquad \frac{x}{4x-8} = \frac{x}{4(x-2)}$$

The denominators do not have any common factors. We call these denominators **relatively prime**.

The LEAST COMMON denominator of **relatively prime** denominators is the **product of all of the factors** in the denominators.

The LCD is
$3 \cdot (x+2) \cdot 4 \cdot (x-2) = 12(x+2)(x-2)$.

YOU TRY IT:

32. Find the LCD of $\dfrac{5}{6x-16}$ and $\dfrac{7}{3x}$.

Writing equivalent rational expressions with monomial denominators

Watch *Video 4: Writing Equivalent Fractions* to complete the following.

Convert each expression to an equivalent expression with the indicated denominator.

1.

2.

EXAMPLE:
Complete the equivalent fraction.

$$\frac{5}{3x^2} = \frac{\quad}{6x^5}$$

$$\frac{5}{3x^2} = \frac{5}{3x^2} \cdot \frac{2x^3}{2x^3}$$
$$\frac{5}{3x^2} = \frac{10x^3}{6x^5}$$

YOU TRY IT:

33. Complete the equivalent fraction.

$$\frac{-2}{3x^3} = \frac{\quad}{12x^8}$$

Writing equivalent rational expressions with polynomial denominators

Watch the video *Exercise: Completing an Equivalent Fraction* to complete the following.

Fill in the blank to make an equivalent fraction with the given denominator.

EXAMPLE: Complete the equivalent fraction.

$$\frac{2x}{x+3} = \frac{}{(x+3)(x-7)}$$

$$\frac{2x}{x+3} = \frac{2x}{x+3} \cdot \frac{x-7}{x-7}$$

$$= \frac{2x(x-7)}{(x+3)(x-7)}$$

YOU TRY IT: Complete the equivalent fraction.

34. $\dfrac{2x(x-1)}{x+2} = \dfrac{}{(2x+3)(x+2)}$

Writing equivalent rational expressions involving opposite factors

Take notes from the Learning Page for the problem

$$\frac{-2}{u+3} = \frac{\square}{-u-3}$$

YOU TRY IT: Complete the equivalent fraction.

35. $\dfrac{2x}{6-x} = \dfrac{}{x-6}$

Adding rational expressions with common denominators and GCF factoring

Additon and Subtraction Properties of Rational Expressions

Let p, q, and r represent polynomials where $q \neq 0$. Then

1. $\frac{p}{q} + \frac{r}{q} =$ ________________

2. $\frac{p}{q} - \frac{r}{q} =$ ________________

Adding rational expressions with common denominators and quadratic factoring

Watch the video *Subtracting Rational Expressions with Like Denominators* to complete the following.

Add or subtract as indicated and simplify if possible.

EXAMPLE:

Simplify $\frac{x}{x^2+x-6} + \frac{3}{x^2+x-6}$.

$$\begin{aligned} \frac{x}{x^2+x-6} + \frac{3}{x^2+x-6} &= \frac{x+3}{x^2+x-6} \\ &= \frac{x+3}{(x+3)(x-2)} \\ &= \frac{\cancel{x+3}}{\cancel{(x+3)}(x-2)} \\ &= \frac{1}{x-2} \end{aligned}$$

YOU TRY IT:

36. Simplify $\frac{4}{x^2-x-20} + \frac{x}{x^2-x-20}$.

Adding rational expressions with denominators ax^n and bx^m

Addition and Subtraction of Rational Expressions with Unlike Denominators

Adding and Subtracting Rational Expressions

Step 1 Factor the ______________ of each rational expression.

Step 2 Identify the __________.

Step 3 Rewrite each rational expression as an ________________________ with the LCD as its denominator.

Step 4 Add or subtract the ______________, and write the result over the common denominator.

Step 5 Simplify, if possible.

Watch *Video 5: Adding Rational Expressions* to complete the following.

Add.

EXAMPLE: Add.

$$\frac{6}{7x} + \frac{4}{x^3}$$

$$\begin{aligned}\frac{6}{7x} + \frac{4}{x^3} &= \frac{6}{7x} \cdot \frac{x^2}{x^2} + \frac{4}{x^3} \cdot \frac{7}{7} \\ &= \frac{6x^2}{7x^3} + \frac{28}{7x^3} \\ &= \frac{6x^2 + 28}{7x^3}\end{aligned}$$

YOU TRY IT: Add.

37. $\frac{5}{6b^4} + \frac{7}{4b}$

Adding rational expressions with linear denominators without common factors: Basic

EXAMPLE:

Simplify $\dfrac{3}{x+2} + \dfrac{5}{x+1}$.

$$\begin{aligned}
\frac{3}{x+2} + \frac{5}{x+1} &= \frac{3}{x+2} \cdot \frac{x+1}{x+1} + \frac{5}{x+1} \cdot \frac{x+2}{x+2} \\
&= \frac{3(x+1)}{(x+2)(x+1)} + \frac{5(x+2)}{(x+1)(x+2)} \\
&= \frac{3(x+1) + 5(x+2)}{(x+1)(x+2)} \\
&= \frac{3x+3+5x+10}{(x+1)(x+2)} \\
&= \frac{8x+13}{(x+1)(x+2)}
\end{aligned}$$

YOU TRY IT:

38. Simplify $-\dfrac{2}{x-1} + \dfrac{3}{x+4}$.

Adding rational expressions with linear denominators without common factors: Advanced

Watch *Video 7: Subtracting Rational Expressions* to complete the following.

Subtract.

Adding rational expressions with linear denominators with common factors: Basic

To do the addition, the ______________ must be the same. So first, we find the ____________________.

Watch the video *Exercise: Subtracting Rational Expressions with Unlike Denominators 1* and complete the box below.

Adding rational expressions with linear denominators with common factors: Advanced

Watch the video *Exercise: Subtracting Rational Expressions with Unlike Denominators 1*

Add or subtract as indicated.

Adding rational expressions with denominators $ax - b$ and $b - ax$

Watch the video *Exercise: Subtracting Rational Expressions with Unlike Denominators 2* to complete the following.

Add or subtract as indicated.

EXAMPLE:

Simplify $\dfrac{x+3}{x-2} + \dfrac{x-5}{2-x}$.

$$\begin{aligned}
\frac{x+3}{x-2} + \frac{x-5}{2-x} &= \frac{x+3}{x-2} + \frac{x-5}{2-x} \cdot \frac{-1}{-1} \\
&= \frac{x+3}{x-2} + \frac{-x+5}{-2+x} \\
&= \frac{x+3-x+5}{x-2} \\
&= \frac{8}{x-2}
\end{aligned}$$

YOU TRY IT:

39. Simplify $\dfrac{x-3}{x-1} - \dfrac{x+4}{1-x}$.

Adding rational expressions involving different quadratic denominators

Watch the video *Adding Rational Expressions* to complete the following.

Avoiding Mistakes

- Do not try to ____________ or cancel the _____ in the numerator and denominator. They are ________________________.
- Likewise, do not to reduce or ____________ the y in the ______________ and ______________. They are ________________________.

What values are the two expressions NOT equivalent for?

EXAMPLE:

Simplify $\dfrac{y+2}{y^2-36} - \dfrac{y}{y^2+9y+18}$.

$$
\begin{aligned}
&\frac{y+2}{y^2-36} - \frac{y}{y^2+9y+18} \\
&= \frac{y+2}{(y-6)(y+6)} - \frac{y}{(y+6)(y+3)} \\
&= \frac{y+2}{(y-6)(y+6)} \cdot \frac{y+3}{y+3} - \frac{y}{(y+6)(y+3)} \cdot \frac{y-6}{y-6} \\
&= \frac{(y+2)(y+3) - y(y-6)}{(y-6)(y+6)(y+3)} \\
&= \frac{y^2+5y+6-y^2+6y}{(y-6)(y+6)(y+3)} \\
&= \frac{11y+6}{(y-6)(y+6)(y+3)}
\end{aligned}
$$

YOU TRY IT:

40. Simplify $\dfrac{x-2}{x-4} + \dfrac{2x^2-15x+12}{x^2-16}$.

Additional Notes:

Additional Notes:

Module 4

Complex fraction without variables: Problem type 1

EXAMPLE:

Simplify $\dfrac{\frac{15}{8}}{\frac{5}{2}}$.

A fraction bar means division so we can write the complex fraction as

$$\frac{\frac{15}{8}}{\frac{5}{2}} = \frac{15}{8} \div \frac{5}{2} = \frac{15}{8} \cdot \frac{2}{5}$$

Simplifying, $\dfrac{\cancel{15}^{3}}{\cancel{8}_{4}} \cdot \dfrac{\cancel{2}^{1}}{\cancel{5}_{1}} = \dfrac{3}{4}$

YOU TRY IT:

41. Simplify $\dfrac{\frac{9}{4}}{\frac{3}{10}}$

Complex fraction without variables: Problem type 2

Watch *Video 2: Simplifying a Complex Fraction using Method I* to complete the following.

Simplify the complex fraction by using Method I.

Watch *Video 4: Simplifying a Complex Fraction using Method II* to complete the following.

Simplify the complex fraction by using Method II.

