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Winning Strategy for Dice Game Farkle

Karlee Westrem
University of North Dakota

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Introduction

The dice game Farkle uses 6 six-sided dice and the objective of the game is to be the first person to score 10,000 points. The dice game involves rolling against the odds and possibly lose your running total points. Generating functions are used to determine the probability of various roll patterns and their expected values of points earned given the number of dice rolled. We are assuming the maximum Farkle score is attained for each roll for computations.

Farkle Rules

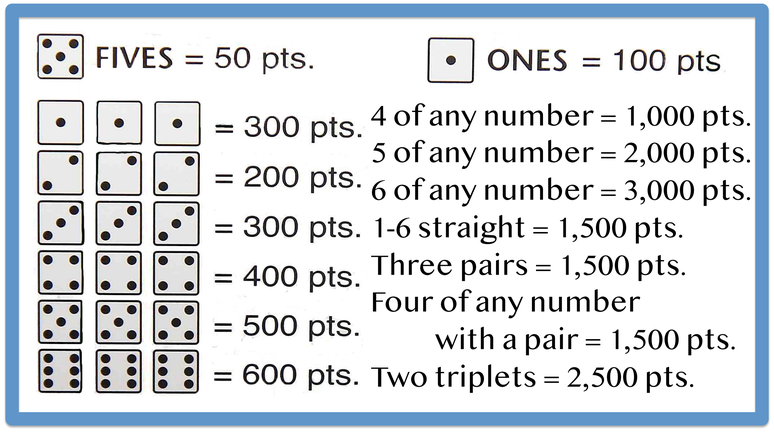
Farkle Facts

- Only single 1's and 5's are worth points.
- Other numbers are worth points only if you get three or more of the same number in a single roll.
- Other combinations of numbers are worth points if you get them in a single roll.
- Dice from multiple rolls cannot be added together.

Play

- When it's your turn, roll all six dice.
- After each roll, set aside dice that are worth points and roll the remaining dice. You must remove at least one die after each roll and keep a running total of the points for that turn.
- If you're lucky enough to set aside all 6 dice, you can roll them all again to continue to build your running total.
- If you cannot set aside any dice after a roll, that's a **Farkle.** You lose your running total of points for that turn and play passes to the left.
- A **Farkle** can happen on your first roll or when you roll the remaining dice.
- To score for the first time, you must have a running total of 500 points before you can stop rolling.
- After your first score of 500 points or more is recorded, you may stop rolling at any time and have your running total added to your accumulated score. Once your points are entered on the score sheet, they are safe, and you cannot lose them.

Below is the Farkle scoring rules.



Techniques

Generating functions are used to solve counting problems. Generating functions are related to the Multinomial Theorem. Given k variables denoted $x_1, x_2, \ldots, x_k$, and using n k-sided dice we have the generating function,

$$G_n(x_1, x_2, \ldots, x_k) = (x_1 + x_2 + \cdots + x_k)^n$$

$$= \underbrace{(x_1 + x_2 + \cdots + x_k)(x_1 + x_2 + \cdots + x_k)\cdots(x_1 + x_2 + \cdots + x_k)}_{\text{n times}}$$

$$= \sum \binom{n}{n_1, n_2, \ldots, n_k} {x_1}^{n_1} {x_2}^{n_2} \cdots {x_k}^{n_k}$$

where $n_1 + n_2 + \ldots + n_k = n \quad \forall i \;\; n_i \geq 0.$

We expand the polynomial to attain the coefficients in front of each term, which tells us the frequency for each roll pattern. The generating function for the number of possible patterns rolling k six-sided dice is,

$$G_k(x_1, x_2, \ldots, x_6) = (x_1 + x_2 + \ldots + x_6)^k$$

where the subscripts denote each side of the die and k is a fixed value. We can build frequency tables to determine the probability for each combination. Rolling 1 six-sided die gives us the frequency table,

Outcome	Frequency	Probability	Score
1	1	1/6	100
5	1	1/6	50
x	4	4/6	0

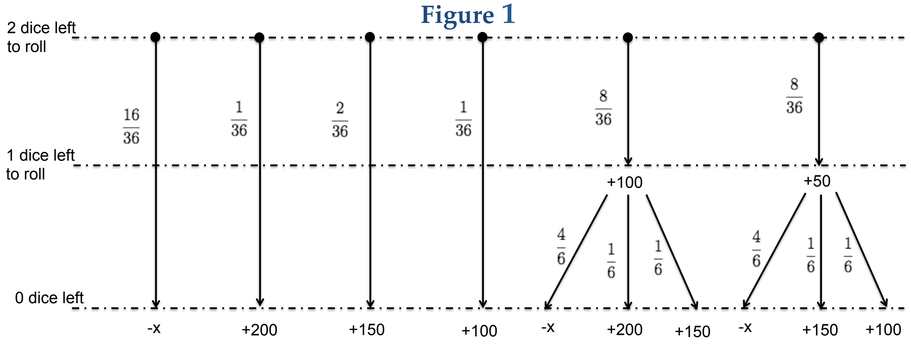
where x denotes rolling a non-scoring single number such as 2, 3, 4, or 6. We can construct outcome tables based on scoring dice rolled.

Score	Frequency	Dice left
100	1	0
50	1	0
Farkle	4	---

Scoring Dice	Frequency	Approx. Likelihood
1	2	33%
0	4	67%

From the outcome tables we are able to map out all possible paths that can be taken during one person's turn. The diagram below describes all possible paths with 2 remaining dice assuming that the individual keeps rolling until they roll a Farkle or score all dice.

Figure 1



Conclusions

Based on the outcome tables, we can determine the approximated percentage of rolling a Farkle for rolling 1 die to 6 dice.

Number of dice being rolled	Approximated Probability
1	67%
2	44%
3	28%
4	16%
5	8%
6	2%

Expected Value

The expected value is the predicted score you are expected to have after a roll. It's the sum of the values multiplied by it's corresponding probability. If you are below the expected value points benchmark, it would be your best interest to continue rolling to accumulate more points. If you have a running total that's greater than the expected value, it's recommended to secure your running total points, except in the case of your first turn and last turn of the game.

Number of dice	Approximated Expected Value
1	37.5
2	112.5
3	300.83
4	843.14
5	2694.58
6	16729.17

Acknowledgements

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