Complex fraction involving univariate monomials

EXAMPLE: Simplify.

$$\frac{\frac{3x}{x-5}}{\frac{7x^2}{x+5}} = \frac{3x}{x-5} \cdot \frac{x+5}{7x^2}$$

$$= \frac{3\cancel{x}^1}{x-5} \cdot \frac{x+5}{7x^{\cancel{2}1}}$$

$$= \frac{3(x+5)}{7x(x-5)}$$

YOU TRY IT: Simplify .

42. $\frac{\frac{x+2}{5x^2}}{\frac{x+2}{x-5}}$

Complex fraction involving multivariate monomials

Watch the video *Introduction to Complex Fractions* to complete the following.

Simplify the complex fraction.

EXAMPLE:

Simplify $\dfrac{\frac{3wy^3}{14z^2x}}{\frac{6y^5}{7z^4x^2}}$.

$$
\begin{aligned}
\frac{\frac{3wy^3}{14z^2x}}{\frac{6y^5}{7z^4x^2}} &= \frac{3wy^3}{14z^2x}\cdot\frac{7z^4x^2}{6y^5}\\
&= \frac{3wy^3}{2\cdot 7z^2x}\cdot\frac{7z^2z^2xx}{2\cdot 3y^3y^2}\\
&= \frac{\cancel{3}wy^{\cancel{3}}}{2\cdot \cancel{7}\cancel{z^2}\cancel{x}}\cdot\frac{\cancel{7}\cancel{z^2}z^2\cancel{x}x}{2\cdot \cancel{3}\cancel{y^3}y^2}\\
&= \frac{xwz^2}{4y^2}
\end{aligned}
$$

YOU TRY IT:

43. Simplify $\dfrac{\frac{15b^5}{4c^3d^3}}{\frac{5ab^2}{8c^2d}}$.

Complex fraction: GCF factoring

Watch the video *Exercise: Simplifying Complex Fractions Using Method I (1)* to complete the following.

Simplify the complex fraction by using Method I.

EXAMPLE: Simplify.

$$
\begin{aligned}
\frac{\frac{3x}{x-5}}{\frac{7}{4x-20}} &= \frac{3x}{x-5}\cdot\frac{4x-20}{7}\\
&= \frac{3x}{x-5}\cdot\frac{4(x-5)}{7}\\
&= \frac{3x}{\cancel{x-5}}\cdot\frac{4\cancel{(x-5)}}{7}\\
&= \frac{12x}{7}
\end{aligned}
$$

YOU TRY IT: Simplify .

44. $\dfrac{\frac{7a}{a+3}}{\frac{1}{4a+12}}$.

Complex fraction made of sums involving rational expressions: Problem type 1

Complex fraction made of sums involving rational expressions: Problem type 2

A **complex fraction** is an expression containing ____________________.

Simplifying a Complex Fraction-Method 1

Step 1 Add or subtract expressions in the __________ to form a ____________.

Add or subtract expressions in the __________ to form a ________________.

Step 2 __________ the rational expression from Step 1.

Step 3 Simplify to ______________, if possible.

Simplifying a Complex Fraction-Method II

Step 1 Multiply the numerator and denominator of the complex fraction by the

____________________ within the expression.

Step 2 Apply the ________________, and simplify the numerator and denominator.

Step 3 Simplify to lowest terms, if possible.

Watch *Video 3: Simplifying a Complex Fraction Using Method I* to complete the following.

Simplify the complex fraction by using Method I.

YOU TRY IT: Simplify .

45. $\dfrac{1-\frac{1}{x-6}}{x-\frac{7}{x-6}}$.

Complex fraction made of sums involving rational expressions: **Problem type 4**

EXAMPLE: Simplify.

$$\frac{2-\frac{2}{x+1}}{2+\frac{2}{x}}$$

Using Method II we multiply the numerator and denominator of the complex fraction by the LCD, $x(x+1)$.

$$\begin{aligned}
\frac{\left(2-\frac{2}{x+1}\right)\cdot x(x+1)}{\left(2+\frac{2}{x}\right)\cdot x(x+1)} &= \frac{2\cdot x(x+1)-\frac{2}{x+1}\cdot x(x+1)}{2\cdot x(x+1)+\frac{2}{x}\cdot x(x+1)}\\
&= \frac{2\cdot x(x+1)-\frac{2}{\cancel{x+1}}\cdot x\cancel{(x+1)}}{2\cdot x(x+1)+\frac{2}{\cancel{x}}\cdot \cancel{x}(x+1)}\\
&= \frac{2x(x+1)-2x}{2x(x+1)+2(x+1)}\\
&= \frac{2x^2+2x-2x}{2x^2+2x+2x+2}\\
&= \frac{2x^2}{2(x^2+2x+1)}\\
&= \frac{\cancel{2}x^2}{\cancel{2}(x^2+2x+1)} = \frac{x^2}{(x+1)^2}
\end{aligned}$$

YOU TRY IT: Simplify .

46. $\dfrac{\frac{4}{y+5}-4}{1-\frac{y+6}{2y+10}}$

Complex fraction made of sums involving rational expressions: **Problem type 6**

Watch *Video 6: Simplifying Complex Fractions Using Method II* to complete the following.

Simplify the complex fraction by using Method II.

EXAMPLE:

Simplify $\dfrac{\frac{2}{x-1}+\frac{2}{x+2}}{\frac{1}{x+2}-\frac{3}{x-1}}$.

$$\begin{aligned}\frac{\frac{2}{x-1}+\frac{2}{x+2}}{\frac{1}{x+2}-\frac{3}{x-1}} &= \frac{\frac{2}{x-1}+\frac{2}{x+2}}{\frac{1}{x+2}-\frac{3}{x-1}} \cdot \frac{(x+2)(x-1)}{(x+2)(x-1)} \\ &= \frac{2(x+2)+2(x-1)}{1(x-1)-3(x+2)} \\ &= \frac{2x+4+2x-2}{x-1-3x-6} \\ &= \frac{4x+2}{-2x-7}\end{aligned}$$

YOU TRY IT:

47. Simplify $\dfrac{\frac{3}{x-4}-\frac{1}{x+4}}{\frac{1}{x+4}+\frac{2}{x-4}}$.

Additional Notes:

Additional Notes:

Module 5

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel, but still count toward your module completion. To prepare for your upcoming exam:

- ☐ Complete this module.
- ☐ At least two days before your focus group, take your ALEKS exam in the MALL.
- ☐ If you score less than 80% you are strongly encouraged to retake the ALEKS exam.
  - ☐ Ask for a ticket to retake from a tutor.
  - ☐ Work in the MALL for one hour.
  - ☐ Have a tutor sign that you have finished your review.
  - ☐ Retake the ALEKS portion of your exam.
- ☐ Take your written exam the day of your focus group. No retakes will be allowed on written exams.

The score on your Scheduled Knowledge Check is the number of topics that you have mastered (including prerequisite topics) out of the number of topics that you should have mastered by this point.

	Score
ALEKS Exam	
ALEKS Exam Retake	
Written Exam	

*Your recorded ALEKS exam score is the higher of your ALEKS Exam score and ALEKS Exam Retake score.

Module 6

Solving a proportion of the form $\frac{x}{a} = \frac{b}{c}$

Watch the video *Introduction to Proportions* to complete the following.

Solve the proportion.

$\boxed{\textit{DEFINITON}}$ The **ratio of a to b** can be written as $\dfrac{a}{b}$ where $b \neq 0$.

An equation that ________________ is called a ___________. For example:

$$\frac{a}{b} = \_\_\_\_\_\_\_\_ \text{ for } \_\_\_\_\_\_\_\_ \text{ and } \_\_\_\_\_\_\_\_$$

EXAMPLE:
Solve $\dfrac{x}{4} = -\dfrac{3}{5}$ for x.

$$\begin{aligned} \frac{x}{4} &= -\frac{3}{5} \\ 20 \cdot \frac{x}{4} &= -\frac{3}{5} \cdot 20 \\ 5x &= -12 \\ x &= -\frac{12}{5} \end{aligned}$$

YOU TRY IT:

48. Solve $\dfrac{x}{2} = \dfrac{1}{6}$ for x.

Solving a proportion of the form $\frac{a}{x+b} = \frac{c}{x}$

Click on the hyperlink for "method of cross products" to complete the following.

Cross products and proportions:

If two ____________________, then their ____________________.

If _______________ then _______________

.

EXAMPLE:

Solve $\dfrac{3}{x+4} = -\dfrac{5}{x-1}$ for x.

$$\begin{aligned} \frac{3}{x+4} &= \frac{-5}{x-1} \\ 3(x-1) &= -5(x+4) \\ 3x-3 &= -5x-20 \\ 8x &= -17 \\ x &= -\frac{17}{8} \end{aligned}$$

YOU TRY IT:

49. Solve $\dfrac{2}{x-1} = \dfrac{1}{x+6}$ for x.

Solving a rational equation that simplifies to linear: Denominators a, x, or ax

Solving a Rational Equation

1. Factor the _______________ of all rational expressions. Identify any values of the variable for

   which any expression is _______________.

2. Identify the __________ of all terms in the equation.

3. _______________ both sides of the equation by the __________.

4. Solve the ______________________________.

5. Check the potential solutions in the original equation. Note that ____________________

   for which the equation is _______________ cannot be a solution to the equation.

Watch *Video 2: Solving a Rational Equation* to complete the following.

Solve the equation.

Solving a rational equation that simplifies to linear: Denominator $x + a$

EXAMPLE:

Solve $\dfrac{3}{p-7} = -2$.

$$\frac{3}{p-7} = -2$$

Multiply both sides of the equation by $p - 7$

$$(p-7)\left(\frac{3}{p-7}\right) = (p-7)(-2)$$

$$\cancel{(p-7)}\left(\frac{3}{\cancel{p-7}}\right) = (p-7)(-2)$$

$$3 = -2p + 14$$

$$2p = 11$$

$$p = \frac{11}{2}$$

YOU TRY IT:

50. Solve $\quad 5 = \dfrac{4}{2y+1}$.

Solving a rational equation that simplifies to linear: Denominators ax and bx

Watch the video *Exercise: Solving a Rational Equation 1* to complete the following.

Solve the rational equation.

EXAMPLE:

Solve $\frac{1}{2} - \frac{3}{2p} = \frac{p-4}{p}$.

$$\frac{1}{2} - \frac{3}{2p} = \frac{p-4}{p}$$

Multiply both sides of the equation by $2p$

$$2p\left(\frac{1}{2} - \frac{3}{2p}\right) = 2p\left(\frac{p-4}{p}\right)$$

$$2p\left(\frac{1}{2}\right) - 2p\left(\frac{3}{2p}\right) = 2\cancel{p}\left(\frac{p-4}{\cancel{p}}\right)$$

$$p - 3 = 2(p-4)$$

$$p - 3 = 2p - 8$$

$$-p = -5$$

$$p = 5$$

YOU TRY IT:

51. Solve $\frac{2}{3y} + \frac{1}{4} = \frac{11}{6y} - \frac{1}{3}$.

Solving a rational equation that simplifies to linear: Unike binomial denominators

EXAMPLE:

Solve

$$\frac{3}{x-2} - 1 = \frac{5}{4x-8}$$

Factor all denominators

$$\frac{3}{x-2} - 1 = \frac{5}{4(x-2)}$$

Multiply both sides of the equation by $4(x-2)$

$$4(x-2)\left(\frac{3}{x-2} - 1\right) = 4(x-2)\left(\frac{5}{4(x-2)}\right)$$

$$4\cancel{(x-2)}\left(\frac{3}{\cancel{x-2}}\right) - 4(x-2)\cdot 1 = \cancel{4(x-2)}\left(\frac{5}{\cancel{4(x-2)}}\right)$$

$$4\cdot 3 - 4x + 8 = 5$$

$$12 - 4x + 8 = 5$$

$$-4x = -15 x = \frac{15}{4}$$

YOU TRY IT:

52. Solve $\dfrac{-1}{x+2} + \dfrac{4}{3x+6} = 2.$

Solving a rational equation that simplifies to linear: Factorable quadratic denominator

Watch *Video 3: Solving a Rational Equation with No Solution* to complete the following.

Solve the rational equation.

EXAMPLE:

Solve $\dfrac{6}{5x+10} - \dfrac{1}{x-5} = \dfrac{4}{x^2-3x-10}$.

We begin by factoring the denominators.

$$\frac{6}{5x+10} - \frac{1}{x-5} = \frac{4}{x^2-3x-10}$$
$$\frac{6}{5(x+2)} - \frac{1}{x-5} = \frac{4}{(x-5)(x+2)}$$

Multiply both sides of the equation by $5(x-5)(x+2)$

$$5(x-5)(x+2)\left(\frac{6}{5(x+2)} - \frac{1}{x-5} = \frac{4}{(x-5)(x+2)}\right)$$
$$\frac{6\cdot 5(x-5)(x+2)}{5(x+2)} - \frac{1\cdot 5(x-5)(x+2)}{x-5} = \frac{4\cdot 5(x-5)(x+2)}{(x-5)(x+2)}$$
$$\frac{6\cdot \cancel{5}(x-5)\cancel{(x+2)}}{\cancel{5}\cancel{(x+2)}} - \frac{1\cdot 5\cancel{(x-5)}(x+2)}{\cancel{x-5}} = \frac{4\cdot 5\cancel{(x-5)}\cancel{(x+2)}}{\cancel{(x-5)}\cancel{(x+2)}}$$
$$6(x-5) - 5(x+2) = 20$$
$$6x - 30 - 5x - 10 = 20$$
$$x - 40 = 20$$
$$x = 60$$

YOU TRY IT:

53. Solve $\dfrac{5}{y^2-7y+12} = \dfrac{2}{y-3} + \dfrac{5}{y-4}$.

Solving a rational equation that simplifies to quadratic: Proportional form, basic

Solving a Rational Equation

1. Factor the _______________ of all rational expressions. Identify any values of the variable for

   which any expression is _______________.

2. Identify the __________ of all terms in the equation.

3. _______________ both sides of the equation by the __________.

4. Solve the ______________________________.

5. Check the potential solutions in the original equation. Note that ____________________

   for which the equation is _______________ cannot be a solution to the equation.

Watch the video *Exercise: Solving a Proportion* to complete the following.

Solve the proportion.

EXAMPLE:

Solve $\dfrac{1}{b-5} = \dfrac{b-3}{3}$.

Use the method of cross products.

$$\begin{aligned}
\frac{1}{b-5} &= \frac{b-3}{3}\\
(b-5)(b-3) &= 1 \cdot 3\\
b^2 - 3b - 5b + 15 - 3 &= 0\\
b^2 - 8b + 12 &= 0\\
(b-6)(b-2) &= 0\\
b &= 6 \text{ or } 2
\end{aligned}$$

YOU TRY IT:

54. Solve $\dfrac{-2}{y-2} = \dfrac{y-3}{8y+11}$.

Solving a rational equation that simplifies to quadratic: Binomial denominators, constant numerators

Solving a rational equation that simplifies to quadratic: Factorable quadratic denominator

Watch the video *Exercise: Solving a Rational Equation 2* to complete the following.

Solve the rational equation.

YOU TRY IT:

55. Solve $\dfrac{y}{y-3} - \dfrac{24}{y^2-9} = \dfrac{4}{y+3}$.

Word problem on proportions: Problem type 1

Watch *Video 2: An Application of Rational Equations: Solving a Proportion* to complete the following.

Franco drove ____________ on ____________ of gas in his Honda hybrid. How many gallons will he need for a ____________ trip across country?

Word problem on proportions: Problem type 2

Watch *Video 3: An Application of Rational Equations: Solving a Proportion* to complete the following.

The ratio of female to male students taking algebra is ______________ If the total number of students taking the algebra class is ____________, how many students are __________?

Word problem involving multiple rates

Watch the video *Exercise: Solving a Rational Equation Application Involving Work* to complete the following.

Karen can was her SUV in ________. Clarann can was the same SUV in ________, how long will it take them to wax the SUV together?

Additional Notes:

Module 7

Finding all square roots of a number

Watch the video *Video 1: Definition of a Square Root* to complete the following.

b is a **square root** of a if ______________________.

Determine the square roots of the given real number.

1.

2.

3.

Square root of a rational perfect square

Watch the video *Video 11: Evaluating Square Roots* to complete the following.

Simplify the expressions.

1.

3.

4.

2.

5.

YOU TRY IT:
Simplify the following.

56. $\sqrt{49} =$

57. $-\sqrt{100} =$

58. $\sqrt{-64} =$

Square root of a perfect square with signs

Watch *Video 2: Defining Square Roots Using Radical Notation* to complete the following.

Simplify the expressions.

1.

2.

3.

4.

5.

If a is a positive real number, then

- $\sqrt{a}$ is the __________ square root of a (also called the ____________________
- $-\sqrt{a}$ is the __________ (or opposite) square root of a (also called the ____________________)
- $\sqrt{0} = 0$

Note: In the expression $\sqrt{a}$, the symbol, $\sqrt{\ }$ is called a _______________. The value of a is called the _______________.

Square roots of integers raised to even exponents

Watch the video *Determining the Principal n^{th} Root of an n^{th} Power* to complete the following.

- If n is a positive __________, then $\sqrt[n]{a^n} =$ __________.
- If n is a positive __________, then $\sqrt[n]{a^n} =$ __________.

Simplify the expressions.

1.

2.

3.

4.

5.

6.

YOU TRY IT:
Simplify the following.

59. $\sqrt{(-5)^2} =$

60. $-\sqrt{3^4} =$

Introduction to simplifying a radical expression with an even exponent

Watch the video *Simplifying the n^{th} Root of Perfect n^{th} Powers* to complete the following.

Simplify the expressions. Assume that all variables represent positive real numbers.

1.

2.

3.

4.

Perfect squares	Perfect Cubes
$(x^1)^2 =$	$(x^1)^3 =$
$(x^2)^2 =$	$(x^2)^3 =$
$(x^3)^2 =$	$(x^3)^3 =$
$(x^4)^2 =$	$(x^4)^3 =$

Square root of a perfect square monomial

YOU TRY IT:
Simplify the following.

61. $\sqrt{x^{12}} =$

63. $\sqrt{49y^8} =$

62. $\sqrt{a^{14}} =$

64. $\sqrt{16x^6} =$

Cube root of an integer

Watch the video *Evaluating Roots* to complete the following.

Evaluate the roots without using a calculator. Identify those that are not real numbers.

1.

3.

5.

2.

4.

6.

YOU TRY IT:
Simplify the following.

65. $\sqrt[3]{125} =$

66. $\sqrt[3]{-8} =$

67. $-\sqrt[3]{216} =$

Finding n^{th} roots of perfect n^{th} powers with signs

Definition of an n^{th} Root

b is an nth root of a if ________________.

Example: 2 is a ___________ root of 4 because ______________________.

Example: 2 is a ___________ root of 8 because ______________________.

Example: 2 is a ___________ root of 16 because ______________________.

Evaluating $\sqrt[n]{a}$

1. If ________ is an ________ integer and ________, then ____________ is the principal (____________) nth root of a.

2. If ________ is an ________ integer then ____________ is the ________________.

3. If ________ is an ____________, then ____________.

YOU TRY IT:
Simplify the following.

68. $\sqrt{-64} =$

69. $\sqrt[3]{-125} =$

Finding n^{th} roots of perfect n^{th} power monomial

YOU TRY IT:
Simplify the following.

70. $\sqrt[3]{27x^{15}} =$

71. $\sqrt[5]{32x^{10}} =$

Finding n^{th} roots of perfect n^{th} power fraction

Take notes from the Learning Page.

Converting between radical form and exponent form

Watch the video *Video 3:Converting Between Radical Notation and Rational Exponents* to complete the following.

Convert each expression to radical notation. Assume all variables represent positive real numbers.

1.

2.

3.

Convert each expression to an expression with rational exponents. Assume all variables represent positive real numbers.

4.

6.

5.

7.

YOU TRY IT:
Write as an exponential expression.

72. $\sqrt[5]{x^2} =$

73. $\sqrt{y^3} =$

Write as a radical expression.

74. $x^{5/3} =$

75. $y^{2/7} =$

Rational exponents: Unit fraction exponents and whole number bases

Watch the video *Definition of "a" to the* $1/n$ *Power* to complete the following.

DEFINITION Let a be a real number, and let $n > 1$ be an integer. Then,

$a^{1/n} =$ ________________ provided that ________________ is a real number.

Write each expression in radical notation and simplify.

1.

4.

2.

5.

3.

6.

YOU TRY IT:
Simplify the following.

76. $16^{1/4} =$

77. $8^{1/3} =$

Simplifying the square root of a whole number greater than 100

Take notes from the Learning Page.

Rational exponents: Unit fraction exponents and bases involving signs

Rational exponents: Non-unit fraction exponent with a whole number base

Watch the video *Definition of "a" to the m/n Power* to complete the following.

DEFINITION Let a be a real number, and let m and n be positive integers that share no common factors other than 1. Then,

1. $a^{1/n} =$ ________________ provided that ________________ is real number.

2. $a^{m/n} =$ ________________ provided that ________________ is a real number.

Write the expression in radical notation and simplify.

1.

2.

3.

4.

YOU TRY IT:
Simplify the following.

78. $8^{2/3} =$

79. $16^{3/4} =$

Additional Notes:

Module 8

Rewriting an algebraic expression without a negative exponent

Watch the video *Video 4: Definition of b to a Negative Exponent* to complete the following.

DEFINITION   Definition of b^{-n}

Let b be a nonzero real number and n be an integer. Then,

$$b^{-n} = \_\_\_\_\_\_\_\_ \text{ or } \_\_\_\_\_\_\_\_$$

Simplify. Write the answers with positive exponents.

1.

2.

3.

Product rule with negative exponents

YOU TRY IT: Simplify.

80. $x^{-3} \cdot x^{-5}$

81. $5c^2d^{-4} \cdot 2c^3 \cdot 6c^{-2}d^4$

Quotient rule with negative exponents: Problem type 1

We'll be using the following rules for exponents.

Quotient rule:
For any number a and any integers m and n, we have the following.

$$\frac{a^m}{a^n} = \underline{\hspace{4cm}}$$

Negative exponent rule:
For any number nonzero number a and any integer m, we have the following.

$$a^{-m} = \underline{\hspace{4cm}}$$

YOU TRY IT: Simplify.

82. $\dfrac{x^{-3}}{x^{-5}}$

83. $\dfrac{10x^4y^{-5}}{20x^{-1}y^{-2}}$

Rational exponents: Negative exponents and fractional bases

Watch the video *Video 9: Problem Recognition Exercises: Evaluating Expressions with Radical Exponents* to complete the following.

Simplify the expressions.

1.

2.

3.

4.

5.

6.

7.

8.

Rational exponents: Product rule

Watch the video *Properties of Rational Exponents* to complete the following.

PROPERTY Let a and b be nonzero real numbers. Let m and n be rational numbers such that a^m, a^n, and b^m are real numbers.

Description	**Property**	**Example**
Multiplying like bases	$a^m a^n =$ ________	$x^{2/3}x^{4/3} =$
Dividing like bases	$\frac{a^m}{a^n} =$ ________	$\frac{5^{3/4}}{5^{1/2}} =$
Power rule	$(a^m)^n =$ ________	$(p^{3/5})^{1/3} =$
Power of a Product	$(ab)^m =$ ________	$(c^{1/3}d^{1/2})^6 =$
Power of a Quotient	$\left(\frac{a}{b}\right)^m =$ ________	$\left(\frac{25}{b^2}\right)^{1/2} =$

Rational exponents: Quotient rule

If you have not completed the chart above, watch the video *Properties of Rational Exponents* to complete it.

Rational exponents: Power of a power rule

We can use the following exponent rule to simplify the expression.

Power of a power rule:

$(a^m)^n =$ ________ for any ____________ number a and any real numbers m and n

Rational exponents: Products and quotients with negative exponents

YOU TRY IT: Simplify the following.

84. $\dfrac{a^{1/4}}{a^{1/3}a^{-1/2}} =$

Rational exponents: Powers of powers with negative exponents

Watch the video *Video 6: Simplifying Expressions with Rational Exponents* to complete the following.

Simplify the expression and write the answer with positive exponents. Assume that c and d represent positive real numbers.

YOU TRY IT:
Simplify the following.

85. $(x^{1/3}y^{-6})^{5/2} =$

86. $(a^{-3}b^{7/4})^{-2/3} =$

Simplifying a radical expression with an even exponent

Watch the video *Video 3: Simplifying Radicals* to complete the following.

Simplify the expressions. Assume that x and y are positive real numbers.

1.

2.

3.

4.

5.

YOU TRY IT: Simplify the following.

87. $\sqrt{24x^8} =$

88. $\sqrt{18x^{10}} =$

Simplifying a radical expression with an odd exponent

YOU TRY IT: Simplify the following.

89. $\sqrt{20x^{11}} =$

90. $\sqrt{27x^{15}}$

Simplifying a higher root of a whole number

Simplified Radical Form

Definition:
A square root expression is in *simplified radical form* when it satisfies each of these conditions.

(1)

(2)

(3)

Definition:
More generally, a radical expression of index n is in *simplified radical form* when it satisfies each of these conditions.

(1)

(2)

(3)

Simplifying a radical expression with two variables

YOU TRY IT:
Simplify the following.

91. $\sqrt{27x^4y^5} =$

92. $\sqrt{40a^3b^{12}} =$

Additional Notes:

Additional Notes:

Module 9

Introduction to simplifying a higher radical expression

Simplified Form of a Radical

Consider any radical expression where the radicand is written as a product of prime factors. The expression is in *simplified form* if all the following conditions are met:

1. The radicand has ____________________ raised to a power greater than or equal to the _______________.

2. The radicand does not contain a ____________________.

3. There are no _______________ in the ____________________ of a fraction.

YOU TRY IT:
Simplify the following.

93. $\sqrt[4]{32x^{17}} =$

94. $\sqrt[5]{243y^{13}} =$

Simplifying a higher radical expression: Univariate

Take notes from the Learning Page.

Simplifying a higher radical expression: Multivariate

Watch the video *Video 4:Simplifying Radicals* to complete the following.

Simplify the expressions. Assume that a, b, and c are positive real numbers.

1.

2.

Introduction to square root addition or subtraction

Watch the video *Adding or Subtracting Radical Expressions 1* to complete the following.

Add or subract as indicated.

YOU TRY IT: Simplify the following.

95. $7\sqrt{5} - 4\sqrt{5} =$

96. $8\sqrt{3} + 2\sqrt{3} =$

Square root addition or subtraction

Definition of *Like* Radicals

Two radical terms are called ***like* radicals** if they have the ______________________________

and ______________________________.

Avoiding Mistakes

The process of adding like radicals with the distributive property is similar to adding

____________________.

The end result is that the _________________________ are added and the radical factor is

_______________.

$$\sqrt{3} + \sqrt{3} = 1\sqrt{3} + 1\sqrt{3} = \underline{\hspace{3cm}}$$

Be careful: True or False: $\sqrt{x} + \sqrt{y} = \sqrt{x+y}$

YOU TRY IT: Simplify the following.

97. $3\sqrt{12} + 2\sqrt{48} =$

98. $3\sqrt{40} - \sqrt{8} + 2\sqrt{50} =$

Square root addition or subtraction with three terms

Watch the video *Exercise: Finding the Perimeter of a Triangle* to complete the following.

Find the exact value of the perimeter, and then approximate the value to 1 decimal place.

Introduction to simplifying a sum or difference of radical expressions: Univariate

Take notes from the Learning Page

Simplifying a sum or difference of radical expressions: Univariate

Watch the video *Video 3: Adding and Subtracting Radicals* to complete the following.

Add or subtract as indicated.

YOU TRY IT: Simplify the following.

99. $2y\sqrt{48y^2} + \sqrt{27y^4} =$

100. $5x\sqrt{20x^2} - x^2\sqrt{80} =$

Simplifying a sum or difference of radical expressions: Multivariate

Watch the video *Adding Radicals* to complete the following.

Add.

YOU TRY IT: Simplify the following.

101. $3ab\sqrt{24a^3} + 5\sqrt{54a^5b^2} =$

Introduction to square root multiplication

Watch the video *Video 1: Introduction to the Multiplication of Radicals* to complete the following.

PROPERTY **Multiplication Property of Radicals**

Let a and b represent real numbers such that $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers.

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \underline{\hspace{4cm}}$$

Multiply. Assume that x represents a positive real number.

1.

2.

3.

Square root multiplication: Advanced

Watch the video *Multiplying Radical Expressions* to complete the following.

Multiply and simplify the result. Assume that all variables represent positive real numbers.

1.

2.

YOU TRY IT:
Simplify the following.

102. $2\sqrt{20} \cdot \sqrt{54} =$

103. $3\sqrt{24} \cdot 2\sqrt{18} =$

Simplifying a product of radical expressions: Univariate

The Multiplication Property of Radicals

Let a and b represent real numbers such that $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are both real. Then

$$\underline{\hspace{3cm}} = \underline{\hspace{4cm}}$$

Simplifying a product of radical expressions: Multivariate

Take notes from the Learning Page.

Simplifying a product of radical expressions: Multivariate, fractional expressions

Take notes from the Learning Page.

Additional Notes:

Module 10

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel, but still count toward your module completion. To prepare for your upcoming exam:

- ☐ Complete this module.
- ☐ At least two days before your focus group, take your ALEKS exam in the MALL.
- ☐ If you score less than 80% you are strongly encouraged to retake the ALEKS exam.
  - ☐ Ask for a ticket to retake from a tutor.
  - ☐ Work in the MALL for one hour.
  - ☐ Have a tutor sign that you have finished your review.
  - ☐ Retake the ALEKS portion of your exam.
- ☐ Take your written exam the day of your focus group. No retakes will be allowed on written exams.

The score on your Scheduled Knowledge Check is the number of topics that you have mastered (including prerequisite topics) out of the number of topics that you should have mastered by this point.

	Score
ALEKS Exam	
ALEKS Exam Retake	
Written Exam	

*Your recorded ALEKS exam score is the higher of your ALEKS Exam score and ALEKS Exam Retake score.

Module 11

Introduction to simplifying a product of higher roots

Watch the video *Exercise: Multiplying Radical Expressions 1* to complete the following.

Multiply the radical expressions.

Simplifying a product involving square roots using the distributive property: Basic

Watch the video *Multiplying Radical Expressions* to complete the following.

Multiply the radical expressions.

YOU TRY IT: Simplify the following.

104. $3\sqrt{5}(2\sqrt{5}+4) =$

105. $2\sqrt{6}(\sqrt{3}-\sqrt{7}) =$

Simplifying a product involving square roots using the distributive property: Advanced

Watch the video *Multiplying Two-Term Radical Expressions* to complete the following.

Multiply.

YOU TRY IT: Simplify the following.

106. $(\sqrt{3}+2\sqrt{10})(4\sqrt{3}-\sqrt{10}) =$

107. $(\sqrt{3}+\sqrt{5})(2\sqrt{2}+4) =$

Simplifying a quotient of square roots

YOU TRY IT: Simplify the following.

108. $\dfrac{\sqrt{3}}{\sqrt{5}} =$

109. $\dfrac{3}{\sqrt{6}} =$

Simplifying a quotient involving a sum or difference with a square root

Watch the video *Simplifying a Radical Expression* to complete the following.

Simplify.

YOU TRY IT: Simplify the following.

110. $\dfrac{4-\sqrt{12}}{6} =$

111. $\dfrac{\sqrt{24}+4\sqrt{3}}{8} =$

Special products of radical expressions: Conjugates and squaring

Watch the video *Video 7: Squaring a Two-Term Radical Expression* to complete the following.

$\boxed{\textit{Formula}}$ **Squaring a Binomial**

$(a+b)^2 =$ ______________________________

$(a-b)^2 =$ ______________________________

Square the radical expression. Assume that the variables represent positive real numbers.

1.

2.

Show the alternative way (other than using the formula above) to simplify $(\sqrt{z}+7)^2$

YOU TRY IT:
Simplify the following.

112. $(\sqrt{x}+\sqrt{5})(\sqrt{x}-\sqrt{5}) =$

113. $(2\sqrt{x}-\sqrt{5})^2 =$

Rationalizing a denominator: Square root of a fraction

Division Property of Radicals

Let a and b represent real numbers such that $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are both real. Then,

________________ = ________________

Rationalizing a denominator using conjugates: Integer numerator

Watch the video *Video 9: Rationalizing the Denominator (2 Terms)* to complete the following.

STRATEGY

Suppose the denominator of an expression has ____________________ in which one or both terms has a ____________________. We can ________________ the ________________ by multiplying ______________________________ by the ________________ of the ________________.

Simplify the denominator.

EXAMPLE:

Rationalize the denominator of $\dfrac{4}{3-2\sqrt{x}}$ and simplify.

$$\begin{aligned}\frac{4}{3-2\sqrt{x}} &= \frac{4}{3-2\sqrt{x}} \cdot \frac{3+2\sqrt{x}}{3+2\sqrt{x}} \\ &= \frac{12+8\sqrt{x}}{9-4x}\end{aligned}$$

YOU TRY IT:

114. Rationalize the denominator of $\dfrac{-3}{4+5\sqrt{y}}$ and simplify.

Rationalizing a denominator using conjugates: Square root in numerator

Watch the video *Video 9: Rationalizing the Denominator-2 Terms* to complete the following.

Rationalize the denominator.

YOU TRY IT: Rationalize the denominator.

115. $\dfrac{5\sqrt{2}-\sqrt{5}}{4\sqrt{2}+\sqrt{5}} =$

Introduction to solving a radical equation

Watch the video *Introduction to Radical Equations* to complete the following.

Solve the equations.

1.

2.

Avoiding Mistakes

When an equation is raised to an ____________________, it is necessary to

____ in the original equation.

YOU TRY IT:
Solve the following.

116. $\sqrt{y} = -7$

117. $\sqrt{x} = 8$

Solving a radical equation that simplifies to a linear equation: One radical, advanced

Watch the video *Procedure to Solve a Radical Equation* to complete the following.

PROCEDURE **Solving Radical Equations**

1.

2.

3.

4.

Solve the equation.

EXAMPLE:
Solve $\sqrt{y+8}+2=4$

$$\begin{aligned} \sqrt{y+8}+2 &= 4 \\ \sqrt{y+8} &= 2 \\ (\sqrt{y+8})^2 &= (2)^2 \\ y+8 &= 4 \\ y &= -4 \end{aligned}$$

Check the solution.

$$\begin{aligned} \sqrt{-4+8}+2 &\stackrel{?}{=} 4 \\ \sqrt{4}+2 &\stackrel{?}{=} 4 \\ 4 &= 4 \end{aligned}$$

$y=-4$ is a solution.

YOU TRY IT:

118. Solve $\sqrt{2x+29}+3=1$

Solving a radical equation that simplifies to a linear equation: Two radicals

Watch the video *Solving a Radical Equation Involving More than One Radical* to complete the following. This video may also be called *Exercise: Solving an Equation Containing One Radical 5*.

Solve the radical equation, if possible.

EXAMPLE:

Solve $\sqrt{6x-2} = \sqrt{2x+10}$

$$\begin{aligned} \sqrt{6x-2} &= \sqrt{2x+10} \\ (\sqrt{6x-2})^2 &= (\sqrt{2x+10})^2 \\ 6x-2 &= 2x+10 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

Check the solution.

$$\begin{aligned} \sqrt{6(3)-2} &\stackrel{?}{=} \sqrt{2(3)+10} \\ \sqrt{16} &\stackrel{?}{=} \sqrt{16} \\ 4 &= 4 \end{aligned}$$

$x = 3$ is a solution.

YOU TRY IT:

119. Solve $\sqrt{3m+4} = \sqrt{7m-16}$

Solving a radical equation that simplifies to a quadratic equation: One radical, basic

EXAMPLE: Solve for y.

$$\begin{aligned}
\sqrt{y+18}+2 &= y\\
\sqrt{y+18} &= y-2\\
(\sqrt{y+18})^2 &= (y-2)^2\\
y+18 &= y^2-4y+4\\
0 &= y^2-5y-14\\
0 &= (y-7)(y+2)\\
y &= -2,7
\end{aligned}$$

Check the solutions.

$$\begin{aligned}
\sqrt{-2+18}+2 &\stackrel{?}{=} -2 & \sqrt{7+18}+2 &\stackrel{?}{=} 7\\
\sqrt{16}+2 &\stackrel{?}{=} -2 & \sqrt{25}+2 &\stackrel{?}{=} 7\\
4+2 &\stackrel{?}{=} -2 & 5+2 &\stackrel{?}{=} 7\\
6 &\neq -2 & 7 &= 7
\end{aligned}$$

$y=7$ is a solution.

YOU TRY IT: Solve for x.

120. $\sqrt{2x+29}+3=x$

Solving a radical equation that simplifies to a quadratic equation: One radical, advanced

Watch the video *Solving a Radical Equation in which One Potential Solution Does not Check* to complete the following.

Solve the equation.

Additional Notes:

Module 12

Algebraic symbol manipulation with radicals

Watch the video *Exersice: Solving an Equation Containing One Radical 3* to complete the following.

Assume all variables represent positive real numbers.

Solve for __________:

YOU TRY IT: Solve for V.

121. $b = \sqrt{\dfrac{3V}{h}}$

Solving an equation with exponent $\frac{1}{a}$: Problem type 2

Watch the video *Video 3: Solving an Equation where the Variable Is Raised to a Rational Exponent* to complete the following.

Solve.

EXAMPLE:

Solve $(3x+5)^{\frac{1}{4}}+6=8$.

$$\begin{aligned}(3x+5)^{\frac{1}{4}}+6&=8\\(3x+5)^{\frac{1}{4}}&=2\\((3x+5)^{\frac{1}{4}})^4&=2^4\\3x+5&=16\\3x&=11\\x&=\frac{11}{3}\end{aligned}$$

YOU TRY IT:

122. Solve $(3x+1)^{\frac{1}{3}}=(7x-6)^{\frac{1}{3}}$

Using i to rewrite square roots of negative numbers

Definition of the Imaginary Number i

$$i = \underline{\hspace{3cm}}$$

Note: From the definition of i, it follows that ____________________.

Definition of $\sqrt{-b}$ for $b>0$

Let b be a positive real number. Then __________________________

YOU TRY IT: Simplify.

123. $\sqrt{-49}$

124. $\sqrt{-24}$

125. $\sqrt{-15}$

Simplifying a product and quotient involving square roots of negative numbers

Watch the video *Simplifying a Product of Imaginary Numbers* to complete the following.

Simplify the expression.

We must write the expression in terms of i first ________________ we multiply or divide.

EXAMPLE:
Simplify the following.

a) $\sqrt{-5} \cdot \sqrt{8}$

$$\begin{aligned} \sqrt{-5} \cdot \sqrt{8} &= i\sqrt{5} \cdot \sqrt{8} \\ &= i\sqrt{40} \\ &= 2i\sqrt{10} \end{aligned}$$

b) $\dfrac{\sqrt{-36}}{\sqrt{-4}}$

$$\begin{aligned} \frac{\sqrt{-36}}{\sqrt{-4}} &= \frac{6i}{2i} \\ &= 3 \end{aligned}$$

YOU TRY IT:
Simplify the following.

126. $\sqrt{-49} \cdot \sqrt{-4}$

127. $\dfrac{\sqrt{-45}}{\sqrt{9}}$

Adding or subtracting complex numbers

Watch the video *Adding and Subtracting Complex Numbers* to complete the following.

Perform the indicated operation. Write the answers in the form $a + bi$.

1.

2.

3.

EXAMPLE:
Simplify the following.

a) $(5 + 3i) + (-2 + 4i)$

$= 3 + 7i$

b) $(5 + 3i) - (-2 + 4i)$

$= 5 + 3i + 2 - 4i$
$= 7 - i$

YOU TRY IT:
Simplify the following.

128. $(-4 + 5i) - (7 - 3i)$

129. $(-4 + 5i) + (7 - 3i)$

Multiplying complex numbers

Watch the video *Video 8: Multiplying Complex Numbers* to complete the following.

Perform the indicated operation. Write your answers in the form $a + bi$.

1.

2.

3.

EXAMPLE:
Simplify $(5 + 3i) \cdot (-2 + 4i)$.

$$\begin{aligned}(5 + 3i) \cdot (-2 + 4i) &= -10 + 20i - 6i + 12i^2 \\ &= -10 + 14i - 12 \\ &= -22 + 14i\end{aligned}$$

YOU TRY IT:

130. Simplify $(-4 + 5i) \cdot (7 - 3i)$

Solving an equation of the form $x^2 = a$ using the square root property

Watch the video *Introduction to the Square Root Property* to complete the following.

Solve the equation. This video shows two ways to solve the equation. Make sure to write down BOTH ways.

PROPERTY **The Square Root Property**

For any real number k, if $x^2 = k$, then ____________________.

YOU TRY IT: Solve.

131. $x^2 = 36$

132. $x^2 = 5$

Solving a quadratic equation using the square root property: Exact answers, basic

Watch the video *Video 2: Solving Quadratic Equations Using the Square Root Property* to complete the following.

Solve the equations.

1.

2.

3.

4.

YOU TRY IT: Solve.

133. $x^2 - 40 = 0$

134. $3x^2 + 6 = 0$

Completing the square

Watch the video *Video 4: Practice completing the Square* to complete the following.

Perfect square trinomial	Factored Form
$m^2 + 2m + 1$	

Perfect square trinomial	Factored Form
	$(x-7)^2$

A trinomial of the form ____________ is a perfect square trinomial if the constant term, n, is equal to the ________________.

$$x^2 + bx + n$$

$$n = \_\_\_\_\_\_\_\_\_\_\_\_$$

Determine the value of n so that the trinomial is a perfect square trinomial. Then factor the result.

1.

2.

3.

4.

YOU TRY IT:
Complete the square.

135. $x^2 + 3x + \underline{\qquad\qquad}$

136. $\sqrt{x} = 8$

Solving a quadratic equation by completing the square: Exact answers

Watch the video *Solving a Quadratic Equation by Completing the Square and Applying the Square Root Property (2)* to complete the following.

Solve the quadratic equation by completing the square and applying the square root property.

EXAMPLE:
Solve $x^2 - 12x + 33 = 0$ by completing the square.

$$x^2 - 12x + 33 = 0$$

$$x^2 - 12x = -33 \quad \text{Add } \left(\frac{12}{2}\right)^2 \text{ to each side}$$

$$x^2 - 12x + 36 = -33 + 36$$

$$(x-6)^2 = 3$$

$$x - 6 = \pm\sqrt{3}$$

$$x = 6 \pm \sqrt{3}$$

YOU TRY IT:

137. Solve $x^2 + 2x + 5 = 0$ by completing the square.

Additional Notes:

Module 13

Solving a quadratic equation using the square root property: **Exact answers, advanced**

Watch the video *Solving Quadratic Equations Using the Square Root Property* to complete the following.

Solve the equations.

1.

2.

EXAMPLE:
Solve: $2(x+1)^2 = 16$.

$$\begin{aligned} 2(x+1)^2 &= 16 \\ (x+1)^2 &= 8 \\ x+1 &= \pm\sqrt{8} \\ x &= -1 \pm 2\sqrt{2} \end{aligned}$$

YOU TRY IT:

138. Solve: $\frac{1}{2}(x-2)^2 - 5 = 0$

Applying the quadratic formula: Exact answers

Watch the video *Introduction to the Quadratic Formula* to complete the following.

FORMULA **The Quadratic Formula**

Given a quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$, the solutions are:

$$x =$$

Solve the equation by using the Quadratic Formula.

YOU TRY IT: Solve using the Quadratic Formula.

139. $x^2 - 3x + 1 = 0$

Solving a quadratic equation with complex roots

Watch the video *Solving a Quadratic Equation by Using the Quadratic Formula* to complete the following.

Solve the equation by using the quadratic formula.

EXAMPLE:
Solve $5x^2 - 4x + 1 = 0$ using the quadratic formula.

$$5x^2 - 4x + 1 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{4 \pm \sqrt{-4}}{10}$$

$$x = \frac{4 \pm 2i}{10}$$

$$x = \frac{2}{5} \pm \frac{1}{5}i$$

YOU TRY IT:

140. Solve $3x^2 + 2x + 1 = 0$ by using the quadratic formula.

Solving a word problem using a quadratic equation with irrational roots

Watch the video *Using the Quadratic Formula in an Application Involving an Object in Free Fall* to complete the following.

A child tosses a ball straight upward with an initial velocity of 60 ft/sec. The height of the ball can be modeled as a function of time by the function,

$h(t) =$ where $h(t)$ is the height in feet and t is the time in seconds.

Determine the times at which the ball is at a height of 36 ft.

EXAMPLE:
If football is kicked straight up with an initial velocity of 128 ft/sec from a height of 5 ft, then its height, h, above the earth is a given by $h = -16t^2 + 128t + 5$. When will the football hit the ground?

We want to know when the height is 0.

$-16t^2 + 128t + 5 = 0$

Multiply each by -1.

$16t^2 - 128t - 5 = 0$

Use the quadratic formula.

$$x = \frac{128 \pm \sqrt{128^2 - 4(16)(-5)}}{2(16)}$$

$$x = \frac{128 \pm \sqrt{16704}}{32}$$

There will only be one solution because cannot have a negative time.

$$x = \frac{128 + \sqrt{16704}}{32} \approx 8.04 \text{ sec}$$

YOU TRY IT:

141. If football is kicked straight up with an initial velocity of 128 ft/sec from a height of 5 ft, then its height, h, above the earth is a given by $h = -16t^2 + 128t + 5$. When will the football be at 37 feet?

Writing a quadratic equation given the roots and the leading coefficient

We use the ______________________, which states that if k is a root of the polynomial $P(x) = 0$, then ______________________ is a factor of the polynomial $P(x)$.

EXAMPLE:
Write the quadratic equation whose roots are -2 and 3, and whose leading coefficient is 7.

-2 is a root so $x + 2$ is a factor and 3 is a root so $x - 3$ is a factor.

$$\begin{aligned} 7(x+2)(x-3) &= 0 \\ 7(x^2 - 3x + 2x - 6) &= 0 \\ 7(x^2 - x - 6) &= 0 \\ 7x^2 - 7x - 42 &= 0 \end{aligned}$$

YOU TRY IT:

142. Write the quadratic equation whose roots are 5 and -2, and whose leading coefficient is 3.

Discriminant of a quadratic equation

Using the Discriminant to Determine the Number and Type of Solutions to a Quadratic Equation

Consider the equation $ax^2 + bx + c = 0$ where a, b, and c are rational numbers and $a \neq 0$. The expression ________________ is called the ________________. Furthermore,

- If $b^2 - 4ac > 0$, then there will be ______________________.
  - a. If $b^2 - 4ac$ is a perfect square, the solutions will be ______________________.
  - b. If $b^2 - 4ac$ is not a perfect square, the solutions will be ______________________.
- If $b^2 - 4ac < 0$, then there will be ______________________.
- If $b^2 - 4ac = 0$, then there will be ______________________.

EXAMPLE:
Compute the value of the discriminant and give the number of real solutions of $3x^2 - 7x + 5 = 0$.

$$\begin{aligned} b^2 - 4ac &= (-7)^2 - 4(3)(5) \\ &= 49 - 60 \\ &= -11 \end{aligned}$$

The number of real solutions is 0 because the discriminant is negative.

YOU TRY IT:

143. Compute the value of the discriminant and give the number of real solutions of

$$-4x^2 + x + 3 = 0.$$

Roots of a product of polynomials

Take notes from the Learnig Page

Additional Notes:

Module 14

Solving an equation that can be written in quadratic form: Problem type 1

Watch the video *Solving a Higher Degree Polynomial Equation in Quadratic Form* to complete the following.

Solve.

EXAMPLE:
Solve $x^4 - 9x^2 + 8 = 0$. If we let $u = x^2$, then $u^2 = (x^2)^2 = x^4$.

$$\begin{aligned} u^2 - 9u + 8 &= 0 \\ (u-8)(u-1) &= 0 \end{aligned}$$

$$\begin{array}{rcr} u - 8 = 0 & \text{or} & u - 1 = 0 \\ u = 8 & \text{or} & u = 1 \\ x^2 = 8 & \text{or} & x^2 = 1 \\ x = 2\sqrt{2}, -2\sqrt{2} & \text{or} & x = 1, -1 \end{array}$$

The solutions are $x = 1, -1, 2\sqrt{2}, -2\sqrt{2}$.

YOU TRY IT:

144. Solve $2(x-1)^2 + 3(x-1) - 20 = 0$.

Solving an equation with positive rational exponent

When solving equations with rational exponents, there are two important facts to keep in mind.

- **Extraneous Solutions**
  Raising both sides of an equation to an ______________ could produce a nonequivalent equation that has ________________________ than the original equation. These extra solutions that do not solve the original equation are called ____________________________. Whenever we raise both sides of an equation to ________________________, we must ____________________________ to the new equation are also solutions to the original equation.
- **The even root property**
  Consider the equation $x^n = c$, where the unknown x and the constant c are real numbers, and n is ___________________.
  - If c is a negative number, ___________________.

    This is because any non-zero number raised to ________________________ must equal a ___________________.
  - If c is a positive number, ____________________________.

EXAMPLE: Solve for x.

$$(x+3)^{\frac{7}{3}} = 1$$

Raise each side to the $\frac{3}{7}$ power.

$$\left((x+3)^{\frac{7}{3}}\right)^{\frac{3}{7}} = 1^{\frac{3}{7}}$$

Simplify.

$$x+3 = 1$$

$$x = -2$$

Check the solution.

$$(-2+3)^{\frac{7}{3}} \stackrel{?}{=} 1$$

$$1^{\frac{7}{3}} \stackrel{?}{=} 1$$

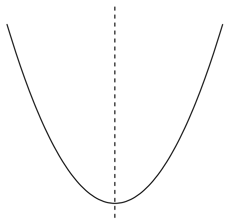
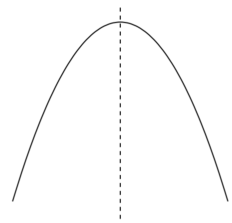
$$1 = 1$$

YOU TRY IT: Solve for m.

145. $(m-4)^{\frac{2}{3}} = 4$

Finding the vertex, intercepts, and axis of symmetry from the graph of a parabola

Here are some facts about parabolas.

Parabola opening upward	**Parabola opening downward**
The *vertex* is the ____________________ on the graph. 	The *vertex* is the ____________________ on the graph. 
The _________________________ is the line that the divides the parabola into _________________________. (This line goes through the _______________.)	

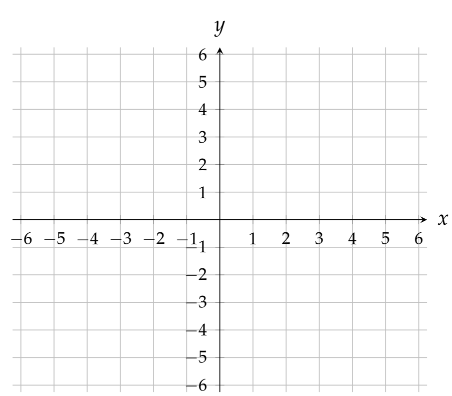
Graphing a parabola of the form $y = ax^2 + bx + c$

Watch the video *Completing the Square and Graphing a Quadratic Function* to complete the following.

Given $g(x) =$ ____________________

a. Write the function in the form $g(x) = a(x-h)^2 + k$.

b. Identify the vertex, axis of symmetry, and maximum or minimum value.



c. Determine the y-intercept.

d. Determine the x-intercept(s).

Finding the x-intercept(s) and the vertex of a parabola

Watch the video *Exercise: Find the vertex by using the vertex formula* to complete the following.

Find the vertex by using the vertex formula. $r(x) =$ ________________

x-coordinate of the vertex:

y-coordinate of the vertex:

Vertex:

EXAMPLE:
Find the vertex of $f(x) = -2x^2 - 16x - 40$ by using the vertex formula.

$$\frac{-b}{2a} = \frac{-(-16)}{2(-2)} = \frac{16}{-4} = -4$$

$$\begin{aligned} f(-4) &= -2(-4)^2 - 16(-4) - 40 \\ &= -32 + 64 - 40 = -8 \end{aligned}$$

So the vertex is $(-4, -8)$.

YOU TRY IT:

146. Find the vertex of $g(x) = 2x^2 - 4x - 9$ using the vertex formula.

Word problem involving the maximum or minimum of a quadratic function

Watch the video *An application of the Vertex Formula: Finding Maximum Height* to complete the following.

A baseball is thrown at an angle of 35° from the horizontal. The height of the ball $h(t)$ in feet can be approximated by

$h(t) =$ where t is the number of seconds after release.

a. How long will it take the ball to reach its maximum height? Round to the nearest tenth of a second.

b. Determine the maximum height. Round to the nearest foot.

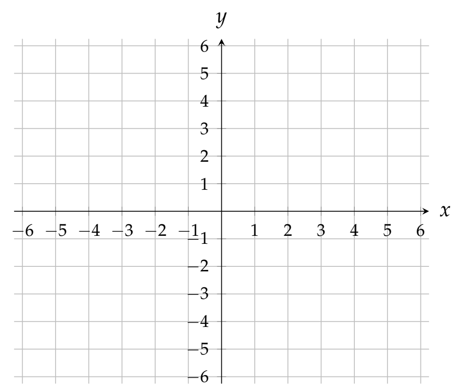
Finding the maximum or minimum of a quadratic function

Watch the video *Video 1: Completing the Square and Graphing a Quadratic Function* to complete the following.

Given $g(x) = x^2 - 6x + 5$

a. Write the function in the form $g(x) = a(x-h)^2 + k$.

b. Identify the vertex, axis of symmetry, and maximum or minimum value.



c. Determine the y-intercept.

d. Determine the x-intercept(s).

Solving a quadratic inequality written in factored form

Watch the video *Exercise: Solving Polynomial Inequalities (2)* to complete the following.

Solve the equation and related inequalities.

a. $3(4-x)(2x+1) = 0$

b. $3(4-x)(2x+1) < 0$

c. $3(4-x)(2x+1) > 0$

YOU TRY IT:

147. Solve $2(x+1)(x-3) > 0$.

Solving a quadratic inequality

Watch the video *Video 2: Solving a Quadratic Inequality Using the Test Point Method* to complete the following.

Test Point Method

Step 1: Solve the related ________ and find the boundary ________.

Step 2: Plot the ________________ on the number line.

Step 3: Test a point from each ________ to determine if the original inequality is ________________.

Step 4: Test the boundary points in the ________________ and write the solution set.

Write the solution set.

EXAMPLE:
Graph the solution to the inequality $x^2 - x < 12$.

We rewrite the inequality, then factor.

$$x^2 - x < 12$$
$$x^2 - x - 12 < 0$$
$$(x-4)(x+3) < 0$$

- We want the values of x that make $(x-4)(x+3)$ less than zero (negative).
- $(x-4)(x+3)$ is equal to zero when $x = 4$ or $x = -3$.

$$\begin{array}{ccccc} & 0 & & 0 & \\ \text{———————} & | & \text{———————} & | & \text{———————} \\ & -3 & & 4 & \end{array}$$

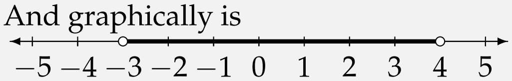
We will test a point in each interval on the number line above.

- For $x = -4$, we have $(-)(-) = +$
- For $x = 0$, we have $(-)(+) = -$
- For $x = 5$, we have $(+)(+) = +$

Note that we do not need the VALUE, just whether it will be positive or negative.

$$\begin{array}{ccccc} +++++++++ & 0 & ------- & 0 & +++++++++ \\ \text{———————} & | & \text{———————} & | & \text{———————} \\ & -3 & & 4 & \end{array}$$

The solution in interval notation is $(-3, 4)$. And graphically is

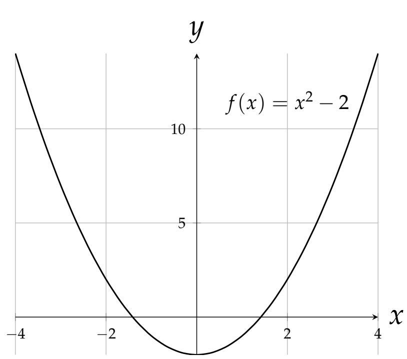


YOU TRY IT:

148. Graph the solution to the inequality

$$2x^2 - 9x \geq 5.$$

An alternative method to the one shown on the previous page is to graph the parabola and determine the answer from the graph. Solve $x^2 - 2 \geq 0$



We can find the x-intercepts of the graph $(\sqrt{2}, 0)$ and $(-\sqrt{2},, 0)$. We want the x values where the graph lies on or above the x-axis.

The solution is $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$.

Solving a rational inequality: Problem type 1

Watch the video *Video 7: Solving a Rational Inequality Using the Test Point Method* to complete the following.

Write the solution set.

EXAMPLE:
Solve $\dfrac{(x+1)(4-x)}{x-2} \leq 0$.

- Determine where $\dfrac{(x+1)(4-x)}{x-2} = 0$.
  This happens when $x = -1$ and $x = 4$.
- $\dfrac{(x+1)(4-x)}{x-2}$ is undefined when $x = 2$.

Test a value in the four intervals.

- Let $x = -2$.
  $\dfrac{(\text{neg})(\text{pos})}{(\text{pos})} = \text{pos}$
- Let $x = 0$.
  $\dfrac{(\text{pos})(\text{pos})}{(\text{neg})} = \text{neg}$
- Let $x = 3$.
  $\dfrac{(\text{pos})(\text{pos})}{(\text{pos})} = \text{pos}$
- Let $x = 5$.
  $\dfrac{(\text{pos})(\text{neg})}{(\text{pos})} = \text{neg}$

$+ + + + + + + 0 - - - - - - U + + + + + + + + + + 0 - - - - - -$

$-1 \qquad 2 \qquad 4$

The solution is $[-1, 2) \cup [4, \infty)$.

YOU TRY IT:

149. Solve $\dfrac{x-1}{(2x-5)(x+2)} \geq 0$.

Solving a rational inequality: Problem type 2

Watch the video *Video 9: Solving a Rational Inequality Using the Test Point Method* to complete the following.

Write the solution set.

Additional Notes:

Module 15

Module 15 contains all of the topics from Modules 1-14. This is to help you review for your upcoming final exam. If you have already mastered these topics, you will not see them in your carousel.

Additional Notes:

Solutions

Module 1

1. $-6x^2 + 19x - 15$

2. $(x-9)(x-3)$

3. $-5(y-3)(2y-1)$

4. $(5x-1)(x+2)$

5. $(2x+3)(x-5)$

6. $(4x-7)(4x+7)$

7. Not factorable.

8. $x = -5, \frac{3}{2}$

9. $x = 0, 5$

10. $x = 3, -7$

11. $x = -\frac{3}{4}, 1$

12. $x = -3, -2$

13. 6 in X 13 in

14. 2

15. 2

16. $\frac{7}{4}$

17. Undefined

18. Undefined

Module 2

19. $\frac{4(x-7)}{9(x+7)}$

20. 2

21. $\frac{x+2}{2-x}$

22. -2

23. $\frac{x+1}{3}$

24. $\frac{3}{7ab}$

25. $-\frac{15}{2}$

26. $-\frac{15}{2}$

27. $\frac{7x}{6}$

28. $\frac{x+4}{x-4}$

29. $4(t+3)$

30. $60x^2y^3z$

Module 3

31. $(x+2)(x-3)(x+7)$

32. $2 \cdot (3x - 8) \cdot 3x = 6x(3x-8)$

33. $-8x^5$

34. $2x(x-1)(2x+3)$

35. $\frac{-2x}{x-6}$

36. $\frac{1}{x-5}$

37. $\frac{10+21b^3}{12b^4}$

38. $\frac{x-11}{(x-1)(x+4)}$

39. $\frac{2x+1}{x-1}$

40. $\frac{3x-1}{x+4}$

Module 4

41. $\frac{15}{2}$

42. $\frac{x-5}{5x^2}$

43. $\frac{6b^3}{acd^2}$

44. $28a$

45. $\frac{1}{x+1}$

46. -8

47. $\frac{2x+16}{3x+4}$

Module 6

48. $x = \frac{1}{3}$

49. $x = -13$

50. $y = \frac{-1}{10}$

51. $y = 2$

52. $y = 2$

53. No solution

54. $y = -7, -4$

55. $y = 4$

Module 7

56. 7

57. -10

58. Not a real number

59. 5

60. -9

61. x^6

62. a^7

63. $7y^4$

64. $4x^3$

65. 5

66. -2

67. -6

68. Not a real number

69. -5

70. $3x^5$

71. $2x^2$

72. $x^{2/5}$

73. $y^{3/2}$

74. $\sqrt[3]{x^5}$

75. $\sqrt[7]{y^2}$

76. 2

77. 2

78. 4

79. 8

Module 8

80. $\frac{1}{x^8}$

81. $60c^3$

82. x^2

83. $\frac{x^5}{2y^3}$

84. $a^{5/12}$

85. $\frac{x^{5/6}}{y^{15}}$

86. $\frac{a^2}{b^{7/6}}$

87. $2x^4\sqrt{6}$

88. $3x^5\sqrt{2}$

89. $2x^5\sqrt{5x}$

90. $3x^7\sqrt{3x}$

91. $3x^2y^2\sqrt{3y}$

92. $2ab^6\sqrt{10a}$

Module 9

93. $2x^4\sqrt[4]{2x}$

94. $3y^2\sqrt[5]{y^3}$

95. $3\sqrt{5}$

96. $10\sqrt{3}$

97. $14\sqrt{3}$

98. $6\sqrt{10}+8\sqrt{2}$

99. $11y^2\sqrt{3}$

100. $6x^2\sqrt{5}$

101. $21a^2b\sqrt{6a}$

102. $12\sqrt{30}$

103. $72\sqrt{3}$

Module 11

104. $30+12\sqrt{5}$

105. $6\sqrt{2}-2\sqrt{42}$

106. $-8+7\sqrt{30}$

107. $2\sqrt{6}+4\sqrt{3}+2\sqrt{10}+4\sqrt{5}$

108. $\frac{\sqrt{15}}{5}$

109. $\frac{\sqrt{6}}{2}$

110. $\frac{2-\sqrt{3}}{3}$

111. $\frac{\sqrt{6}+2\sqrt{3}}{4}$

112. $x-5$

113. $4x-4\sqrt{5x}+5$

114. $\frac{-12+15\sqrt{y}}{16-25y}$

115. $\frac{5-\sqrt{10}}{3}$

116. No solution

117. 64

118. No Solution

119. $m=5$

120. $x=10$

Module 12

121. $V=\frac{b^2h}{3}$

122. $x=\frac{7}{4}$

123. $7i$

124. $2i\sqrt{6}$

125. $i\sqrt{15}$

126. -14

127. $i\sqrt{5}$

128. $-11+8i$

129. $3+2i$

130. $-13+47i$

131. $x=\pm 6$

132. $x=\pm\sqrt{5}$

133. $x=\pm 2\sqrt{10}$

134. $\pm i\sqrt{2}$

135. $\frac{9}{4}$

136. 64

137. $x=-1\pm 2i$

Module 13

138. $x=2\pm\sqrt{10}$

139. $\frac{3\pm\sqrt{5}}{2}$

140. $x=-\frac{1}{3}\pm i\frac{\sqrt{2}}{3}$

141. $x=4\pm\sqrt{14}$ sec
$x\approx .26$ sec, 7.74 sec

142. $3x^2-9x-30=0$

143. Discriminant: 49
Number of real solutions: 2

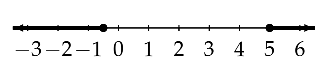
Module 14

144. $x=\frac{7}{2},-3$

145. $x=-4,12$

146. $(1,-11)$

147. $(-\infty,-1)\cup(3,\infty)$

148. 

149. $(-2,1]\cup(\frac{5}{2},\infty)$